Repairable systems
Repairable system

• A reparable system is obtained by gluing individual non-repairable systems each around a single failure

• To describe this gluing process we need to review the concept of stochastic process
Stochastic process

- Stochastic process is a collection of random variables $X_t: X(t, \omega)$
- It is defined by its
  - **State Space** – the range of possible values of $X_t$, $\omega$
  - The **index set** of the state space, $t$
  - The **dependence structure** among random variables $X_t$ that make up the whole stochastic process: how to transform $X(t, \ )$ to $X(t+h, \ )$
Stochastic process
Failure occurrences

- For each index \( i \), we have a new random variable \( T_i \) representing the \textbf{Time to the \( i \)-th Failure}
- Each \( T_i \) has its own
  - Probability density function \( f_i \),
  - Cumulative distribution function \( F_i \) and
  - Hazard rate \( h_i \)
Failure occurrences

- We introduce a new random variable which is the **Time Between Failures**
  
  \[ X_1 = T_1 \text{ and } X_i = T_i - T_{i-1} \]

- Note: giving Time of Failure you can easily derive the Time Between Failures and vice versa
Failure occurrences

• Each Time Between Failures $X_i$ (or $T_i$) is a random variable for each state of the process

• These variables may be
  • Dependent or independent
  • Identically distributed or not identically distributed
Independent random variables

- Two random variables $X$ and $Y$ say, are said to be **independent** if and only if the value of $X$ has no influence on the value of $Y$ and vice versa.
Identically distributed random variables

• Two random variables $X$ and $Y$ are said **identically distributed** if they have the same cumulative distribution function $F$ (or density function).
Repairable systems

• The assumption of independent and identically distributed times between failures is usually invalid for software repairable systems

• Why?
Repairable systems

• In classical **hardware theory**, we simply **replace** failed components with **identical working** new ones
  
  • We might have that **all density functions** to be **identical** and their cumulative distribution function as well
• Generally speaking: substituting a part of a car does not change the car performance (density function) because a mechanic cannot intervene on its design!

• In few cases, a mechanic may also replace a failed component with one of better quality though
Repairable systems

• Once a software fault is completely removed it will not cause the same failure again, but ...

• Dependency: Removing faults may cause new failures: the variable *Times Between Failures* $X_i$ may be dependent
Repairable systems

- Once a software fault is removed it will never cause the same failure again, but ...
  - Removing faults may cause **new failures**: $T_i$ may be dependent
  - By fixing a failure we may also **improve the design** to minimize the likelihood of recurrence of the faults that have caused the failure
  - Software reliability can be also improved by **testing** whereas for hardware one has to use better material, improved design, and increased strength etc.
Repairable systems

• Altogether, we expect that the probability density function of \( X_i \) would be **different** from the one of \( X_{i-1} \)
  • For example, by improving design \( E[X_{i-1}] \) tends to be less than the one of \( E[X_i] \)
Minimal/Perfect repair

- **Minimal repair (as bad as old):** the repair done on a system leaves the system in exactly the same condition as it was just before the failure.

- **Perfect repair (as good as new):** the system is brought to a new state after the repair.
• If every repair is a **perfect repair** then times between failures are independent and identically distributed
Examples

- Perfect repair: replacement of a failed system to a brand new one
- Minimal repair: changing a flat tire on a car
Reliability growth

- In software we can improve design and implementation having a **lower probability** for the remaining failures after a repair (sometimes even for hardware we can change a component with a better one)
- The series of PDF can change!
- **Reliability can grow!**
Reliability Growth

- Reliability Growth means:
  - The PdF of $T_{i+1}$ is different from the PdF of $T_i$
  - $E[T_{i+1}] \geq E[T_i]$

- Expected Failure times tend to increase!

- Reliability Growth is not normally considered in hardware reliability
  - It is a goal of software maintenance!
Software systems are updated many times during their life cycle.

Each revision introduces some side effects which result in a local increase of failures.

Reliability Growth models are good for one revision period rather than the whole life cycle.

Some research glues different models over different revisions.