UNIVERSITÀ DEGLI STUDI DI MILANO BICOCCA
DIPARTIMENTO DI INFORMATICA, SISTEMISTICA E COMUNICAZIONE

Abstraction and Meta-knowledge: where does knowledge come from?

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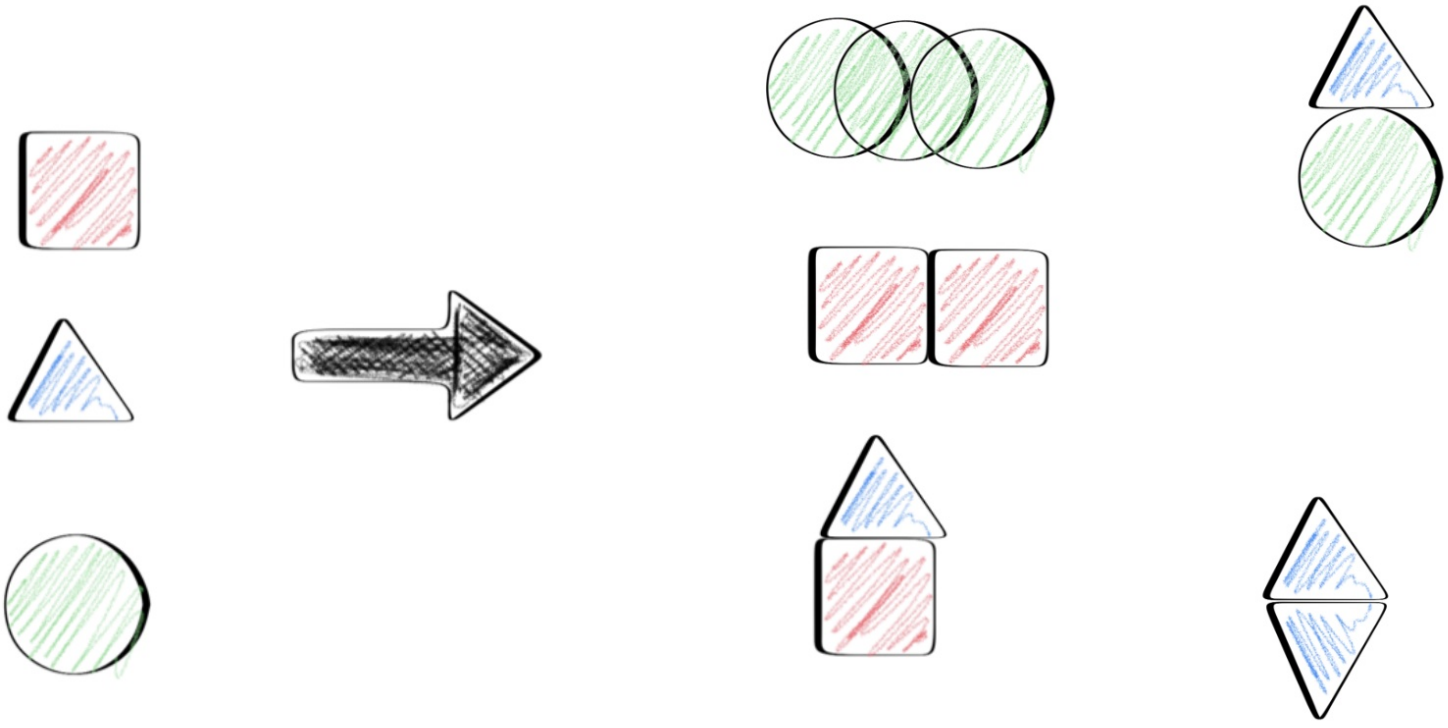
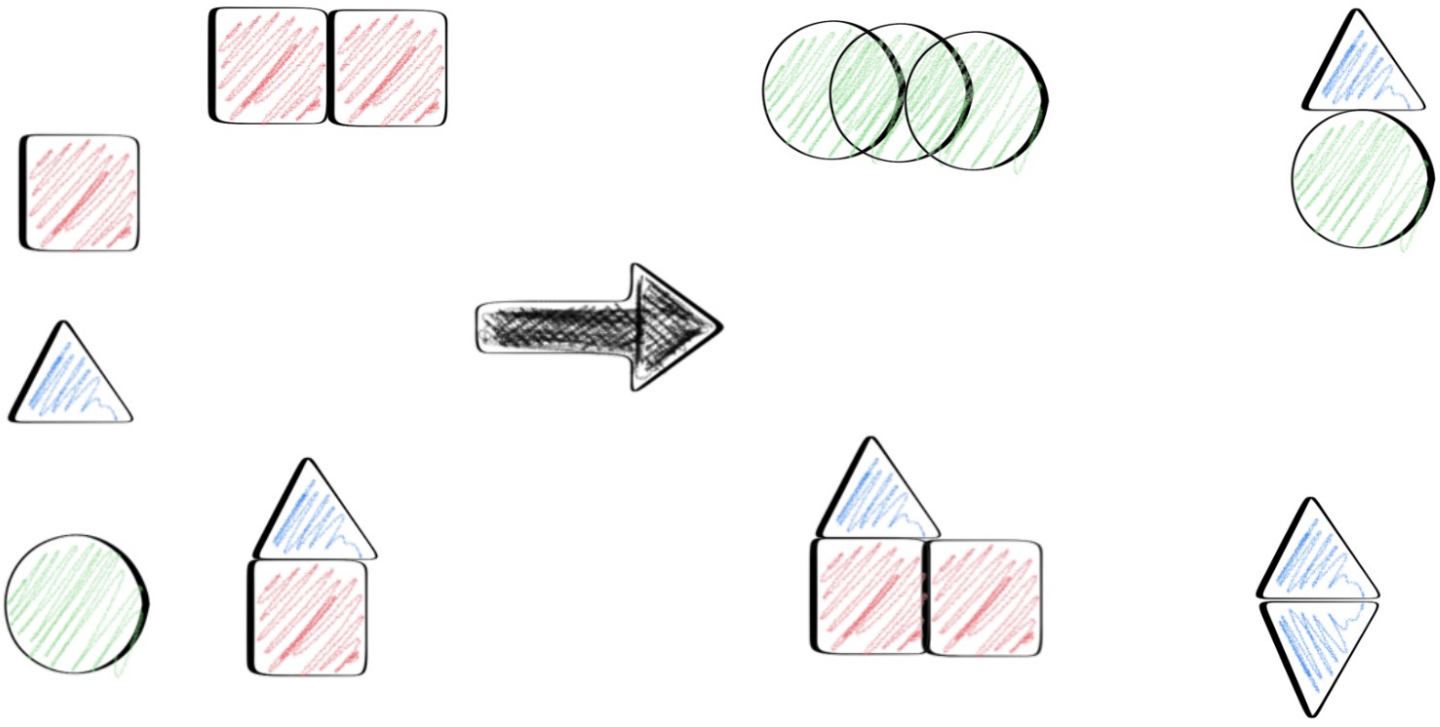
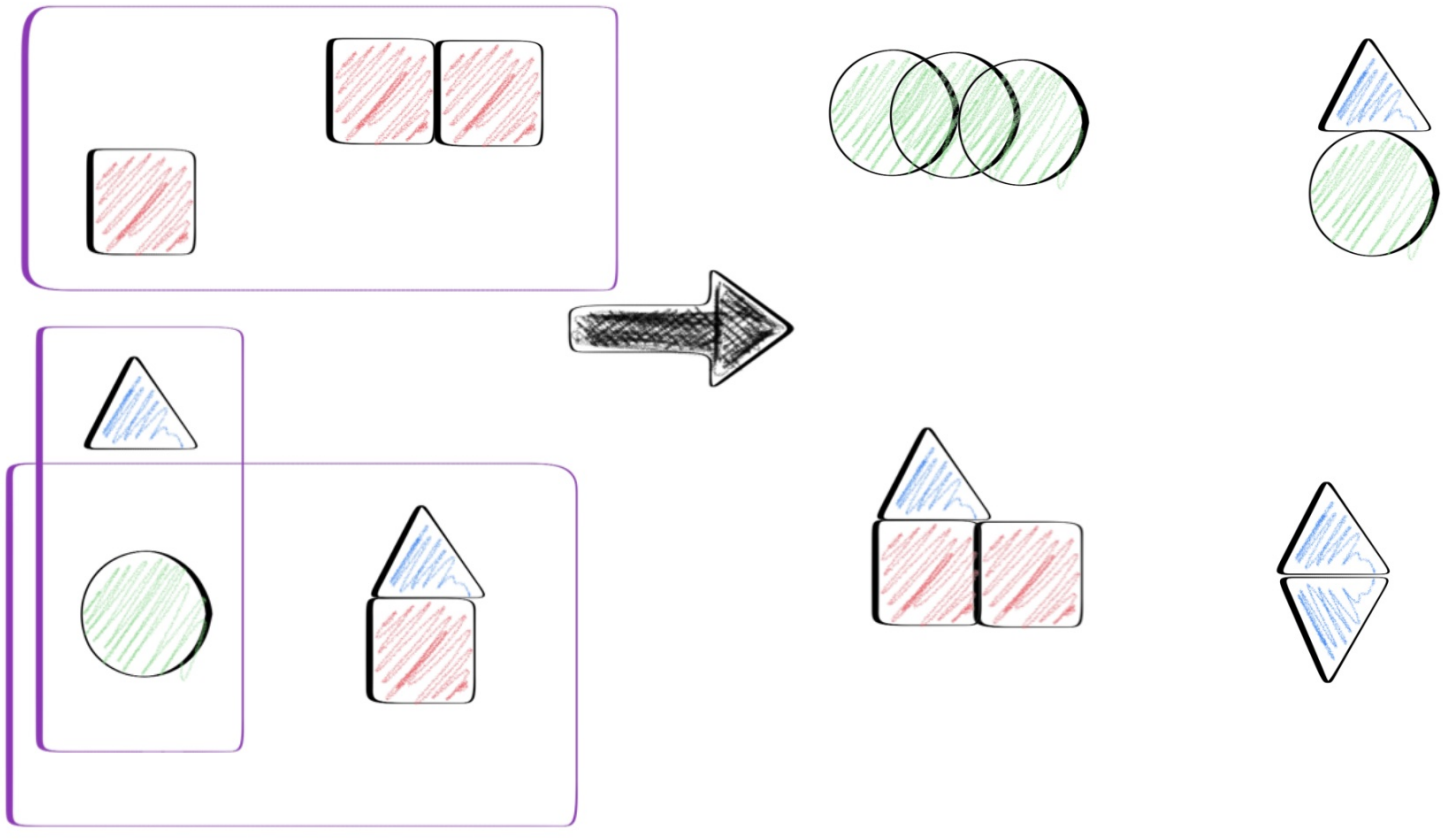
Knowledge



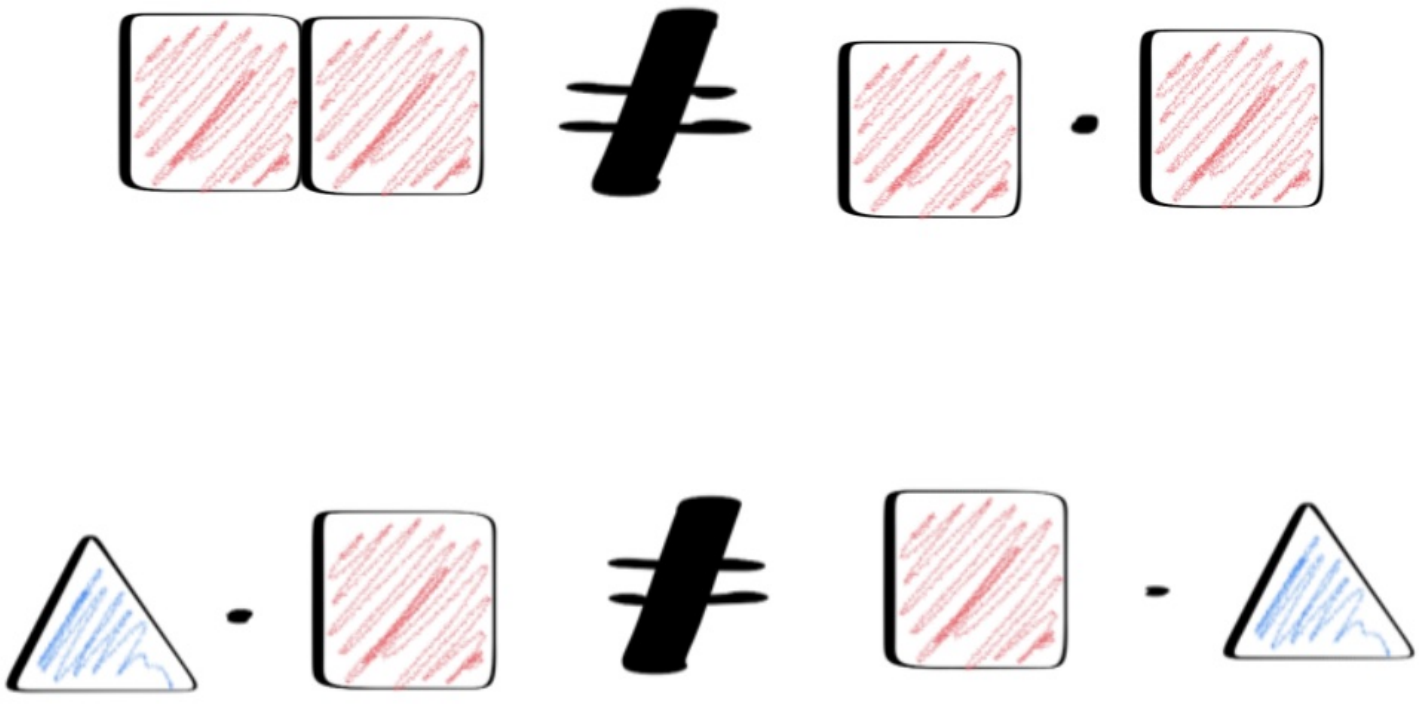
Domain Knowledge



Sources



Weights and Order



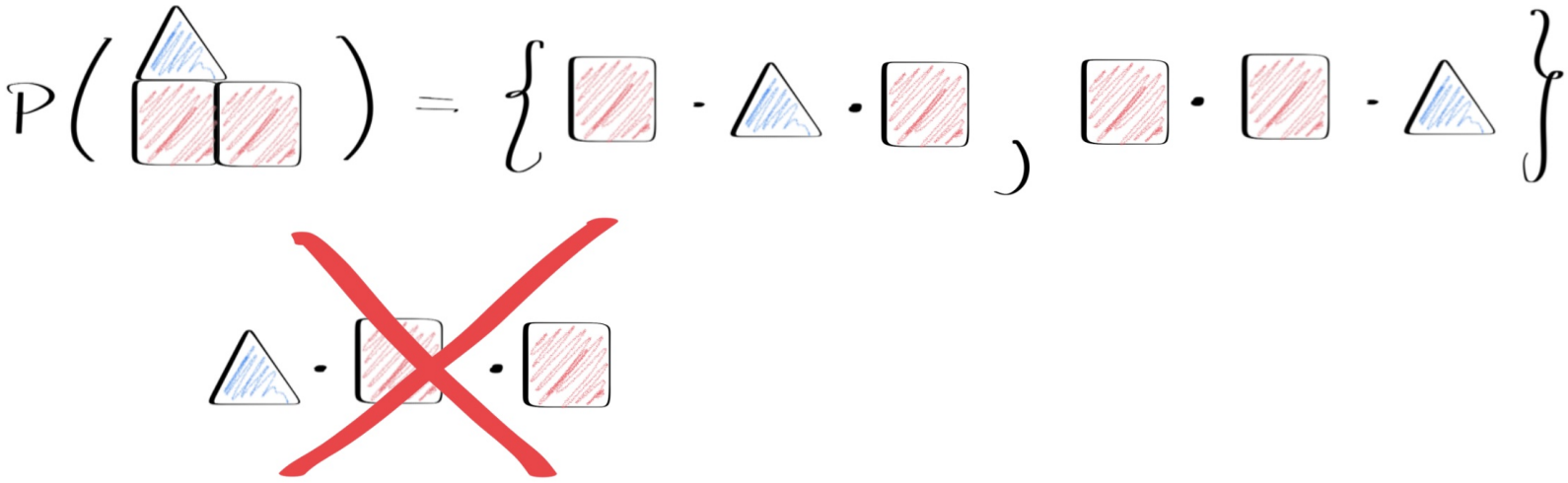
Commonalities

- many different ways to derive a consequence
- compute over all

- order of the derivations irrelevant
- allow for lemmas

- each derivation counts once (?)

Provenance



Provenance Semirings

$\mathcal{S} = (S, \oplus, \otimes, \mathbf{0}, \mathbf{1})$:

- S: carrier set
- $\oplus$ associative, commutative, (idempotent), neutral $\mathbf{0}$
- $\otimes$ associative with neutral $\mathbf{1}$; annihilating $\mathbf{0}$
- $s \otimes (t \oplus u) = (s \otimes t) \oplus (s \otimes u)$

Weakest semiring: languages

$$(2^{\Sigma^*}, \cup, \cdot, \emptyset, \{\varepsilon\})$$

Abstract Provenance

Provenance of a consequence is a (regular) language
each word describes one way to derive it

Instantiating $\cdot$ with $\otimes$ and $\cup$ with $\oplus$ solves it
for any arbitrary semiring

BUT this can be infinite in general

Special Cases

- $\otimes$ commutative: polynomials with coefficient 1
- $+$ $\otimes$ idempotent: sums of products of variables
- $+$ absorptive: incomparable sets of variables

- finitely generating
- ordered
- ...

Computational Issues

What does it mean to have a derivation?

When can we compute it?

- which theories?
- which semirings?

How expensive is it?

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