Counterparts in Language and Space

Similarity and $\mathcal{S}$-Connection

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Abstract We aim to combine the semantics of spatial natural language specified as a linguistically motivated ontology, the Generalized Upper Model, with spatial logics or ontologies that specify space according to certain conceptualisations, based on regions, shapes, orientations, distances, or object properties.

Such combinations, however, introduce uncertainties of various kinds, caused by different levels of detail in the definition of one of the spatial ontologies, under-specifications within parts of an ontology, or different viewpoints of the topics the ontologies address.

To model these problems formally, we extend the combination technique of $\mathcal{E}$-connections by adding (heterogeneous) similarity measures. Local similarity compares objects within one domain, whilst comparing objects across domains leads to similarity measures that are motivated by and based on counterpart-theoretic semantics. The new formalism is called $\mathcal{S}$-connection.

Keywords. Ontology, Natural Language Semantics, Similarity, Counterparts, $\mathcal{E}$-Connection, $\mathcal{S}$-Connection

1. Introduction

"Tesco is the second building from there", "Take the left where the trees are on the corner", "Boots is past Plymouth university on the right hand side", "I'm going down 50 meters past the pine forest towards the wheat fields"—natural language describes spatial situations in a flexible way: within one description, it changes fluently in terms of granularity, combines different modes of spatial relationships, gives as much information as necessary needed for a specific purpose, refers to situation-dependent knowledge given by the dialogue discourse, or specifies attributes of spatial entities [1, 2, 3]. Spatial logics, in contrast, specify axiomatically only select aspects of the environment, but they do this with a relatively high degree of precision concerning those aspects. Spatial qualitative calculi as one group of spatial logics, for instance, differ in terms of the spatial entities and kinds of relationships they describe, as well as reasoning support. Specifications within a calculus may correspond to aspects about regions, orientations, shapes, distances, movements, topology, or metric spaces [4, 5].

Both, linguistic and logical formalisations of space, however, are applied at different levels within spatially aware information systems interfaced with a natural language

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dialogue system [6]. Hence, relations between both these representations, linguistic and logical, that provide descriptions of the environment from different viewpoints, have to be aligned and integrated with each other.

In this paper, we provide a method that formally connects both viewpoints on the basis of $\mathcal{E}$-connections. While giving examples of how natural language, specified in a linguistically motivated ontology, is related to different spatial logics, we will elucidate the impact of uncertainties and similarities influencing this relationship. Connections of these viewpoints are strongly influenced by external factors, and so the relationship between instances in different domains can only be determined to a certain degree. A framework that supports a formalisation of such relationships is given, enriching the technique of $\mathcal{E}$-connections with (heterogeneous) similarity measures. These so-called $\mathcal{S}$-connections are motivated by and based on counterpart-theoretic semantics.

2. Linguistic Spatial Semantics

Language has a broad but structured range of ways for relating entities of different kinds to each other, both semantically and syntactically [7], and can therefore be partly specified as a formal theory or ontology. A linguistic categorisation particularly for spatial descriptions has been developed in the Generalized Upper Model (GUM) [8], which has been successfully applied in a natural language system [6] and which is evaluated against linguistic corpora with more than 600 entries for English and German. Its structure is governed by results from linguistic evidence, empirical research, and grammatical indications: it classifies language into groups of categories and relations according to their semantics. Hence, GUM is strictly based on the requirement that the distinctions that should be covered are those that are derived from linguistic evidence. This implies that GUM captures precisely those aspects given by the semantics, but not by the pragmatic principles and distinctions associated with particular lexicogrammatical items and structures.[2]

GUM's spatial categorisation is not based on groups of prepositions, but on the way language characterises spatial relationships either grammatically or inherently.[3] Natural language utterances about spatial contexts are specified accordingly as instances in GUM. Those distinction not covered by the linguistic structure are therefore not represented in GUM. Talmy [1] points out that language schematises spatial information only into *underspecified* qualitative concepts. These concepts then need to be adapted and interpreted with respect to specific spatial situations. This *underspecification* renders the connection between linguistic descriptions and formal spatial theories with uncertainty.

Given the ontological structure of GUM, the most expressive categorisation of linguistic aspects are those describing dynamic or static spatial configurations and, in particular, different kinds of spatial relationships [8]. In fact, different modes of spatial relationships give the strongest indication about relative positions or motions of spatial entities and their attributes [9]. These relationships, however, can only be seen in the context of the linguistic entities participating in the relationship. Lexical terms, however, are less

[2] A detailed overview of GUM would go beyond the scope of this paper; see [8] for details.

[3] Although GUM is based on the semantics of English and German, it is rooted in a language-based approach to cognition across different languages [7]. Language-dependent differences in spatial semantics should therefore result in refinements or extensions of GUM.

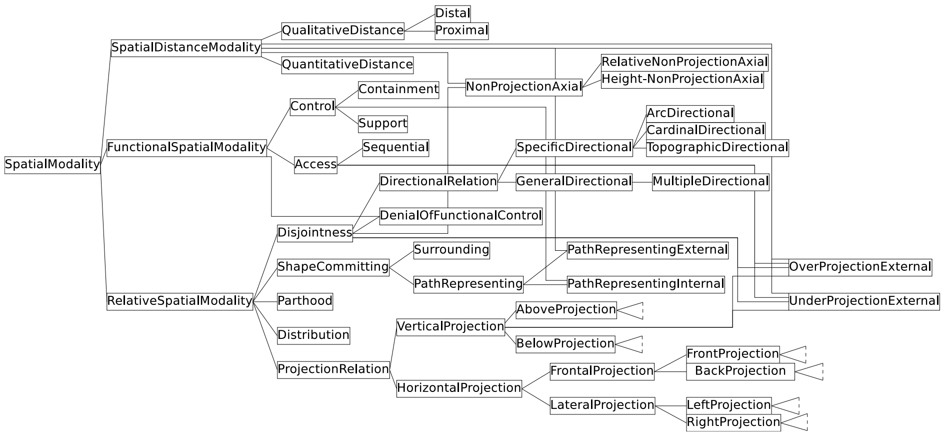


Figure 1. Spatial modalities represent modes of spatial relationships between entities. (ProjectionRelation leafs are further distinguished into internal and external projections, subsumed by Parthood or Disjointness and SpatialDistanceModality respectively.)

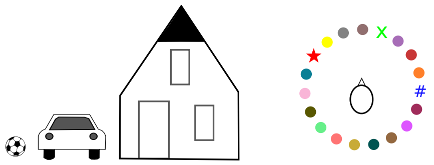
indicating the meaning of a spatial linguistic description, as they can be conceptualised in many ways according to the spatial relationship in use (cf. [10] on the meaning of "place"). We will therefore focus on these relationships.

GUM[4], as a formal theory, is specified in first-order logic. However, large parts of it can also be expressed in description logics such as $\mathcal{SROIQ}$ [11] (underlying the Web Ontology Language OWL 2.0). Its signature contains *categories* (unary predicates) and *relations* (binary predicates). The spatial extension of GUM introduces all categories and relations necessary for specifying utterances of spatial descriptions. Different kinds of spatial relationships are specified by the category SpatialModality. This category consists of several subtypes, which are defined by their use in natural language and possible entities they relate to. Related objects are then specified by the relations locatum in static and actor in dynamic spatial descriptions and the relatum [12], i.e. the locatum/actor has a certain spatial position with respect to the relatum (corresponding to *figure* and *ground* in [1] or *trajectory* and *landmark* in [13]).

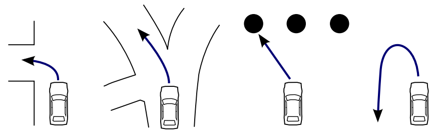
All spatial descriptions indicate the type of relationship being described, typically expressed by a spatial preposition, an adverb, an adjective, parts or implications of the verb, that defines a specific SpatialModality. The most general distinction between spatial modalities is made by distance-, functional-, and property-dependent positions between entities. There are, however, intersections between these three general categories. An overview of GUM's spatial modalities is shown in Fig. 1.

The structure of these spatial modalities are given precisely but solely on the basis of linguistic evidence. Further distinctions made by spatial logics then have to be derived by situation-dependent, context-sensitive, or world knowledge, i.e. external factors. Possible *realisations* of specific linguistic descriptions in models of a spatial logic can therefore only be defined by elements that satisfy a certain similarity.

[4] `http://www.ontospace.uni-bremen.de/ontology/GUM-3-space.owl`



2.1: Ambiguities among static left projections



2.2: Ambiguities among dynamic left projections

Figure 2. Ambiguities for GUM's LeftProjection spatial modality

3. Connections between Spatial Language and Logics

The following examples illustrate the way linguistic descriptions tend to underspecify their possible spatial realisations. As a consequence, these descriptions can be related to different models of spatial logics.

ProjectionRelations in GUM define directional relationships between entities. They represent relationships between entities based on orientations. One of its subclasses, LeftProjection, defines spatial relations as used in the examples "Three steps to the left", "Turn to the left", "It is to the left of you", or "In the left part"; it denotes:

1. static locations, on/in the left side or half-plane of the relatum,
2. static locations with respect to the orientation of the relatum,
3. re-orientations towards the direction or an angle to the left,
4. (re-)directions of motions, to the left side of the moving entity, or
5. combinations of movements and re-orientations to the left of the moving entity or an external left [8].

Although the linguistic surface can reduce the range of realisations, not all possible distinctions are made. As GUM's specification of spatial language has been designed to cover all possible meanings in a flexible (linguistic) way, interpretations of specific utterances have to be determined in spatial situations by external (non-linguistic) factors [14]. Possible realisations of ProjectionRelations might therefore be defined in spatial logics that specify orientations, such as [15, 16, 17]. However, which concrete model corresponds to the linguistic description and vice versa depends on external aspects. Whether one or more connections between language and space are necessary, and to what degree they hold, has to be determined based on indications from these external aspects.

Fig. 2 illustrates spatial situations, in which LeftProjection can be used to describe relationships between entities. In the left part of Fig. 2.1, for instance, LeftProjection is defined in "The ball is *to the left of* the car and the car is *to the left of* the house". From the perspective of someone sitting inside the car, however, "The house is *to the left of* the car" is also acceptable without falsifying the previous example. Hence, LeftProjection has to be interpreted according to the spatial perspective. Furthermore, "The house is *to the left of* the ball" from the perspective of the car might be less acceptable depending on the Figure vs. Ground phenomenon [12], i.e. contextual aspects influence the interpretations of "left" as well.

Although a geometric relation according to a 90 degree angle or half-plane could be a logical definition for LeftProjection in this example, 'left' can be used to reflect further realisations. In the right part of Fig. 2.1, multiple objects are arranged as a circle. Here, one entity is to the left of the other. Various possibilities for "Drive to the left"

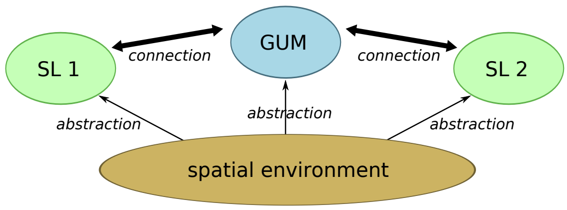


Figure 3. Connections between linguistic description (GUM) and spatial logics (SL 1, SL 2)

are illustrated in Fig. 2.2. Which specific direction is meant may depend on the course of the road, external entities or the intrinsic orientation of the car. In contrast to a linguistic LeftProjection, spatial logics define 'leftness' in an axiomatic way. In [15], a left-like relationship is divided into five possible regions according to orientations between two entities. In [16], 'left' is defined as a range of degrees of a point-based orientation with variable granularity. In [17], a *leftside(x,y,z)* relation is defined among three entities, which is specified as non-collinear. Given the examples in Fig. 2, spatial logics provide different realisations for particular relationships.

Taking into account only the linguistic input from the clauses above, nothing more than a LeftProjection ("left") is defined and possible realisations have to be determined by the context. In particular, these diverse interpretations of "left" cannot be covered by a logical relation left(a,b) together with spatial axioms such as transitivity, antisymmetry, and irreflexivity. Parts of the circle objects, for instance, violate transitivity. That the $\star$ is to the left of the x and the x is to the left of the $\sharp$ does not indicate that the $\star$ is to the left of the $\sharp$ (but rather opposite of it). And in case this would be an acceptable implication because of the circle-like arrangement, then left(a,b) would actually be symmetric (and the $\sharp$ to the left of the $\star$) and reflexive (and the $\star$ to the left of itself). Instead, the linguistic description has to be related to different models of spatial logics. Those objects in a model of a spatial logic that we take to be most adequate as a realisation of the linguistic description, we call the (spatial) *counterparts*. Hence, language specifies space according to linguistic evidence whereas logic specifies space according to its underlying theory of space. Formal relationships (connections) between both layers then have to be defined in order to determine counterparts.

Merging all kinds of spatial information into one theory that formulates all connections between language and space, however, would adversely affect effective reasoning techniques, decidability, expressiveness, modularity, and flexibility. The semantics of a spatial description can instead refer to distinct spatial models of spatial logics while underspecifying external factors (e.g. world knowledge, contextual and environmental information, or the dialogue history). Spatial language and logic can then be formally related by indicating their similarities. For instance, a LeftProjection may be realised as one of the examples in Fig. 2. As a result, the spatially-aware system should be able to determine at least the most likely connection.

In summary, language is connected to different spatial logics with regard to certain environments (see Fig. 3). This connection can be specified together with a similarity value determined by external factors, such as the context, domain-knowledge, environment, properties of spatial objects, alignment, and discourse. Most closely connected entities are called counterparts. $\mathcal{E}$-connections between language and spatial logics together with similarity values can realise this connection, as described in the next section.

4. Counterparts, Connections, and Similarity

David Lewis provided the first formal theory of counterparts [18], a two-sorted first-order theory, whose sorts are objects and worlds, and which has four predicates: $W(x)$ says that x is a world, $I(x, y)$ that x is in the world y, $A(x)$ that x is an actual object, and $C(x, y)$ that x is a counterpart of y.

He described the basic intuition underlying the idea of counterparthood as follows:

> Your counterparts resemble you closely in content and context in important ways. They resemble you more closely than do the other things in their worlds. But they are not really you. For each of them is in his own world, and only you are here in the actual world. [18], p. 27–28

The general idea of counterpart relations being based on a notion of *similarity across worlds* also lies at the heart of heterogeneous knowledge representation, and was a major motivation for the design of 'modular languages', $\mathcal{E}$-connections in particular [19].[5]

4.1. $\mathcal{E}$-Connections as Counterpart Theory

In $\mathcal{E}$-connections, a finite number of formalisms talking about distinct domains are 'connected' by relations between entities in different domains, capturing different aspects or representations of the 'same object'. For instance, an 'abstract' object o of a description logic $\mathcal{L}_1$ can be related via a relation R to its life-span in a temporal logic $\mathcal{L}_2$ (a set of time points) as well as to its spatial extension in a spatial logic $\mathcal{L}_3$ (a set of points in a topological space, for instance). Essentially, the language of an $\mathcal{E}$-connection is the (disjoint) union of the original languages enriched with operators capable of talking about the link relations. The possibility of having multiple relations between domains is essential for the versatility of this framework, the expressiveness of which can be varied by allowing different language constructs to be applied to the connecting relations.[6]

$\mathcal{E}$-connections have also been adopted as a framework for the integration of ontologies in the Semantic Web [22], and, just as DLs themselves, offer an appealing compromise between expressive power and computational complexity: although powerful enough to express many interesting concepts, the coupling between the combined logics is sufficiently loose for proving general results about the transfer of decidability: if the connected logics are decidable, then their connection will also be decidable. More importantly in our present context, they allow the heterogeneous combination of logical formalisms without the need to adapt the semantics of the respective components.

Note that the requirement of disjoint domains is not essential for the expressivity of $\mathcal{E}$-connections. What is essential, however, is the disjointness of the *formal languages* of the component logics. What this boils down to is the following simple fact: while more expressive $\mathcal{E}$-connection languages allow to express various degrees of *qualitative* identity, for instance by using number restrictions on links to establish partial bijections, they lack means to express 'proper' *numerical* trans-module identity. This issue, clearly,

[5] A general overview and discussion of counterpart-theoretic semantics can be found in [20].

[6] Thus analysed, the main difference between distributed description logics (DDLs) [21] and various $\mathcal{E}$-connections then lies in the expressivity of the 'link language' $\mathcal{L}$ connecting the different ontologies: while the link language of basic DDL is a certain sub-Boolean fragment of many-sorted $\mathcal{ALC}$, the basic link language of $\mathcal{E}$-connections is many-sorted $\mathcal{ALCI}$ (i.e. $\mathcal{ALC}$ with inverses).

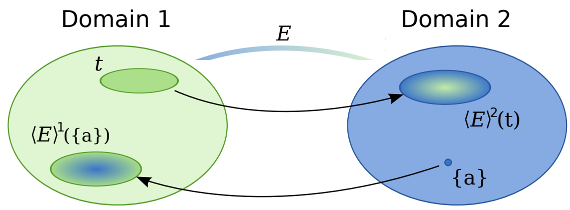


Figure 4. A two-dimensional connection.

is closely related to the problem of trans-world identity well known from counterpart theory; we will expand on this below when introducing $\mathcal{S}$-connections.

For lack of space, we can here only roughly sketch the formal definitions, but compare [19]: we assume that the **languages** L_1 and L_2 of two logics $\mathcal{L}_1$ and $\mathcal{L}_2$ are pairwise disjoint. To form a connection $\mathcal{C}^{\mathcal{E}}(\mathcal{L}_1, \mathcal{L}_2)$, fix a non-empty set $\mathcal{E} = \{E_j \mid j \in J\}$ of binary relation symbols. The **basic $\mathcal{E}$-connection language** is then defined by enriching the respective languages with operators for talking about the link relations. A structure

$$\mathfrak{M} = \langle \mathfrak{W}_1, \mathfrak{W}_2, \mathcal{E}^{\mathfrak{M}} = (E_j^{\mathfrak{M}})_{j \in J} \rangle,$$

where $\mathfrak{W}_i = (W_i, .^{\mathfrak{W}_i})$ is an interpretation of $\mathcal{L}_i$ for $i \in \{1, 2\}$ and $E_j^{\mathfrak{M}} \subseteq W_1 \times W_2$ for each $j \in J$, is called an **interpretation** for $\mathcal{C}^{\mathcal{E}}(\mathcal{L}_1, \mathcal{L}_2)$. Given concepts C_i of logics $\mathcal{L}_i$, $i = 1, 2$, denoting subsets of W_i, the semantics of the basic $\mathcal{E}$-connection operators is[7]

$$(\langle E_j \rangle^1 C_2)^{\mathfrak{M}} = \{x \in W_1 \mid \exists y \in C_2^{\mathfrak{M}} \;: (x, y) \in E_j^{\mathfrak{M}}\}$$
$$(\langle E_j \rangle^2 C_1)^{\mathfrak{M}} = \{y \in W_2 \mid \exists x \in C_1^{\mathfrak{M}} \;: (x, y) \in E_j^{\mathfrak{M}}\}$$

Fig. 4 displays the connection of an ontology with a spatial logic of regions such as $\mathbf{S4_u}$, with a single link relation E interpreted as the relation 'is the spatial extension of'. As follows from the complexity results of [19], $\mathcal{E}$-connections add substantial expressivity and interaction to the component formalism.

In [14], the problem of relating GUM [8] with spatial calculi, using the example of the double-cross calculus DCC [15] for projective relations (orientations), is analysed. The general relation between GUM and DCC is analysed to be a loose coupling as can be adequately modelled by an $\mathcal{E}$-connection. However, two entirely independent layers need to be added for a 'complete' formal representation of a spatial configuration: domain knowledge including naïve physics information is added in a KB $\mathcal{D}$, while contextual information (such as intrinsic orientations, reference system, etc.) is added by a KB $\mathcal{O}$. Both these layers of information are typically formalised in different (heterogeneous) logics. The resulting layered formalism is called **perspectival** $\mathcal{E}$-connections. However, while these extended $\mathcal{E}$-connections formally reflect different layers of a representation, they do not take into account loose couplings in the sense of link-relations that are based on notions of probability or similarity. We next generalise $\mathcal{E}$-connections in this direction.

[7] Note the close resemblance of this definition with the definition of the semantics of existential restrictions in DLs, with the important exception that the former is 'two-sorted'.

4.2. $\mathcal{S}$-Connections: Similarity-based $\mathcal{E}$-Connections

Research on similarity is of a rather broad nature, including work in areas such as philosophy and general cognitive science, (description) logics, bio-informatics, and information retrieval, among others. Technically, the notions of probability, fuzziness, and similarity are closely related, as [23] discusses. For instance, there is no (conceptual or technical) problem with attaching fuzzy-values or probabilities to link-relations: we can say that y is in the spatial extension $E(x)$ of point x with probability $p \in [0, 1]$, etc.[8]

Here, we concentrate on modelling a notion of *heterogeneous similarity*, i.e. similarity of objects drawn from conceptually different domains, specified by means of (heterogeneous) similarity measures which are closely modelled on the notions of distance functions and metrics. The notion of similarity-based $\mathcal{E}$-connections defined below thus combines the ideas of $\mathcal{E}$-connections [19], distance logics [26], and similarity logics [27].

4.3. (Heterogeneous) Similarity Spaces

By $\mathbb{R}^+_{0,\infty}$ we denote the positive real numbers including zero and the symbol ∞, denoting infinity. For $i = 1, 2$, we set $\bar{i} = 1$ if $i = 2$ and $\bar{i} = 2$ if $i = 1$.

Definition 1 *A **similarity space** $\mathbb{S} = \langle \mathcal{S}, \int \rangle$ (sim-space for short) consists of a set $\mathcal{S}$ together with a **similarity measure** $\int$, i.e. a function $\int : \mathcal{S} \times \mathcal{S} \mapsto \mathbb{R}^+_{0,\infty}$ satisfying $\int(x, x) = 0$ for all $x \in \mathcal{S}$. In case $\forall x, y \in \mathcal{S} : \int(x, y) = 0 \iff x = y$ holds, we call $\int$ **discrete**. If $\int$ satisfies $\forall x, y \in \mathcal{S} : \int(x, y) = \int(y, x)$, we call $\int$ **symmetric**, and if it satisfies $\forall x, y, z \in \mathcal{S} : \int(x, y) + \int(y, z) \geq \int(x, z)$, we call it **triangular**. If $\mathbb{S}$ is discrete, symmetric and triangular, and $\infty \notin \mathsf{range}(\int)$, it is also called a **metric**, and $\langle \mathcal{S}, \int \rangle$ is called a **metric space**.*

Here, $\int(x, y) = 0$ means that x is *perfectly similar* to y.[9] However, note that perfect similarity implies identity only in the case of discrete spaces. $\int(x, y) < \int(x, z)$ means that x is *more similar* to y than to z, and $\int(x, y) = \int(x, z)$ means that x is *equally similar* to y and z. Moreover, we say that x is *discernibly similar* to y if $\int(x, y) < \infty$ and *indiscernibly similar* otherwise, i.e. if $\int(x, y) = \infty$. For $X, Y \subseteq \mathcal{S}$ sets (rather than just elements), similarity is defined by extending $\int$ as follows:

$$\int(X, Y) := \begin{cases} \inf\{\int(x, y) \mid x \in X, y \in Y\}, & \text{if } X, Y \neq \emptyset \\ \infty, & \text{otherwise} \end{cases}$$

If in fact the minimum exists for all non-empty sets X and Y, $\mathcal{S}$ is also called a $\min$-space, compare [27]. Clearly, whenever a space is finite, it is a $\min$-space.

When relating *different* sets of objects, such as when connecting linguistic ontologies and spatial logics, the above definitions need to be adapted. For simplicity, we here restrict our attention to the case of only two such sets.

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[8] This natural idea has been studied for instance in the work of Suzuki on graded accessibility relations [24]. Also, Williamson [25] pursued similar semantic ideas when developing his propositional logics of clarity.

[9] Contrary to other formal approaches to similarity, closeness in the similarity space (i.e. a low value of the similarity measure) corresponds to high similarity: this intuition derives from the spatial interpretation of metric spaces.

Definition 2 *A (2-dim)* ***heterogeneous similarity space*** *(*hsim*-space for short) is a quadruple* $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1 \rangle$ *consisting of, for* $i = 1, 2$, sim*-spaces* $\mathbb{S}_i = \langle \mathcal{S}_i, \int_i \rangle$*, and* ***heterogeneous similarity measures*** $\int_i^{\bar{i}} : \mathcal{S}_i \times \mathcal{S}_{\bar{i}} \mapsto \mathbb{R}^+_{0,\infty}$. $\mathbb{H}$ *is* ***het-symmetric*** *if for all* $x \in \mathcal{S}_i$ *and all* $y \in \mathcal{S}_{\bar{i}}$ *we have* $\int_i^{\bar{i}}(x, y) = \int_{\bar{i}}^{i}(y, x)$ *(for* $i = 1, 2$*). It is* ***het-triangular*** *if for all* $x, z \in \mathcal{S}_i$ *and* $y \in \mathcal{S}_{\bar{i}}$ *we have* $\int_i^{\bar{i}}(x, y) + \int_{\bar{i}}^{i}(y, z) \geq \int_i(x, z)$ *(for* $i = 1, 2$*).*

In the heterogeneous case, *perfect similarity* now means that $x \in \mathcal{S}_1$ and $y \in \mathcal{S}_2$ are indistinguishable from the perspectives of both similarity measures, $\int_1$ and $\int_2$.[10]

4.4. Counterparts in Similarity-based $\mathcal{E}$-Connections

Note that, in this setting, the problems of transworld identity and counterparthood can be neatly separated: transworld identity may be taken to be synonymous with perfect similarity as defined above. Counterparthood understood as *maximal similarity* is a looser notion, and may be explicated by the following principle (see [28]).

> For $x \in \mathcal{S}_i$ and $y \in \mathcal{S}_{\bar{i}}$, y is a counterpart of x only if nothing in $\mathcal{S}_{\bar{i}}$ is more similar to x as it is in $\mathcal{S}_i$ than is y as it is in $\mathcal{S}_{\bar{i}}$.

We take this principle as the defining criterion for counterparthood in similarity spaces:

Definition 3 (Counterparts) *Let* $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1 \rangle$ *be a* hsim*-space. We call* $b_{\bar{i}} \in \mathcal{S}_{\bar{i}}$ *an* $\bar{i}$***-counterpart of*** $a_i \in \mathcal{S}_i$ *if* $\int_i^{\bar{i}}(a_i, b_{\bar{i}}) = \inf\{\int_i^{\bar{i}}(a_i, b) \mid b \in \mathcal{S}_{\bar{i}}\} < \infty$*, which we also write as* $\mathsf{Cp}_i^{\bar{i}}(a_i, b_{\bar{i}})$*. This gives us two relations:* $\mathsf{Cp}_i^{\bar{i}} \subseteq \mathcal{S}_i \times \mathcal{S}_{\bar{i}}, i = 1, 2$*. Moreover, for* $X \subseteq \mathcal{S}_i$*, we denote by* $\mathsf{Cp}_i^{\bar{i}}(X)$ *the set* $\{y \in \mathcal{S}_{\bar{i}} \mid \exists x \in X. \mathsf{Cp}_i^{\bar{i}}(x, y)\}$.

Note that counterparts thus defined may or may not be unique. Moreover, $b_{\bar{i}}$ may be an $\bar{i}$-counterpart of a_i without a_i being an i-counterpart of $b_{\bar{i}}$; counterparthood is *directional*. Although counterparts need not be unique, in applications it is often desirable to select amongst the elements with maximal similarity a unique element, according to certain *external* criteria. We here solve this problem by incorporating into the structures an explicit choice function selecting a counterpart.

Definition 4 (Counterpart choice) *A* hsim*-space* ***with choice*** *is a triple* $\langle \mathbb{H}, \lambda_1, \lambda_2 \rangle$*, where* $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1 \rangle$ *is a* hsim*-space, and, for* $i = 1, 2$, $\lambda_i : \mathcal{S}_i \longrightarrow \mathsf{Cp}_i^{\bar{i}}(\mathcal{S}_i)$ *are* ***choice functions*** *such that, for all* $x \in \mathcal{S}_i$*, we have that* $\lambda_i(x) \subseteq \mathsf{Cp}_i^{\bar{i}}(x)$ *is a singleton.*

Of course, often the λ_i are uniquely determined by the similarity measures $\int_i^{\bar{i}}$, in which case we call λ_i a **deterministic choice function**. Apart from the elements with maximal similarity, i.e. the counterparts, it is also of interest to be able to refer to elements of a foreign domain that are similar *to some degree* (i.e. discernibly similar). This can be achieved by simulating the notion of *link relation* from $\mathcal{E}$-connections as follows:

Definition 5 (Link-relation) *Given a* hsim*-space* $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1 \rangle$*, we define the* ***induced link relations*** $E^1_{\mathbb{H}}, E^2_{\mathbb{H}}, E_{\mathbb{H}} \subseteq \mathcal{S}_1 \times \mathcal{S}_2$ *by setting, for all* $x \in \mathcal{S}_1$ *and* $y \in \mathcal{S}_2$*:*

$$E^1_{\mathbb{H}}(x, y) \iff \textstyle\int_1^2(x, y) < \infty; \quad E^2_{\mathbb{H}}(x, y) \iff \int_2^1(y, x) < \infty;$$

[10] The notion of *discrete* similarity measure makes no immediate sense in the heterogeneous case as identity is not available. However, the notion can be 'simulated' by replacing identity with an independently defined notion of trans-module identity, 'equalising' cross-domain elements whilst respecting the similarity measures.

$$E_{\mathbb{H}}(x,y) \iff \min\big(\int_1^2(x,y), \int_2^1(y,x)\big) < \infty (= E_{\mathbb{H}}^1 \cup E_{\mathbb{H}}^2).$$

Intuitively, the relation $E_{\mathbb{H}}(x,y)$ holds if x and y are discernibly similar from at least one 'viewpoint', and $E_{\mathbb{H}}^i(x,y)$ holds if x and y are discernibly similar from the point of view of $\int_i^{\bar{i}}$. We can now recover standard $\mathcal{E}$-connections in the following sense:

Proposition 6 *For every $\mathcal{E}$-connection model $\mathfrak{M} = \langle \mathfrak{W}_1, \mathfrak{W}_2, E^{\mathfrak{M}} \rangle$ there is a* hsim*-space $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1 \rangle$ such that $E_{\mathbb{H}} = E^{\mathfrak{M}}$.*

PROOF. Fix $\mathfrak{M} = \langle \mathfrak{W}_1, \mathfrak{W}_2, E^{\mathfrak{M}} \rangle$. Essentially, we need to show that induced link relations can be arbitrary relations: set, for $x \in \mathcal{S}_1$ and $y \in \mathcal{S}_2$

$$\int_1^2(x,y) = \int_2^1(y,x) = \begin{cases} 0, \text{ if}(x,y) \in E^{\mathfrak{M}} \\ \infty, \text{ otherwise} \end{cases}$$

Clearly, $E^{\mathfrak{M}} = E_{\mathbb{H}}$. $\square$

4.5. Similarity Bridge Logic

So far, we have only (generically) described the model-theory of similarity based $\mathcal{E}$-connections. Whilst the component logics can be assumed to be given, we need to describe possibilities to (syntactically) define the bridge logic of such $\mathcal{E}$-connections. As we have mentioned above, the spectrum of languages that can be used for this can be varied almost arbitrarily. We here describe a language that we consider basic in that it reflects the essential features of the underlying structures. We assume two logics $\mathcal{L}_1$ and $\mathcal{L}_2$ are given, with disjoint sort structure. For $\mathcal{L}_i, i = 1,2$, assume object names a_i (denoting elements of the domains) and terms A_i (denoting subsets of the domains) belonging to the respective logics are given.

Fix a hsim-space with choice $\mathbb{H} = \langle \mathbb{S}_1, \mathbb{S}_2, \int_1^2, \int_2^1, \lambda_1, \lambda_2 \rangle$, and assume, for $i = 1,2$, the logics $\mathcal{L}_i$ are interpreted in models $\mathfrak{M}_i$ over sim-spaces $\mathbb{S}_i$, i.e. $\mathsf{dom}(\mathfrak{M}_i) \supseteq \mathsf{dom}(\mathbb{S})$.

Definition 7 *The **basic similarity bridge logic** $\mathcal{B}_{\mathsf{sim}}(\mathcal{L}_1, \mathcal{L}_2)$ contains:*

- ***projection operators**: $\langle E \rangle^{\bar{i}} A_i$ and $\langle E \rangle^{\bar{i}} a_i$, for $i = 1,2$ and $E \in \{E_{\mathbb{H}}^1, E_{\mathbb{H}}^2, E_{\mathbb{H}}\}$.*
These are the basic $\mathcal{E}$-connection-operators (with the standard semantics), with link-relations E inherited from the similarity measures as defined in Def. 5.

- ***counterpart operators**: $\langle C \rangle^{\bar{i}} A_i$ and $\langle C \rangle^{\bar{i}} a_i$, $i = 1,2$.*
Given the term A_i of logic $\mathcal{L}_i$, the operator $\langle C \rangle^{\bar{i}} A_i$ yields the set of all counterparts of elements of A_i, i.e.

$$(\langle C \rangle^{\bar{i}} A_i)^{\mathfrak{M}} = \{y \in \mathcal{S}_{\bar{i}} \mid \exists x \in A_i^{\mathfrak{M}_i} \text{ and } \mathsf{Cp}_i^{\bar{i}}(x,y)\},$$

and similarly for object names.

- ***choice operators***: $\langle\lambda\rangle^{\bar{i}} a_i, i = 1, 2$.

These pick out the unique counterpart of a_i as a singleton subset whenever there are counterparts, and returns $\bot^{\bar{i}}$ otherwise, i.e.

$$(\langle\lambda\rangle^{\bar{i}} a_i)^{\mathfrak{M}} = \begin{cases} \{\lambda_i(a_i{}^{\mathfrak{M}_i})\}, & \textit{if defined} \\ \bot^{\bar{i}}, & \textit{otherwise} \end{cases}$$

- ***heterogeneous similarity operators***: $\langle\downdownarrows\rangle^i(A_1, A_2)$, $\langle\upuparrows\rangle^i(A_1, A_2)$, $i = 1, 2$.

Intuitively, $\langle\downdownarrows\rangle^1(A_1, A_2)$ gives a term of $\mathcal{L}_1$, consisting of all those members of $\mathbb{S}_1$ that are closer to something in A_1 than to any of A_2's counterparts in $\mathbb{S}_1$ (similarity is evaluated locally*). Conversely, $\langle\upuparrows\rangle^1(A_1, A_2)$ gives a term of $\mathcal{L}_1$, consisting of all those members of $\mathbb{S}_1$ all of whose counterparts are closer to some of A_1's counterparts than to any element in A_2 (similarity is evaluated* externally *for the counterparts). Formally, the semantics is as follows, for $i = 1, 2$:*

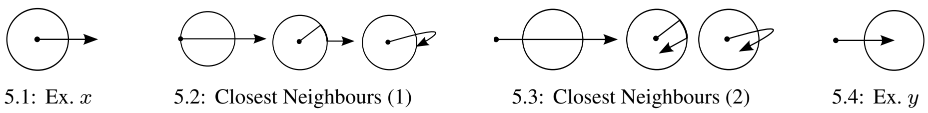
$$\big(\langle\downdownarrows\rangle^i(A_1, A_2)\big)^{\mathfrak{M}} = \{y \in \mathcal{S}_i \mid \textstyle\int_i\big(y, A_i{}^{\mathfrak{M}_i} \cap \mathcal{S}_i\big) < \int_i\big(y, \mathsf{Cp}^{i}_{\bar{i}}(A_{\bar{i}}{}^{\mathfrak{M}_{\bar{i}}}) \cap \mathcal{S}_i\big)\} \textit{ and}$$

$$\big(\langle\upuparrows\rangle^i(A_1, A_2)\big)^{\mathfrak{M}} = \{y \in \mathcal{S}_i \mid \textstyle\int_{\bar{i}}\big(\mathsf{Cp}^{\bar{i}}_i(y), \mathsf{Cp}^{\bar{i}}_i(A_i{}^{\mathfrak{M}_i}) \cap \mathcal{S}_{\bar{i}}\big) < \int_{\bar{i}}\big(\mathsf{Cp}^{\bar{i}}_i(y), A_{\bar{i}}{}^{\mathfrak{M}_{\bar{i}}} \cap \mathcal{S}_{\bar{i}}\big)\}$$

As in standard $\mathcal{E}$-connections, we assume that these operators yield new terms of the respective logics to which the operators of those logics can then be further applied. This process, inductively, defines the **basic similarity language** of **$\mathcal{S}$-connections**.[11]

5. $\mathcal{S}$-Connection for Directions and Regions in Language and Space

An example how $\mathcal{S}$-connections can be used to relate natural language and spatial logics is outlined in the following. Here, GUM is '$\mathcal{S}$-connected' with the 9^+*-intersection for topological relations between a directed line segment (DLine) and a region* (9^+-calculus) [29]. Similarities between examples of linguistic motion descriptions in GUM and related 9^+-calculus examples are presented. The linguistic descriptions (a) "They went out of the park", (b) "They left the park" are defined by source:GeneralDirectional and (c) "They entered the park" is defined by destination:GeneralDirectional in GUM. GeneralDirectional defines directions of motions or orientations determined by the relatum and specified by the relations source and destination. (For reasons of space and simplicity, the reader is referred to [8] for further documentation.)



5.1: Ex. x 5.2: Closest Neighbours (1) 5.3: Closest Neighbours (2) 5.4: Ex. y

Figure 5. Directed line segments and possible relations with a region.

While actor and relatum are linguistically described by "they" and "park" respectively, their counterparts in the 9^+-calculus are the *DLine* for the motion of "they" and

[11] For simplicity, we have here defined only the 'concept language' of $\mathcal{S}$-connections. Assertions and KBs can be defined in the same way as for $\mathcal{E}$-connections, with the addition of object statements allowing to explicitly declare the similarity between named objects such as $\mathsf{sim}^{\bar{i}}_i(a_i, a_{\bar{i}}) = 3$, with the obvious semantics.

the *region* for “park”. A sample of 9^+-models are illustrated in Fig. 5. The topological dependence in Fig. 5.1 between the DLine and the region is defined as the most similar realisation for a and b. Given the neighbourhood graph for x by the 9^+-calculus, Fig. 5.2 shows its direct neighbours. Some of them are also elements with high similarity for GUM’s source:GeneralDirectional. Fig. 5.3 shows neighbours directly related to the first neighbours in Fig. 5.2. Those are, however, rather indiscernibly similar with a and b. As a and b are equally instantiated in GUM, they are not distinguishable and $\int_{GUM}(a,b) = 0$. A set of similar 9^+-elements for a and b are illustrated in Fig. 6, ordered by decreasing similarity. The first one (denoted x) is the counterpart. Clearly, a and b are equally similar to x, and so $\mathsf{sim}_1^2(a,x) = \mathsf{sim}_1^2(b,x)$.

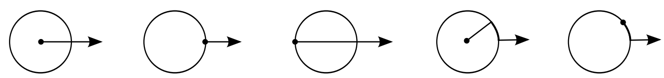


Figure 6. 9^+-calculus counterparts for a “They left the park”

Conversely, the counterpart of c “They entered the park” is y illustrated in Fig. 5.4. Here, the DLine has exactly the opposite direction of the DLine in x. y is also indiscernibly similar to a and b. Hence, the $\mathcal{S}$-connections between GUM and the 9^+-intersections differ in similarities of linguistic descriptions and topological relationships, as indicated by the neighbourhood relation and equal specifications in GUM. An excerpt from these similarity relations and $\mathcal{S}$-connections is illustrated in Fig. 7.

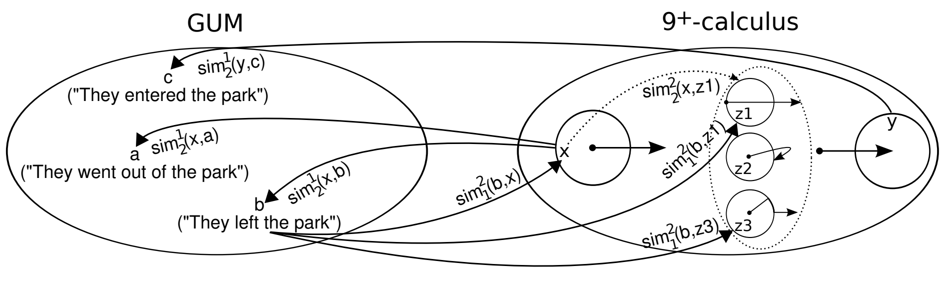


Figure 7. Example of $\mathcal{S}$-connections between GUM and 9^+-calculus. Similar counterparts are $\mathsf{sim}_1^2(b,x)$ (from GUM to SL), $\mathsf{sim}_2^1(x,b) = \mathsf{sim}_2^1(x,a)$ (from SL to GUM), and $\mathsf{sim}_2^2(x,z1)$ (similarities within SL).

6. Discussion

We have introduced $\mathcal{S}$-connections as an extension of $\mathcal{E}$-connections adding similarity measures across domains and corresponding formal apparatus to interpret these measures. We have shown that this framework is well-suited to deal with the problem of relating linguistic semantics and spatial logics whilst respecting the uncertainties or underspecifications that are involved in their relationship. Various examples illustrating how language underspecifies spatial information are given together with aspects causing such underspecifications. However, further investigations will need to elaborate on specific definitions of such measures and on algorithms for calculating them, based on external linguistic and spatial factors, as described for instance in [30].

As concerns the general theory of $\mathcal{S}$-connections, there are many interesting open problems. Most obviously, decidability and complexity issues for various component and bridge logics should be addressed, and an axiomatisation of the basic logic of $\mathcal{S}$-connections should be given (extending the results of [31]). Other interesting areas are the following: (i) analyse structural properties on the interplay between 'local' and 'global' (i.e. heterogeneous) similarity measures; (ii) formulate various notions of qualitative (transmodule) identity compatible with similarity measures; (iii) investigate notions such as transitivity of similarity that have a different flavour in the setting of $\mathcal{S}$-connections.

To elaborate just on the last point, note that the triangular inequality gives us a particular (quantitative) version of transitivity of similarity. Namely, if a is x-similar to b and b is y-similar to c, then a is at least $x + y$-similar to c. Stricter transitivity assertions could, of course, be defined, and would correspond to global '*elasticity*' restrictions on the similarity space. However, similarities between entities in a spatial model will not always directly entail corresponding similarities between spatial language and spatial logic configurations, as indicated by the example in Fig. 5. Therefore, a careful analysis of appropriate transitivity principles for the interplay between spatial language and spatial logics will be necessary.

Acknowledgements

We gratefully acknowledge the financial support of the Deutsche Forschungsgemeinschaft through the Collaborative Research Centre SFB/TR 8. The authors would also like to thank J. Bateman, T. Mossakowski, and T. Tenbrink for fruitful discussions.

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