



What is an Ontology of Computational Complexity?

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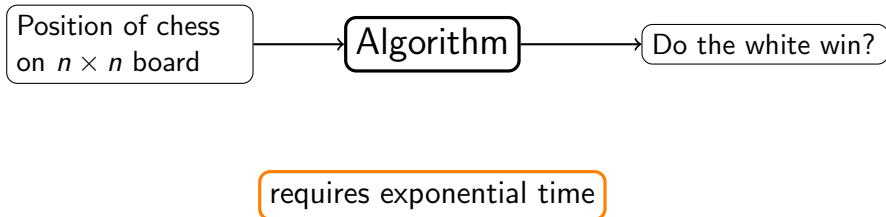
Abstraction Workshop

July 11, 2025

Decision Problems



Decision Problems



$$\text{NP} \subset \text{NExpTime} \subset 2\text{NExpTime} \subset \dots$$

$$\subsetneq \quad \supsetneq \quad \subsetneq \quad \supsetneq \quad \subsetneq \quad \supsetneq$$

$$\text{P} \subset \text{ExpTime} \subset 2\text{ExpTime} \subset 3\text{ExpTime} \subset \dots$$

$$\subsetneq \quad \supsetneq \quad \supsetneq \quad \subsetneq \quad \supsetneq \quad \supsetneq \quad \supsetneq$$

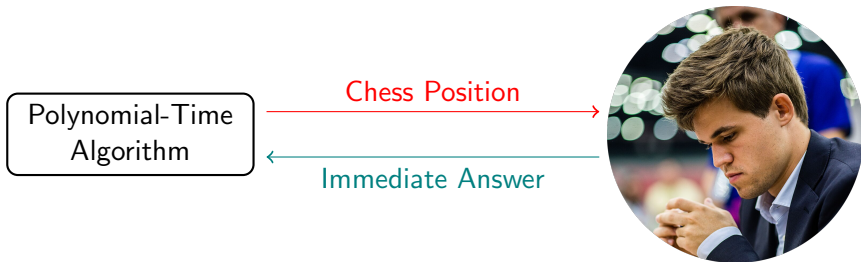
$$\text{L} \subset \text{PSPACE} \subset \text{ExpSpace} \subset 2\text{ExpSpace} \subset \dots$$

$$\supsetneq \quad \subsetneq \quad \supsetneq \quad \supsetneq$$

$$\text{NL} \subset \text{NPSpace} \subset \text{NExpSpace} \subset \dots$$

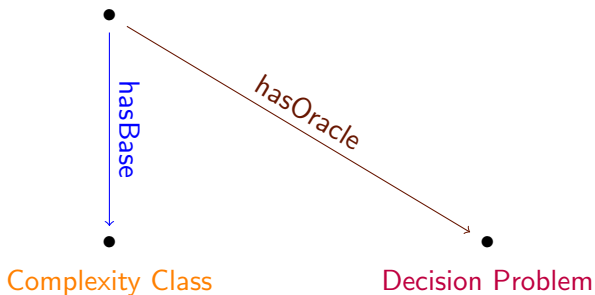
$$\text{AC} \quad \text{ACC} \quad \text{NC} \quad \text{BPP} \quad \text{BQP} \quad \text{MA} \quad \text{QMA} \quad \dots$$

Oracles



Resulting class: $\text{P}^{\text{Chess Problem}}$

Complexity Class



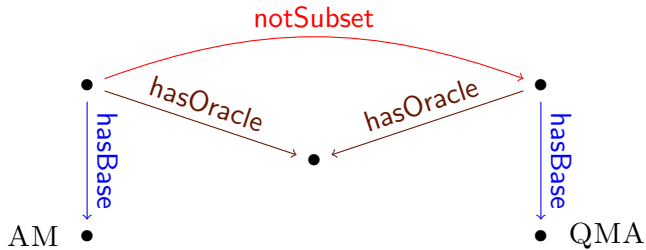
$$\text{Class}(x) \wedge \text{Problem}(y) \rightarrow \exists z [\text{hasBase}(z, x) \wedge \text{hasOracle}(z, y)]$$

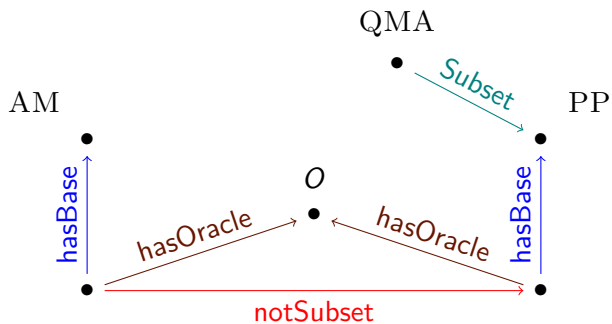
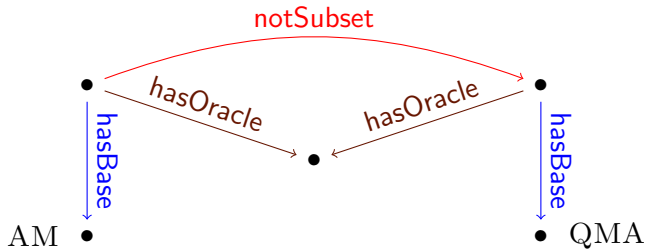
[Once] I spent a week trying to put AM outside QMA relative to an oracle, only to learn that this followed trivially from two known results: that of Vereshchagin [Ver92] that AM is outside PP relative to an oracle, and that of Kitaev and Watrous (unpublished, but mentioned in [Wat00]) that QMA is in PP .

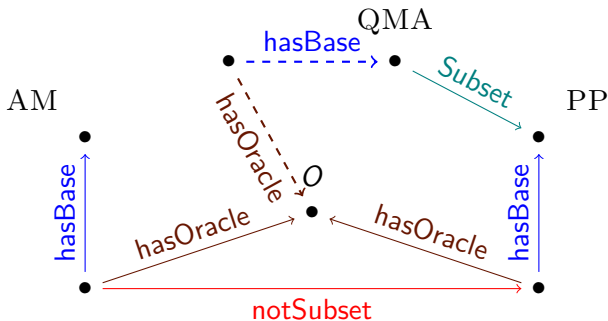
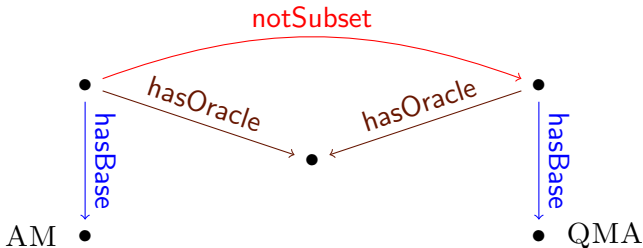


Scott Aaronson

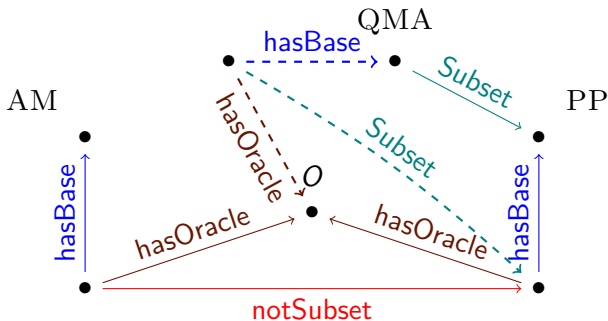
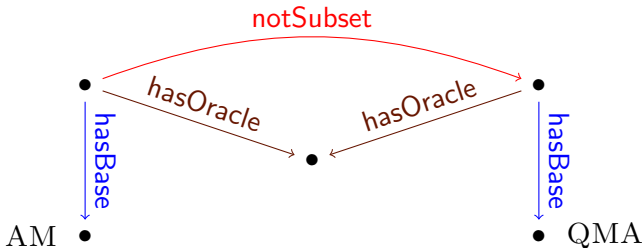
complexityzoo.net



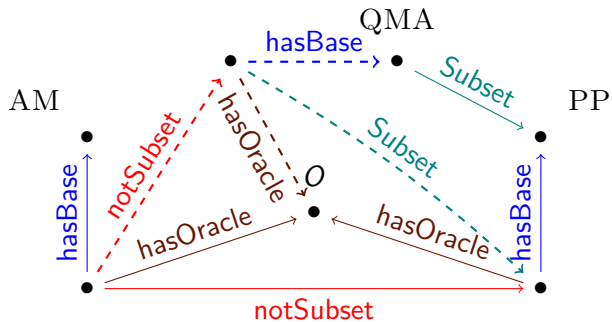
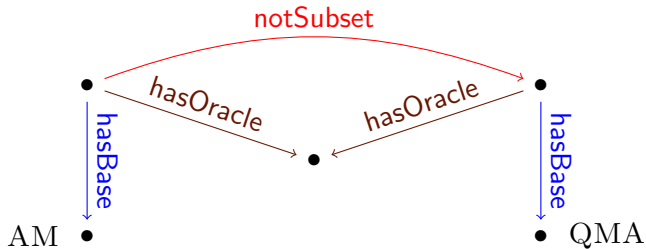




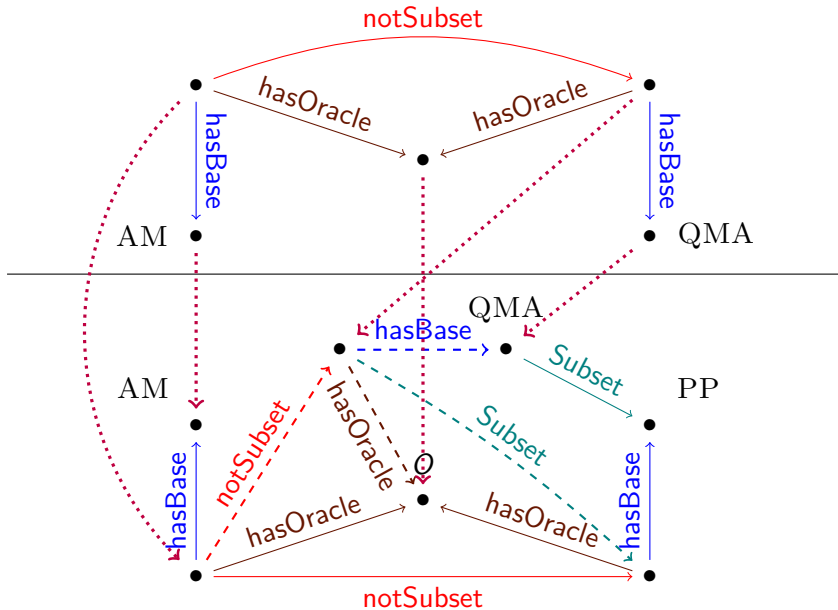
$$\text{Class}(x) \wedge \text{Problem}(y) \rightarrow \exists z [\text{hasBase}(z, x) \wedge \text{hasOracle}(z, y)]$$



$$(\text{hasOracle} \circ \text{hasOracle}^-) \sqcap (\text{hasBase} \circ \text{Subset} \circ \text{hasBase}) \sqsubseteq \text{Subset}$$



$$\text{notSubset} \circ \text{Subset}^- \sqsubseteq \text{notSubset}$$



An Ontology of Complexity

Dots: problems and classes.

Lines: relations between them.

Axioms: description logic + first-order logic.

Is a dot more abstract than a complexity class?