Playing equilibria: What's the outcome function, again?

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Abstract. We propose a logic where the players may not know the outcome function of the strategic game being played. It is an epistemic extension of games of propositional control. The main contribution of the paper is to lay down a logical framework for the combined analysis of (1) game theoretic properties of strategic games and (2) epistemic properties. The logic enables reasoning about agents and coalitions knowingly bringing about a particular consequence or a particular solution concept.

1 Introduction

In game theory, uncertainty typically stems from the ignorance of which actions have been performed by other agents. Some logics have been designed so as to model some aspects of imperfect knowledge in a dynamic and strategic setting. See for instance [6, 7], [2], [3], [9]. In this note however, we build a logic to reason about strategic games where the players have uncertainty about other players' moves, and might *also* not know the outcome function of the game being played.

We base our framework upon the logic of games and propositional control [12] where every agent controls a set of propositional atoms. The repertoire of action (or choices) of an agent i then corresponds to the set of valuations of the atoms controlled by i. In addition to this notion of action, the logic comprises preferences. It allows to characterise a large variety of solution concepts.

In this note, to account for the players' lack of knowledge about the game being actually played, we add a notion of *moment*, which can be conceived as a possible world. Agents control the exact same set of atoms at every moment, and hence always have the same action repertoire. However, the *outcome function*, that is the consequence of a particular action profile, may differ from one moment to another. Since an agent may not be able to distinguish two different moments, part of agents' ignorance is about the outcome function of the game being played itself. It can be exemplified as follows.

Example 1. Ray is blind and cannot see if the the lamp is on or off. He has two possible strategies: (1) switching the light or (2) remaining passive. If the lamp is on (resp. off), switching the light will lead to a moment where the light is off (resp. on). Remaining passive will not change the moment: if the lamp is off (resp. on), it will stay in a moment where it is off (resp. on). Hence, Ray knows his actions but ignores their outcomes.

Just like in [3], we are interested in a particular kind of knowledge that we shall call *commitment-dependent*. It is a somewhat a refinement of 'static' standard knowledge about the moments. When an agent cannot distinguish between two moments s_1 and

s_2, it also cannot *a priori* rule out any outcome of either two moments. However, by committing to a particular choice, the agent also refines its knowledge about the possible next outcomes.

In fact, in a dynamic environment, commitment-dependent knowledge is relevant even if the agents have perfect knowledge about the moment.

Example 2. Ann is facing a lamp that is turned off and she sees so. She has two possible strategies: (1) switching the light on or (2) remaining passive. She does not know whether the light will be on at the next step as both outcomes are possible depending on her action. However she will know once she commits to a strategy, and that, even before actually performing the action.

We present briefly the frames of propositional control in the next section and we introduce our logic for reasoning about games and knowledge in Section 3. In Section 4 we discuss the interplay between strategies and knowledge. In Section 5, we give an example of application in which we use the logic for analysing whether a group of players can knowingly play a Nash equilibrium.

2 Propositional control

Coalition Logic of Propositional Control (CL-PC) [14] is a logic that deals with agency and contingent ability of players in strategic games. Every player controls the truth values of a particular set of atoms At_i. The set of possible valuations over At_i is easily understood as representing the set of actions in the repertoire of the player i.

Definition 1 (frames). *A* frame of propositional control *is a tuple* $\langle N, (At_i)\rangle$*, such that:*

- $N = \{1, 2, \dots n\}$ *is a nonempty finite set of players;*
- At_i *is the set of atoms controlled by agent i.*

We require that $At_i \cap At_j = \emptyset$ *for* $i \neq j$.

We define the set of controlled atoms $At = At_1 \cup \dots \cup At_n$. Every variable is controlled by one and only one agent: the sets At_i form a partition of At. We refer to At_C as the union of the controlled atoms At_i of every agent i in C.

Definition 2 (valuations and reification). *Given a coalition* $C \subseteq N$*, a* C-valuation θ_C *is a function* $\theta_C : At_C \longrightarrow \{\mathtt{tt}, \mathtt{ff}\}$*. Also, we define:*

$$\pi(\theta_C) \triangleq \bigwedge_{p \in At_C, \theta_C(p) = \mathtt{tt}} p \wedge \bigwedge_{q \in At_C, \theta_C(q) = \mathtt{ff}} \neg q.$$

Game theoretically, a C-valuation (viz. a valuation of all the variables in At_C) can be identified with a coalitional action. Like an action profile, an N-valuation specifies one choice for every player. The function π allows us to reify these valuations in the object language.

We note Θ the set of N-valuations. When it is clear from the context, we shall write θ instead of θ_N. Hence, θ_C can be conceived of the restriction of θ to At_C. Given θ and

θ' in Θ, we write $\theta \equiv_C \theta'$ to mean $\theta_C = \theta'_C$. We shall sometimes slightly abuse notation and decompose a valuation θ. Let $\{C_1, \ldots C_k\}$ a partition of N; we denote by θ_N the tuple $(\theta_{C_1}, \ldots, \theta_{C_k})$.

CL-PC allows to reason about strategic game *forms* when the set of outcomes is the set of N-valuations.

In [12], a logic of games is built upon CL-PC. It consists in adding a set of consequences (the outcomes of the valuations) and preferences over the consequences.

Definition 3. *A* game of propositional control with consequences *(notation: GPCC) is a tuple $\langle N, (At_i), S, o, (\preceq_i)\rangle$, such that:*

- *$\langle N, (At_i)\rangle$ is a frame of propositional control;*
- *S is a nonempty finite set of atoms such that $At \cap S = \emptyset$;*
- *o maps a θ_N valuation to an element of S;*
- *$\preceq_i$ is a preference relation over S for every agent i.*

In the next section, we present an epistemic extension of the models and propose a logic that allows to express properties of a strategic game where the players may not know the outcome function.

3 Games of propositional control under imperfect information

The language $\mathcal{L}(N, (At_i), S)$ is inductively defined by the following grammar:

$$\varphi ::= \top \mid a \mid \neg\varphi \mid \varphi \vee \varphi \mid \Diamond_C\varphi \mid \blacklozenge_i\varphi \mid \mathbb{K}_i\varphi \mid \mathbb{E}_C\varphi \mid \mathbb{C}_C\varphi$$

where a is an atom of $At \cup S$, $C \subseteq N$ is a coalition and $i \in N$ is a player.

The formula $\Diamond_C\varphi$ reads that providing that the players outside C hold on with their current choice, the coalition C can ensure φ. $\blacklozenge_i\varphi$ reads that the player i prefers φ (or is indifferent). $\mathbb{K}_i\varphi$ means that agent i knows that φ.

We aim at modelling players that could be incapable of knowing which game they are actually playing; We need a mechanism that allows to deal with several GPCC. We note $\mathcal{G}[N, (At_i), S]$ the set of GPCC over the sets N, At_i $(i \in N)$ and S.

The models are merely a set of strategic games in $\mathcal{G}[N, (At_i), S]$ with a relation of indistinguishability over it for every player. We have a set of moments S and a function *gpcc* that associates a GPCC to every moment. (Note that this is reminiscent of *effectivity structures* in Coalition Logic [10], which consist in a set of moments and a function associating a game form to every moment.) We also have an equivalence relation $\sim_i$ over S meant as a relation of indistinguishably of the player i.

In this note, we will assume that a player has the same preferences over consequences in two indistinguishable moments.

We will need a way of referring to the semantical objects in a specific game. Given a GPCC G we will denote its outcome function by o^G and its preference profile by $\preceq^G$. Hence, a GPCC G is a tuple $\langle N, (At_i), S, o^G, (\preceq_i^G)\rangle$, where N, At_i and S are fixed parameters.

We can now define the models rigorously.

Definition 4 (models of games and knowledge). *A* model of games and knowledge *is a tuple $M = (N, (At_i), S, gpcc, (\sim_i))$, where:*

- *N is a finite non-empty set of players;*
- *At_i is a finite set of atoms such that $At_i \cap At_j = \emptyset$ whenever $i \neq j$;*
- *S is a set of moments;*
- *$gpcc : S \longrightarrow \mathcal{G}[N, (At_i), S]$;*
- *$\sim_i$ is an equivalence relation over S such that for every $\{x, y, q_1, q_2\} \subseteq S$, if $q_1 \sim_i q_2$ then $x \preceq^{gpcc(q_1)} y$ iff $x \preceq^{gpcc(q_2)} y$.*

Everything is rather self-explaining. Remark simply that the last item takes care about our assumption about the uniformity of preferences in indistinguishable moments.

In the models defined previously, $\sim_i$ are the individual epistemic relations. From them we can define the notions of group indistinguishability that will give rise for every coalition C to (i) mutual knowledge (operator: $\mathbb{E}_C$), and (ii) common knowledge ($\mathbb{C}_C$).

We say there is a mutual knowledge within a group C that φ holds when every player in C knows that φ. We say there is common knowledge within a group C that φ when it is mutual knowledge that φ, it is mutual knowledge that it is mutual knowledge that φ, and so on. Formally, these concepts correspond to the following relations:

mutual knowledge: $\sim_{\mathbb{E}_C} = \cup_{k \in C} \sim_k$
common knowledge: $\sim_{\mathbb{C}_C} = \sim_{\mathbb{E}_C}^{+}$, where $^{+}$ is the reflexive and transitive closure

Definition 5 (truth values of $\mathcal{L}(N, (At_i), S)$). *The truth value of a formula of $\mathcal{L}(N, (At_i), S)$ is wrt. a model $M = (S, gpcc, \sim_i)$, a moment s in S and an N-valuation of the controlled atoms ($\theta \in \Theta$). It is inductively given by:*

$M, s, \theta \models \top$			
$M, s, \theta \models x$	*iff*	$o^{gpcc(s)}(\theta) = x$	, $x \in S$
$M, s, \theta \models p$	*iff*	$\theta(p) = \mathtt{tt}$	, $p \in At$
$M, s, \theta \models \neg\varphi$	*iff*	$M, s, \theta \not\models \varphi$	
$M, s, \theta \models \varphi \vee \psi$	*iff*	$M, s, \theta \models \varphi$ *or* $M, s, \theta \models \psi$	
$M, s, \theta \models \Diamond_C\varphi$	*iff*	*there is a* $\theta' \in \Theta$ *such that* $\theta' \equiv_{N \setminus C} \theta$ *and* $M, s, \theta' \models \varphi$	
$M, s, \theta \models \blacklozenge_i\varphi$	*iff*	*there is a* $\theta' \in \Theta$ *such that* $o^{gpcc(s)}(\theta) \preceq_i^{gpcc(s)} o^{gpcc(s)}(\theta')$ *and* $M, s, \theta' \models \varphi$	
$M, s, \theta \models \mathbb{K}_i\varphi$	*iff*	*for all* s' *s.t.* $s \sim_i s'$ *and for all* $\theta' \in \Theta$ *s.t.* $\theta' \equiv_i \theta$ *we have* $M, s', \theta' \models \varphi$.	
$M, s, \theta \models \mathbb{E}_C\varphi$	*iff*	*for all* s' *s.t.* $s \sim_{\mathbb{E}_C} s'$ *and for all* $\theta' \in \Theta$ *s.t.* $\theta' \equiv_C \theta$ *we have* $M, s', \theta' \models \varphi$;	
$M, s, \theta \models \mathbb{C}_C\varphi$	*iff*	*for all* s' *s.t.* $s \sim_{\mathbb{C}_C} s'$ *and for all* $\theta' \in \Theta$ *s.t.* $\theta' \equiv_C \theta$ *we have* $M, s', \theta' \models \varphi$.	

The truth of φ in all models is defined by $\models \varphi$. The classical operators $\wedge$, $\rightarrow$, $\leftrightarrow$ can be defined as usual. We also define $\Box_C\varphi \triangleq \neg\Diamond_C\neg\varphi$ and $\blacksquare_i\varphi \triangleq \neg\blacklozenge_i\neg\varphi$.

We define the operator of brute choice as follows

$$[C]\varphi \triangleq \Box_{N \setminus C}\varphi.$$

It corresponds to the fact that if the players in C commit to their current strategy, then φ holds whatever other agents do.

4 Knowledge and strategies

In this section, we first explain why the logic of the modalities of knowledge presented so far is different from standard epistemic logic. However, we will see that the modalities $\Box_i\mathbb{K}_i$, $\Box_C\mathbb{C}_C$ and $\Box_C\mathbb{E}_C$ obey the standard principles of epistemic logic. After presenting an alternative account of coalitional power with imperfect information, we show how the logic can model many properties about the interaction between knowledge and power of coalitions.

4.1 A logic of uniform choices

The resulting logic, combining the operators of individual and group knowledge is *not* a standard epistemic logic.

In standard epistemic logic we would expect the formula $\mathbb{E}_{\{i,j\}}\varphi \rightarrow \mathbb{K}_i\varphi$ to be a valid principle. However, it is easy to build a model of games and knowledge such that $\mathbb{E}_{\{i,j\}}\varphi \wedge \neg\mathbb{K}_i\varphi$ or $\mathbb{C}_{\{i,j\}}\varphi \wedge \neg\mathbb{K}_i\varphi$ are satisfied. This is typically the case when $\sim_j \subseteq \sim_i$. This has to do with the evaluation of the operators of knowledge being uniform over the choice of the coalition.

To have a better grasp of the distinction, we can see easily that the formula $\mathbb{E}_C\varphi \leftrightarrow \bigwedge_{k\in C}\mathbb{K}_k\varphi$ is not sound in our models, though a valid principle of standard epistemic logic. It would be if we were to replace the truth value of the operator $\mathbb{E}_C$ by this alternative semantics:

$$M, s, \theta \models_{alt} \mathbb{E}_C\varphi \text{ iff for all } s' \text{ s.t. } s \sim_{\mathbb{E}_C} s' \text{ and for all } \theta' \in \Theta \text{ s.t. } \underline{\theta' \equiv_i \theta \text{ for some } i \in C} \\ \text{we have } M, s', \theta' \models \varphi.$$

But then, it just does not correspond to what we want to express about a group of agents that have mutual knowledge of how to achieve something together. It would merely reflect a notion of mutual knowledge that some agents of the coalition can achieve the state of affairs individually.[1]

In [7], the authors propose the notion of *constructive* knowledge.

> "The agents A *constructively know* that [the coalition B can achieve φ] if they can present a strategy for B that guarantees achieving φ." [7, p. 426]

Constructive knowledge is then a matter of knowing (or identifying) a choice of a group of agents. Within our semantics, it corresponds to a *(epistemically) uniform choice*: the set of pairs moment/valuation such that the moments are indistinguishable by the group and the valuations may only differ by the truth value of the atoms that are not controlled by a member of the group.

Our modalities of knowledge for a coalition C are relativised to the moment, but also to the hypothetical commitment of the players in C: they depend on the current actions (valuations) of the players in C. The notion of knowledge reflected by these operators is one of commitment-dependent knowledge. As we have seen in Examples 1 and 2, the knowledge of Ray and Ann changes when they commit to a particular choice.

[1] An analogous observation can be done for common knowledge.

If we wanted to define mutual knowledge in terms of individual knowledge, we would need to use a *local definition* where, given a particular commitment (here $\pi(\theta_C)$), the formula $\mathbb{E}_C\varphi$ is equivalent to a formula that does not contain $\mathbb{E}_C$ but refers to $\pi(\theta_C)$. The appropriate definition of mutual knowledge is then:

$$\pi(\theta_C) \rightarrow \left(\mathbb{E}_C\varphi \leftrightarrow \bigwedge_{k\in C} \mathbb{K}_k(\pi(\theta_C) \rightarrow \varphi)\right) \qquad (def(E))$$

4.2 The underlying epistemic logic

Since evaluations of formulae are wrt. a N-valuation (strategy profile), the knowledge operators are intended to reflect the fact that agents are committed to their strategies. The notion of knowledge of a player i formalised by the operator $\mathbb{K}_i$ is not a knowledge relative to a moment but rather relative to a pair moment / strategy profile (or equivalently – since the knowledge does not depend on other player's strategy – relative to a moment / player i's strategy). Thus, we are just concerned by equivalent i-valuations in i-indistinguishable moments.

The knowledge of player i relative to a moment can be captured by the defined modality

$$K_i\varphi \triangleq \Box_i\mathbb{K}_i\varphi.$$

It corresponds to the knowledge i has whatever its strategy. In this context, the interpretation of the epistemic modalities resembles Stalnaker's *safe knowledge* [11]: $K_i\varphi$ implies that i knows that φ and continues to know φ if any commitment is taken.[2]

Mutual and common knowledge are defined along the same pattern as individual knowledge. Mutual knowledge relative to a moment is defined:

$$E_C\varphi \triangleq \Box_C\mathbb{E}_C\varphi$$

and common knowledge relative to a moment is defined:

$$C_C\varphi \triangleq \Box_C\mathbb{C}_C\varphi.$$

Proposition 1. *The following standard principles of epistemic logic are sound:*

$$\begin{array}{l} E_C\varphi \leftrightarrow \bigwedge_{k\in C} K_k\varphi \\ (C_C\varphi \wedge C_C(\varphi \rightarrow \psi)) \rightarrow C_C\psi \\ C_C\varphi \rightarrow \varphi \\ C_C\varphi \rightarrow C_C C_C\varphi \\ C_C(\varphi \rightarrow E_C\varphi) \rightarrow (\varphi \rightarrow C_C\varphi) \end{array}$$

4.3 Knowledge and logics of power

In [13], van der Hoek and Wooldridge propose an extension of ATL with knowledge. The logic however cannot distinguish between knowing the mere existence of a strategy to achieve something from *knowing how* to achieve something.

To enable this, the authors of [6] introduce a collection of logics for reasoning about "knowing how to play". They analyse several situations of interaction such as:

[2] We thank Hans van Ditmarsch for suggesting this connection.

1. the agent i has a strategy and he knows that playing it will lead to...
2. the agent i knows that the player j has a strategy...
3. the group A has mutual knowledge of a collective strategy...
4. the group A has common knowledge of a collective strategy...
5. the group A has mutual/common knowledge of a collective strategy of group B such that...

To have a grasp of these notions, the logic ATOL is proposed in [6]. Its language comprises a collection of primitive *ad hoc* operators of the form

$$\langle\langle A\rangle\rangle_{\mathcal{K}_B}\varphi$$

which means that the coalition B has the knowledge of type $\mathcal{K}$ (we use $\mathcal{K}$ as a generic notation for individual, mutual, or common knowledge) to identify a strategy of the coalition A to achieve φ. (Operators are in fact more specific than that as they are also specific to one temporal operator. But we are not concerned with temporal aspects here.)

In [7], the authors propose a logic (CSL) with the same abilities that ATOL but using a much nicer syntax. However, this is done to the price of a non-standard semantics. Indeed, the truth-values of the logic is not relative to a possible world but relative to a set of possible world.

It is shown in [8] that the language of CSL admits a normal form. By restricting the language of CSL to its normal form, it is possible to evaluate the formulae in a more standard way. However, as the authors write, it is a technical trick that is much appreciated to use in theoretical analysis, but a restricted language washes away some of the conceptual impact.

4.4 In our logic

Before going on, we must point out two significant differences with the logics of the previous proposals. First, unlike the proposals in the setting of ATL, our logic does not deal explicitly with time. This may be a drawback but it should be noted that the problems of interaction between strategic behaviour and knowledge in which we are interested to clarify are also present in the one-shot strategies setting of ATL-like logics.

Second, the language of our logic is somewhat built from 'more elementary bricks'. This is all good as it makes it more flexible. We will actually see that we can give definitions of numbers of concepts that make a lot of sense. A major drawback however, is that the formulae can be complex in size. It is not problematic from a modelling point of view as the *defined notions* can (and should) be used. What might be more problematic is that it will have to be *translated back* into the basic language if we need to reason with them. Computer scientists are used to that, though. It is merely like writing in a high level programming language like C or Java and compiling it to obtain a machine code in assembly language.

Finally, maybe the main contribution of the present note is to present a logic that combines reasoning about strategic knowledge and reasoning about game equilibria. (See Section 5.)

Knowing how to play ATL-like logics only deal with what the players can do. In our logic we can talk about what players do. This is related to the STIT logics [1, 4]. Our notions of knowledge are inherently linked to the notion of agency, due to the following valid principles:

$$\mathcal{K}_C\varphi \to [C]\varphi \qquad (inclk)$$

The operators of knowledge $\mathbb{K}_i$, $\mathbb{E}_C$ and $\mathbb{C}_C$ capture a notion of knowledge after a *hypothetical* commitment to a particular strategy.

Players' knowledge is dependent on their hypothetical commitment to a strategy. Back to Example 2, Ann can know that the lamp will be on at the next step by simply committing to her choice of switching the light on. But it does not mean that it will be on, as she still could take the other choice. Hence, the way we model the notion "knowing how to play" in our logic is by counterfactual reasoning. The player somewhat assesses its options ($\Diamond_i$) and see whether it leads to some state of affairs.

Knowing how to play φ is having a uniform choice to bring about φ.

This is reminiscent of the constructs of abilities within the deliberative STIT theory [5]. The formula $[i]\varphi$ (already defined on page 4) represents actual agency of the agent i for φ. It formalises a brute choice of i leading to φ. Ability (or power) for a state of affairs φ is formalised by $\Diamond_i[i]\varphi$ (or $\langle N\setminus\{i\}\rangle[i]\varphi$ in the language of STIT). The power of a coalition is

$$\Diamond_C[C]\varphi.$$

The notion of "knowing how to play" is analogous. It is grasped by the formula

$$\Diamond_C\mathcal{K}_C\varphi$$

It means that the players in C have a group strategy such that if they commit to it, they commonly know that φ. This construct was first used in [3].

Identifying a strategy In some situations, a group C_1 could have the information to identify a strategy of the coalition C_2 such that C_2 would see to it that φ if they play it.

The formula

$$\mathcal{K}_{C_1}(\pi(\theta_{C_2}) \to [C_2]\varphi)$$

says that coalition C_1 can identify (with knowledge of type $\mathcal{K}$) C_2's strategy θ_{C_2} as a winning strategy for φ.

5 Example of application: Knowing how to play a Nash equilibirum

In [12], a variety of strategic equilibria have been formalised: Pareto optimality, core membership, strong Nash equilibrium, etc. Only the solution concept of Nash equilibrium will be used here. In this section we show on an example how to reason about coalitions knowing how to play a Nash equilibrium.

The first step is to define the Nash equilibria in the logical language. It is handy to introduce the notion of (weak) best response by an agent i.

$$WBR_i \triangleq \bigvee_{x \in K} (x \wedge \Box_i \blacklozenge_i x).$$

A player i plays a best response in an N-valuation if, x being the outcome, for every deviation of i, i prefers x.

$$NE \triangleq \bigwedge_{i \in N} WBR_i.$$

A valuation is a Nash equilibrium if every player plays a best response.

Consider now the following voting procedure. There are five possible candidates: two Republicans McCon and Rooney, two Democrats Claton and Obomo and one independent Nadar. The set of moments is

$$S = \{claton, obomo, mccon, rooney, nadar, om, or, cm, cr\}.$$

claton corresponds to a moment in which Claton has been elected, etc. We are not interested in what is going on in these moments. We are more concerned about the moments *om*, *or*, *cm* and *cr* where a vote is supposed to take place. *om* correspond to a moment in which Obomo and McCon have been chosen respectively as the Democrat and the Republican candidate, etc.

There are two voters

$$N = \{Ronald, Donald\}.$$

Ronald's political opinions lean towards those of the Republican party, and *Donald* is some kind of a Democrat. *Ronald* is well informed about the Republican primaries and knows whether Rooney or McCon is candidate for the presidential vote. However, he ignores which one of Obomo or Claton is the Democrat candidate. Symmetrically, *Donald* knows who is the Democrat candidate but not the Republican one.

We have

$$\sim_{Ronald} = \{(om, cm), (or, cr)\}^*$$

and

$$\sim_{Donald} = \{(om, or), (cm, cr)\}^*$$

where * is the equivalence closure.

Each voter controls one atom. Setting it true counts as supporting his custom political party, setting it false counts as voting for the other party:

$$At_{Ronald} = \{rep\} \text{ and } At_{Donald} = \{dem\}.$$

At *om*, *or*, *cm* and *cr*, both voters have to choose between Republican and Democrat. If they both vote for the Democrat party (i.e., valuation $\{dem \mapsto \mathtt{tt}, rep \mapsto \mathtt{ff}\}$), the Democrat candidate is elected. If they both vote for the Republican party (i.e., $\{dem \mapsto \mathtt{ff}, rep \mapsto \mathtt{tt}\}$), the Republican candidate is elected. If they fail to coordinate, Nadar is elected (i.e., $\{dem \mapsto \mathtt{tt}, rep \mapsto \mathtt{tt}\}$ or $\{dem \mapsto \mathtt{ff}, rep \mapsto \mathtt{ff}\}$).

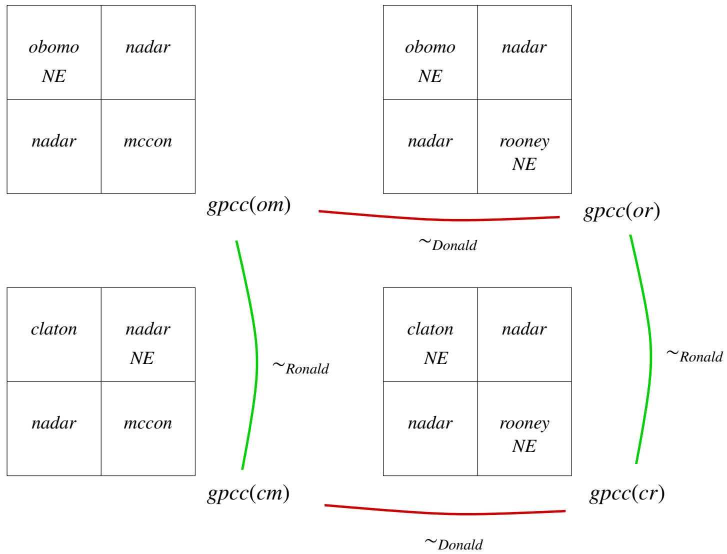

Fig. 1. Representation of the voting procedure. Donald plays rows, Ronald plays columns. In all four strategic games, first row is $dem \mapsto \mathtt{tt}$ and second row is $dem \mapsto \mathtt{ff}$. First column is $rep \mapsto \mathtt{ff}$ and second column is $rep \mapsto \mathtt{tt}$. Preferences are not represented, but each *NE* indicates a Nash equilibrium.

We will make abstraction of the preferences about the moments *om*, *cm*, *or* and *cr*. We are only interested in the preferences about the moments representing the elected candidate of the elections. Suppose *Ronald*'s preferences are as follows:

$$claton =_{Ronald} nadar <_{Ronald} obomo <_{Ronald} rooney <_{Ronald} mccon$$

and *Donald*'s preferences are

$$mccon <_{Donald} nadar =_{Donald} claton <_{Donald} rooney <_{Donald} obomo.$$

The Nash equilibria of the four strategic games that are possible after the primaries are represented by *NE* on Figure 1.

We can verify the following properties of strategy identification and knowing how to play. For all $\theta \in \Theta$ we have:

- *Ronald* cannot know when they are playing a collective strategy leading to a Nash equilibrium.
$$M, or, \theta \not\models \mathbb{K}_{Ronald}[\{Donald, Ronald\}]NE$$
- Nevertheless, he can identify such a strategy.

$$M, or, \theta \models \mathbb{K}_{Ronald}((dem \wedge \neg rep) \rightarrow [\{Donald, Ronald\}]NE)$$

– In fact, *Donald* can identify the same strategy too.

$$M, or, \theta \models \mathbb{K}_{Donald}(dem \wedge \neg rep) \rightarrow [\{Donald, Ronald\}]NE$$

– As a consequence they have the mutual knowledge that the strategy $\{dem \mapsto \texttt{tt}, rep \mapsto \texttt{ff}\}$ leads to a Nash equilibrium.

$$M, or, \theta \models \mathbb{E}_{\{Ronald,Donald\}}((dem \wedge \neg rep) \rightarrow [\{Donald, Ronald\}]NE)$$

– Specifically at $\{dem \mapsto \texttt{tt}, rep \mapsto \texttt{ff}\}$, they mutually know they are playing a Nash equilibrium.

$$M, or, \{dem \mapsto \texttt{tt}, rep \mapsto \texttt{ff}\} \models \mathbb{E}_{\{Ronald,Donald\}}NE$$

– They have the mutual knowledge of how to play to achieve a Nash equilibrium.

$$M, or, \theta \models \Diamond_{\{Ronald,Donald\}}\mathbb{E}_{\{Ronald,Donald\}}NE$$

– However, it is not common knowledge.

$$M, or, \{dem \mapsto \texttt{tt}, rep \mapsto \texttt{ff}\} \not\models \mathbb{C}_{\{Ronald,Donald\}}NE$$

– In fact, they do not have the common knowledge of how to play to achieve a Nash equilibrium.

$$M, or, \theta \not\models \Diamond_{\{Ronald,Donald\}}\mathbb{C}_{\{Ronald,Donald\}}NE$$

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