DELPHIC: Practical DEL Planning via Possibilities

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Abstract. Dynamic Epistemic Logic (DEL) provides a framework for epistemic planning that is capable of representing non-deterministic actions, partial observability, higher-order knowledge and both factual and epistemic change. The high expressivity of DEL challenges existing epistemic planners, which typically can handle only restricted fragments of the whole framework. The goal of this work is to push the envelop of practical DEL planning, ultimately aiming for epistemic planners to be able to deal with the full range of features offered by DEL. Towards this goal, we question the traditional semantics of DEL, defined in terms on Kripke models. In particular, we propose an equivalent semantics defined using, as main building block, so-called *possibilities*: non well-founded objects representing both factual properties of the world, and what agents consider to be possible. We call the resulting framework DELPHIC. We argue that DELPHIC indeed provides a more compact representation of epistemic states. To substantiate this claim, we implement both approaches in ASP and we set up an experimental evaluation to compare DELPHIC with the traditional, Kripke-based approach. The evaluation confirms that DELPHIC outperforms the traditional approach in space and time.

1 Introduction

Multiagent Systems are employed in a wide range of settings, where autonomous agents are expected to face dynamic situations and to be able to adapt in order to reach a given goal. In these contexts, it is crucial for agents to be able to reason on their physical environment as well as on the *knowledge* that they have about other agents and the knowledge those possess.

Bolander and Andersen [5] introduced *epistemic planning* as a planning framework based on Dynamic Epistemic Logic (DEL), where *epistemic states* are represented as Kripke models, *event models* are used for representing epistemic actions, and *product updates* define the application of said actions on states. On the one hand, the resulting framework is very expressive, and it allows one to naturally represent non-deterministic actions, partial observability of agents, higher-order knowledge and both factual and epistemic changes. On the other

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hand, decidability of epistemic planning is not guaranteed in general [5]. This has led to a considerable body of research adopting the DEL framework to obtain (un)decidability results for fragments of the epistemic planning problem (see [6] for a detailed exposition), typically by constraining the event models of actions. Nonetheless, even when such restriction are in place, epistemic planners directly employing the Kripke-based semantics of possible worlds face high complexities, hence considerable efforts have been put in studying action languages that are more amenable computationally [4,10,14].

In contrast with the traditional approach in the literature, in this paper we depart from the Kripke-based semantics for DEL and adopt an alternative representation called *possibilities*, first introduced by Gerbrandy and Groeneveld [12]. As we are going to show experimentally, this choice is motivated primarily by practical considerations. In fact, as we expand in Sect. 3, possibilities support a concise representation of factual and epistemic information and yield a light update operator that promises to achieve better performances compared to the traditional Kripke-based semantics. This is due to the fact that possibilities are *non-well-founded objects*, namely objects that have a *circular* representation (see Aczel [1] for an exhaustive introduction on non-well-founded set theory). In fact, due to their non-well-founded nature, possibilities naturally reuse previously calculated information, thus drastically reducing the computational overhead deriving from redundant information. Conceptually, whenever an agent does not update his knowledge upon an action, then the possibilities representing its knowledge are directly reused (see Examples 3 and 6).

This paper presents a novel formalization of epistemic planning based on possibilities. Although these objects have been previously used in place of Kripke models to represent epistemic states [10], previous semantics lacked a general characterization of actions. In this paper, we complement the possibility-based representation of states by formalizing two novel concepts: *eventualities*, representing epistemic actions, and *union update*, providing an update operator based on possibilities and eventualities. The resulting planning framework, called DELPHIC (*DEL-planning with a Possibility-based Homogeneous Information Characterisation*), benefits from the compactness of possibilities and promises to positively impact the performance of planning. This suggests that DELPHIC is a viable but also convenient alternative to Kripke-based representations. We support this claim by implementing both frameworks in ASP and by setting up an experimental evaluation of the two implementations aimed at comparing the traditional Kripke semantics for DEL and DELPHIC. The comparison confirms that DELPHIC outperforms the traditional approach in terms of both space and time. We point out that time and space gains are obtained in the *average case*, as there exist extreme (*worst*) cases where the two semantics produce epistemic states with the same structure. This follows by the fact that the DELPHIC framework is semantically equivalent to the Kripke-based one (Theorem 1). As a result, the plan existence problems of both frameworks have the same complexity.

Partial evidences of the advantages of adopting possibilities were already experimentally witnessed in [10]. However, the planning framework therein corresponds only to a fragment of the DEL framework. Indeed, as mentioned above, an actual possibility-based formalization of actions is there absent, in favour of a direct, ad-hoc encoding of the transition functions of three prototypical types of actions described in the action language $m\mathcal{A}^*$ [4], namely *ontic*, *sensing* and *announcements* actions. As already mentioned, we overcome this limitation by equipping DELPHIC with eventualities, which we relate to DEL event models.

In conclusion, we provide a threefold contribution: *(i)* we introduce DELPHIC as a general DEL planning framework based on possibilities; *(ii)* we formally show that DELPHIC constitutes an alternative but semantically equivalent framework for epistemic planning, compared to the Kripke-based framework; *(iii)* we experimentally show that the underlying model employed by DELPHIC indeed offers promising advantages in performance, in terms of both time and space.

The paper is organised as follows. In Sect. 2, we recall the necessary preliminaries on DEL; in Sect. 3, we formally define DELPHIC and we show its equivalence with the Kripke-based framework and in Sect. 4 we show our experimental evaluation.

2 Preliminaries

In this section we provide the required preliminaries on DEL [9] by illustrating its fundamental components: epistemic models in Sect. 2.1, event models in Sect. 2.2, and the product update in Sect. 2.3. Although the notion of possibility is part of the preliminaries [12], we defer these to Sect. 3, as this allows us to illustrate the components of DELPHIC by following a similar structure.

2.1 Epistemic Models

Let us fix a countable set $\mathcal{P}$ of propositional atoms and a finite set $\mathcal{AG} = \{1, \ldots, n\}$ of agents. The language $\mathcal{L}_{\mathcal{P},\mathcal{AG}}$ of *multi-agent epistemic logic on $\mathcal{P}$ and $\mathcal{AG}$ with common knowledge/belief* is defined by the following BNF:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box_i \varphi,$$

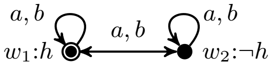
where $p \in \mathcal{P}$, $i \in \mathcal{AG}$, and $G \subseteq \mathcal{AG}$. Formulae of the form $\Box_i \varphi$ are read as "agent i knows/believes that φ". We define the dual operators $\Diamond_i$ as usual. The semantics of DEL formulae is based on the concept of *possible worlds. Epistemic models* are defined as *Kripke models* [15] and they contain both factual information about possible worlds and epistemic information, i.e., which worlds are considered possible by each agent.

Definition 1 (Kripke Model). *A* Kripke model *for $\mathcal{L}_{\mathcal{P},\mathcal{AG}}$ is a triple $M = (W, R, V)$ where:*

- *$W \neq \varnothing$ is the set of possible worlds.*
- *$R : \mathcal{AG} \to 2^{W \times W}$ assigns to each agent i an accessibility relation $R(i)$.*
- *$V : \mathcal{P} \to 2^W$ assigns to each atom a set of worlds.*

We abbreviate the relations $R(i)$ with R_i and use the infix notation wR_iv in place of $(w, v) \in R_i$. An *epistemic state* in DEL is defined as a *multi-pointed Kripke model* (*MPKM*), *i.e.,* as a pair (M, W_d), where $W_d \subseteq W$ is a non-empty set of designated worlds.

Example 1 (Coin in the Box). Agents a and b are in a room where a box is placed. Inside the box there is a coin. None of the agent knows whether the coin lies heads (h) or tails up ($\neg h$). Both agents know the perspective of the other. This is represented by the following MPKM (where the circled bullet represent the designated world).



Definition 2 (Truth in Kripke Models). *Let $M = (W, R, V)$ be a Kripke model, $w \in W$, $i \in \mathcal{AG}$, $p \in \mathcal{P}$ and $\varphi, \psi \in \mathcal{L}^{C}_{\mathcal{P},\mathcal{AG}}$ be two formulae. Then,*

$$
\begin{array}{lll}
(M, w) \models p & \textit{iff} & w \in V(p) \\
(M, w) \models \neg\varphi & \textit{iff} & (M, w) \not\models \varphi \\
(M, w) \models \varphi \wedge \psi & \textit{iff} & (M, w) \models \varphi \textit{ and } (M, w) \models \psi \\
(M, w) \models \Box_i\varphi & \textit{iff} & \forall v \textit{ if } wR_iv \textit{ then } (M, v) \models \varphi
\end{array}
$$

Moreover, $(M, W_d) \models \varphi$ iff $(M, v) \models \varphi$, for all $v \in W_d$.

We recall the notion of bisimulation for MPKMs [7].

Definition 3 (Bisimulation). *A* bisimulation *between MPKMs $((W, R, V), W_d)$ and $((W', R', V'), W'_d)$ is a binary relation $B \subseteq W \times W'$ satisfying:*

- Atoms*: if $(w, w') \in B$, then for all $p \in \mathcal{P}$, $w \in V(p)$ iff $w' \in V'(p)$.*
- Forth*: if $(w, w') \in B$ and wR_iv, then there exists $v' \in W'$ such that $w'R'_iv'$ and $(v, v') \in B$.*
- Back*: if $(w, w') \in B$ and $w'R'_iv'$, then there exists $v \in W$ such that wR_iv and $(v, v') \in B$.*
- Designated*: if $w \in W_d$, then there exists a $w' \in W'_d$ such that $(w, w') \in B$, and vice versa.*

We say that two MPKMs s and s' are *bisimilar* (denoted by $s \underline{\leftrightarrow} s'$) when there exists a bisimulation between them. It is well known that bisimilar states satisfy the same formulae, hence encode the same information.

2.2 Event Models

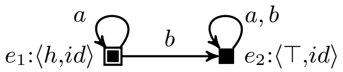
In DEL, actions are modeled by *event models* [3,8], which capture action preconditions and effects from the perspectives of multiple agents at once. Intuitively, *events* represent possible outcomes of the action, accessibility relations describe which events are considered possible by agents, preconditions capture the applicability of events, and postconditions specify how events modify worlds.

Definition 4 (Event Model). *An* event model *for $\mathcal{L}_{\mathcal{P},\mathcal{AG}}$ is a quadruple $\mathcal{E} = (E, Q, pre, post)$ where:*

- *$E \neq \varnothing$ is a finite set of events.*
- *$Q : \mathcal{AG} \rightarrow 2^{E \times E}$ assigns to each agent i an accessibility relation $Q(i)$.*
- *$pre : E \rightarrow \mathcal{L}_{\mathcal{P},\mathcal{AG}}$ assigns to each event a* precondition.
- *$post : E \rightarrow (\mathcal{P} \rightarrow \mathcal{L}_{\mathcal{P},\mathcal{AG}})$ assigns to each event a* postcondition *for each atom.*

We abbreviate $Q(i)$ with Q_i and use the infix notation eQ_if in place of $(e, f) \in Q_i$. An *epistemic action*[1] in DEL is defined as a *multi-pointed event model* (*MPEM*), *i.e.,* as a pair $(\mathcal{E}, E_d)$, where $E_d \subseteq E$ is a non-empty set of designated events. An action is *purely epistemic* if, for each $e \in E$, $post(e)$ is the identity function id; otherwise it is *ontic.*

Example 2. Suppose that, in the scenario of Example 1, agent a peeks inside the box to learn how the coin has been placed while b is distracted. Two events are needed to represent this situation: e_1 (the designated event) represents the perspective of agent a, who is looking inside the box; e_2 represents the fact that agent b does not know what a is doing. In the figure below, a pair $\langle pre(e), post(e) \rangle$ represents the precondition and the postconditions of event e.



We give a notion of bisimulation for actions, which will be needed to show an important relationship with our model.

Definition 5 (Bisimulation for actions). *A* bisimulation *between MPEMs $((E, Q, pre, post), E_d)$ and $((E', Q', pre', post'), E'_d)$ is a binary relation $B \subseteq E \times E'$ satisfying:*

- Formulae*: if $(e, e') \in B$, then $pre(e) = pre'(e')$ and, for all $p \in \mathcal{P}$, $post(e)(p) = post'(e')(p)$.*
- Forth*: if $(e, e') \in B$ and eQ_if, then there exists $f' \in W'$ such that $e'Q'_if'$ and $(f, f') \in B$.*
- Back*: if $(e, e') \in B$ and $e'Q'_if'$, then there exists $f \in W$ such that eQ_if and $(f, f') \in B$.*
- Designated*: if $e \in E_d$, then there exists a $e' \in E'_d$ such that $(e, e') \in B$, and vice versa.*

We say that two MPEMs α and α' are *bisimilar* (denoted by $\alpha \underline{\leftrightarrow} \alpha'$) when there exists a bisimulation between them.

2.3 Product Update

The product update of a MPKM with a MPEM results into a new MPKM that contains the updated information of agents. Here we adapt the definition of van Ditmarsch and Kooi [8] to deal with multi-pointed models. An MPEM $(\mathcal{E}, E_d)$

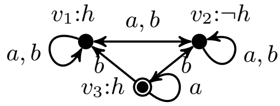
[1] We use "epistemic action" with a broad meaning, simply referring to actions in epistemic planning, irrespective of their effects.

is *applicable* in (M, W_d) if for each world $w \in W_d$ there exists an event $e \in E_d$ such that $(M, w) \models pre(e)$.

Definition 6 (Product Update). *The* product update *of a MPKM* (M, W_d) *with an applicable MPEM* $(\mathcal{E}, E_d)$*, with* $M = (W, R, V)$ *and* $\mathcal{E} = (E, Q, pre, post)$*, is the MPKM* $(M, W_d) \otimes (\mathcal{E}, E_d) = ((W', R', V'), W'_d)$*, where:*

$$\begin{aligned} W' &= \{(w, e) \in W \times E \mid (M, w) \models pre(e)\} \\ R'_i &= \{((w, e), (v, f)) \in W' \times W' \mid wR_iv \text{ and } eQ_if\} \\ V'(p) &= \{(w, e) \in W' \mid (M, w) \models post(e)(p)\} \\ W'_d &= \{(w, e) \in W' \mid w \in W_d \text{ and } e \in E_d\} \end{aligned}$$

Example 3. The product update of the MPKM of Example 1 with the MPEM of Example 2 is the MPKM below, where $v_3 = (w_1, e_1)$, $v_1 = (w_1, e_2)$ and $v_2 = (w_2, e_2)$. Now, agent a knows that the coin lies heads up, while b did not change its perspective. Importantly, notice that w_1 (resp., w_2) and v_1 (resp., v_2) encode the same information, but they are *distinct* objects.



2.4 Plan Existence Problem

We recall the notions of planning task and plan existence problem in DEL [2].

Definition 7 (DEL-Planning Task). *A* DEL-planning task *is a triple* $T = (s_0, \mathcal{A}, \varphi_g)$*, where: (i)* s_0 *is the initial MPKM; (ii)* $\mathcal{A}$ *is a finite set of MPEMs; (iii)* $\varphi_g \in \mathcal{L}^C_{\mathcal{P},\mathcal{AG}}$ *is a* goal formula.

Definition 8. *A* solution *(or* plan*) to a DEL-planning task* $(s_0, \mathcal{A}, \varphi_g)$ *is a finite sequence* $\alpha_1, \ldots, \alpha_\ell$ *of actions of* $\mathcal{A}$ *such that:*

1. $s_0 \otimes \alpha_1 \otimes \cdots \otimes \alpha_\ell \models \varphi_g$*, and*
2. *For each* $1 \leq k \leq \ell$*,* α_k *is applicable in* $s_0 \otimes \alpha_1 \otimes \cdots \otimes \alpha_{k-1}$*.*

Definition 9 (Plan Existence Problem). *Let* $n \geq 1$ *and* $\mathcal{T}$ *be a class of DEL-planning tasks.* PlanEx*(*$\mathcal{T}$*,* n*) is the following decision problem: "Given a DEL-planning task* $T \in \mathcal{T}$*, where* $|\mathcal{AG}| = n$*, does* T *have a solution?"*

3 DELPHIC

We introduce the DELPHIC framework for epistemic planning. DELPHIC is built around the concept of *possibility* (Definition 10), first introduced by Gerbrandy and Groeneveld to represent epistemic states. We develop a novel representation for epistemic actions inspired by possibilities, which we term *eventualities* (Definition 15). Then, we present a novel characterisation of update, called *union update* (Definition 19), based on possibilities and eventualities.

3.1 Possibilities

Possibilities are tightly related to *non-well-founded sets*, *i.e.,* sets that may give rise to infinite *descents* $X_1 \in X_2 \in \dots$ (*e.g.,* $\Omega = \{\Omega\}$ is a n.w.f. set). We refer the reader to Aczel [1] for a detailed account on non-well-founded set theory.

Definition 10 (Possibility). *A* possibility u *for* $\mathcal{L}_{\mathcal{P},\mathcal{AG}}$ *is a function that assigns to each atom* $p \in \mathcal{P}$ *a truth value* $\mathsf{u}(p) \in \{0, 1\}$ *and to each agent* $i \in \mathcal{AG}$ *a* set of possibilities $\mathsf{u}(i)$*, called* information state*.*

Definition 11 (Possibility Spectrum). *A* possibility spectrum *is a finite set of possibilities* $\mathsf{U} = \{\mathsf{u}_1, \dots \mathsf{u}_k\}$ *that we call* designated possibilities*.*

Possibility spectrums represent epistemic states in DELPHIC and are able to represent the same information as MPKMs. Intuitively, each possibility u represent a possible world and the components $\mathsf{u}(p)$ and $\mathsf{u}(i)$ correspond to the valuation function and the accessibility relations of the world, respectively. Finally, the possibilities in a possibility spectrum represent the designated worlds. We formalize this intuition in Proposition 1.

Definition 12 (Truth in Possibilities). *Let* u *be a possibility,* $i \in \mathcal{AG}$*,* $p \in \mathcal{P}$ *and* $\varphi, \psi \in \mathcal{L}^C_{\mathcal{P},\mathcal{AG}}$ *be two formulae. Then,*

$$\begin{array}{ll}
\mathsf{u} \models p & \textit{iff } \mathsf{u}(p) = 1 \\
\mathsf{u} \models \neg\varphi & \textit{iff } \mathsf{u} \not\models \varphi \\
\mathsf{u} \models \varphi \wedge \psi & \textit{iff } \mathsf{u} \models \varphi \textit{ and } \mathsf{u} \models \psi \\
\mathsf{u} \models \Box_i \varphi & \textit{iff } \forall \mathsf{v} \textit{ if } \mathsf{v} \in \mathsf{u}(i) \textit{ then } \mathsf{v} \models \varphi
\end{array}$$

Moreover, $\mathsf{U} \models \varphi$ *iff* $\mathsf{v} \models \varphi$*, for all* $\mathsf{v} \in \mathsf{U}$*.*

Comparing Possibilities and Kripke Models. Gerbrandy and Groeneveld [12] show how possibilities and Kripke models correspond to each other. In what follows, we extend this result by analyzing the relation between possibility spectrums and MPKMs. First, following [12], we give some definitions.

Definition 13 (Decoration of Kripke Model). *The* decoration *of a Kripke model* $M = (W, R, V)$ *is a function* δ *that assigns to each world* $w \in W$ *a possibility* $\mathsf{w} = \delta(w)$*, such that:*

- $\mathsf{w}(p) = 1$ *iff* $w \in V(p)$*, for each* $p \in \mathcal{P}$*;*
- $\mathsf{w}(i) = \{\delta(w') \mid wR_iw'\}$*, for each* $i \in \mathcal{AG}$*.*

Intuitively, decorations provide a link between Kripke-based representations and their equivalent possibility-based ones: given w in M, the decoration of M returns the possibility that encodes w (its valuation and accessibility relation).

Definition 14 (Picture and Solution). *If* δ *is the decoration of a Kripke model* $M = (W, R, V)$ *and* $W_d \subseteq W$*, then* (M, W_d) *is the* picture *of the possibility spectrum* $\mathsf{W} = \{\delta(w) \mid w \in W_d\}$*.* W *is called* solution *of* (M, W_d)*.*

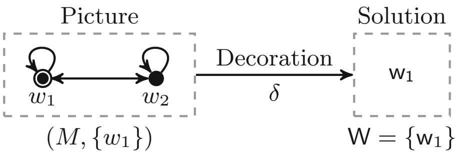


Fig. 1. Relation between picture, decoration and solution.

Namely, the solution of a MPKM (M, W_d) is the possibility spectrum W that contains the possibilities calculated by the decoration function, one for each designated world in W_d. Finally, (M, W_d) is the picture of W. Notice that, in general, *different* MPKMs may share the *same* solution. This observation will be formally stated in Proposition 1. We now give an example (see also Fig. 1).

Example 4. The decoration δ of the MPKM of Example 1 assigns the possibilities $\mathsf{w}_1 = \delta(w_1)$, $\mathsf{w}_2 = \delta(w_2)$. Since $W_d = \{w_1\}$, we have that $\mathsf{W} = \{\mathsf{w}_1\}$ is the solution of (M, W_d), where:

- $\mathsf{w}_1(h) = 1$ and $\mathsf{w}_1(a) = \mathsf{w}_1(b) = \{\mathsf{w}_1, \mathsf{w}_2\}$;
- $\mathsf{w}_2(h) = 0$ and $\mathsf{w}_2(a) = \mathsf{w}_2(b) = \{\mathsf{w}_1, \mathsf{w}_2\}$.

Notice that, in Example 4, although the possibility w_2 is not explicitly part of W, it is "stored" *within* w_1. That is, we do not lose the information about w_2.

Given the above definitions, we are now ready to formally compare possibility spectrums with MPKMs. The following result generalize the one by Gerbrandy and Groeneveld [12, Proposition 3.4]:

Proposition 1.

1. *Each MPKM has a unique decoration;*
2. *Each possibility spectrum has a MPKM as its picture;*
3. *Two MPKMs have the same solution iff they are bisimilar.*

From item 3 of the above Proposition, we obtain the following remark:

Remark 1. Let $s = (M, W_d)$ be a MPKM and let s' be its bisimulation contraction (*i.e.,* the smallest MPKM that is bisimilar to s). Since s and s' share the same solution W, it follows that possibility spectrums naturally provide a more compact representation w.r.t. MPKMs.

Finally, we show that the solution of a MPKM preserves the truth of formulae.

Proposition 2. *Let* (M, W_d) *be a MPKM and let* W *be its solution. Then, for every* $\varphi \in \mathcal{L}_{\mathcal{P},\mathcal{AG}}$, $(M, W_d) \models \varphi$ *iff* $\mathsf{W} \models \varphi$.

Proof. Let δ *be the decoration of* (M, W_d)*. We denote with* $eq(\psi)$ *the fact that* $(M, w) \models \psi$ *iff* $\delta(w) \models \psi$*, for all* $w \in W$.

Consider now $w \in W$ *and let* $\mathsf{w} = \delta(w)$*. We only need to show that* $eq(\varphi)$ *holds for any* $\varphi \in \mathcal{L}_{\mathcal{P},\mathcal{AG}}$*. The proof is by induction of the structure of* φ*. For the base case, let* $\varphi = p$*. By Definition 13, we immediately have that, for any* $p \in \mathcal{P}$ *and* $w \in W$, $(M, w) \models p$ *iff* $\mathsf{w} \models p$ *(*i.e., $eq(p)$*). For the inductive step, we have:*

– *Let* $\varphi=\neg\psi$. *From* $eq(\psi)$ *we get* $eq(\neg\psi)$;
– *Let* $\varphi=\psi_1\wedge\psi_2$. *From* $eq(\psi_1)$, $eq(\psi_2)$ *we get* $eq(\psi_1\wedge\psi_2)$;
– *Let* $\varphi=\Box_i\psi$ *and assume* $eq(\psi)$. *Then we have:*

$$(M,w)\models\Box_i\psi \overset{\textbf{Def. 2}}{\Leftrightarrow} \forall v \textit{ if } wR_iv,\ \textit{then } (M,v)\models\psi$$
$$\overset{\textbf{Def. 13},\, eq(\psi)}{\Leftrightarrow} \forall \mathsf{v} \textit{ if } \mathsf{v}\in\mathsf{u}_i,\ \textit{then } \mathsf{v}\models\psi \overset{\textbf{Def. 12}}{\Leftrightarrow} \mathsf{w}\models\Box_i\psi$$

3.2 Eventualities

In DELPHIC, we introduce the novel concept of *eventuality* to model epistemic actions that is compatible with possibilities. In the remainder of the paper, we fix a fresh propositional atom $pre \notin \mathcal{P}$ and let $\mathcal{P}' = \mathcal{P} \cup \{pre\}$. In the following definition, pre encodes the precondition of an event, while the remaining atoms in $\mathcal{P}$ encode postconditions.

Definition 15 (Eventuality). *An* eventuality e *for* $\mathcal{L}_{\mathcal{P},\mathcal{AG}}$ *is a function that assigns to each atom* $p' \in \mathcal{P}'$ *a formula* $\mathsf{e}(p') \in \mathcal{L}_{\mathcal{P},\mathcal{AG}}$ *and to each agent* $i \in \mathcal{AG}$ *a* set of eventualities $\mathsf{e}(i)$, *called* information state.

Note that an eventuality is essentially a possibility that associates to each atom a formula (instead of a truth value).

Definition 16 (Eventuality Spectrum). *An* eventuality spectrum *is a finite set of eventualities* $\mathsf{E}=\{\mathsf{e}_1,\dots \mathsf{e}_k\}$ *that we call* designated eventualities.

Eventuality spectrums represent epistemic actions in DELPHIC. Moreover, we can easily show that they are able to represent the same information as MPEMs. Intuitively, each eventuality e represents an event and the components $\mathsf{e}(pre)$ and $\mathsf{e}(p)$ represent the precondition and the postconditions of the event, respectively. Finally, the eventualities in an eventuality spectrum represent the designated events. We formalize this intuition in Proposition 3.

Comparing Eventualities and Event Models. We now analyze the relationship between eventuality spectrums and MPEMs. We introduce the notions of decoration, picture and solution for event models.

Definition 17 (Decoration of an Event Model). *The* decoration *of an event model* $\mathcal{E} = (E, Q, pre, post)$ *is a function* δ *that assigns to each* $e \in E$ *an eventuality* $\mathsf{e} = \delta(e)$, *where:*

– $\mathsf{e}(pre) = pre(e)$ *and* $\mathsf{e}(p) = post(e)(p)$, *for each* $p \in \mathcal{P}$;
– $\mathsf{e}(i) = \{\delta(e') \mid eQ_ie'\}$, *for each* $i \in \mathcal{AG}$.

Definition 18 (Picture and Solution). *If* δ *is the decoration of an event model* $\mathcal{E} = (E, Q, pre, post)$ *and* $E_d \subseteq E$, *then* $(\mathcal{E}, E_d)$ *is the* picture *of the eventuality spectrum* $\mathsf{E} = \{\delta(e) \mid e \in E_d\}$ *and* E *is the* solution *of* $(\mathcal{E}, E_d)$.

The above definitions are the counterparts of the notions of decoration and picture given in Definitions 13 and 14.

Example 5. The decoration δ of the MPEM of Example 2 assigns the eventualities $\mathsf{e}_1 = \delta(e_1)$ and $\mathsf{e}_2 = \delta(e_2)$. Since $E_d = \{e_1\}$, we have that $\mathsf{E} = \{\mathsf{e}_1\}$ is the solution of $(\mathcal{E}, E_d)$, where:

- $\mathsf{e}_1(pre) = h$; $\mathsf{e}_1(h) = h$; $\mathsf{e}_1(a)=\{\mathsf{e}_1\}$ and $\mathsf{w}_1(b)=\{\mathsf{w}_2\}$;
- $\mathsf{e}_2(pre) = \top$; $\mathsf{e}_2(h) = h$; $\mathsf{e}_2(a) = \mathsf{w}_2(b) = \{\mathsf{e}_2\}$.

The following results formally compare eventuality spectrums with MPEMs.

Proposition 3.

- *Each MPEM has a unique decoration;*
- *Each eventuality spectrum has a MPEM as its picture;*
- *Two MPEMs have the same solution iff they are bisimilar.*

Thus, analogously to the case of possibility spectrums, we can see that eventuality spectrums provide us with a compact representation of epistemic actions.

3.3 Union Update

We are now ready to present the novel formulation of update of DELPHIC. We say that an eventuality e is *applicable* in a possibility u iff $\mathsf{u} \models \mathsf{e}(pre)$. Then, an eventuality spectrum E is *applicable* in a possibility spectrums U iff for each $\mathsf{u} \in \mathsf{U}$ there exists an applicable eventuality $\mathsf{e} \in \mathsf{E}$.

Definition 19 (Union Update). *The* union update *of a possibility* u *with an applicable eventuality* e *is the possibility* $\mathsf{u}' = \mathsf{u} \uplus \mathsf{e}$*, where:*

$$\mathsf{u}'(p) = 1 \textit{ iff } \mathsf{u} \models \mathsf{e}(p)$$
$$\mathsf{u}'(i) = \{\mathsf{v} \uplus \mathsf{f} \mid \mathsf{v} \in \mathsf{u}(i), \mathsf{f} \in \mathsf{e}(i) \textit{ and } \mathsf{v} \models \mathsf{f}(pre)\}$$

The union update *of a possibility spectrum* U *with an applicable eventuality spectrum* E *is the possibility spectrum*

$$\mathsf{U} \uplus \mathsf{E} = \{\mathsf{u} \uplus \mathsf{e} \mid \mathsf{u} \in \mathsf{U}, \mathsf{e} \in \mathsf{E} \textit{ and } \mathsf{u} \models \mathsf{e}(pre)\}.$$

Example 6. The union update of the possibility spectrum W of Example 4 with the eventuality spectrum of Example 5 is $\mathsf{W} \uplus \mathsf{E} = \{\mathsf{w}_1 \uplus \mathsf{e}_1\} = \{\mathsf{v}_3\}$, where $\mathsf{v}_3(h) = 1$, $\mathsf{v}_3(a) = \{\mathsf{v}_3\}$ and $\mathsf{v}_3(b) = \{\mathsf{w}_1 \uplus \mathsf{e}_2, \mathsf{w}_2 \uplus \mathsf{e}_2\} = \{\mathsf{w}_1, \mathsf{w}_2\}$.

Notice that, since $\mathsf{w}_1 \uplus \mathsf{e}_2 = \mathsf{w}_1$ and $\mathsf{w}_2 \uplus \mathsf{e}_2 = \mathsf{w}_2$ the union update allows to reuse previously calculated information.

Comparing Union Update and Product Update. Intuitively, it is easy to see that the possibility spectrum of Example 6 represents the same information of the MPKM of Example 3. We formalize this intuition with the following lemma, witnessing the equivalence between product and union updates (full proof in the arXiv Appendix).

Lemma 1. *Let* $(\mathcal{E}, E_d)$ *be a MPEM applicable in a MPKM* (M, W_d)*, with solutions* E *and* W*, respectively. Then the possibility spectrum* $\mathsf{W}' = \mathsf{W} \uplus \mathsf{E}$ *is the solution of* $(M', W'_d) = (M, W_d) \otimes (\mathcal{E}, E_d)$.

3.4 Plan Existence Problem in DELPHIC

We conclude this section by giving the definitions of planning task and plan existence problem in DELPHIC.

Definition 20 (DELPHIC-Planning Task). *A* DELPHIC-planning task *is a triple* $T = (\mathsf{W}_0, \Sigma, \varphi_g)$*, where: (i)* W_0 *is an initial possibility spectrum; (ii)* Σ *is a finite set of eventuality spectrums; (iii)* $\varphi_g \in \mathcal{L}^C_{\mathcal{P},\mathcal{AG}}$ *is a* goal formula.

Definition 21. *A* solution *(or* plan*) to a* DELPHIC*-planning task* $(\mathsf{W}_0, \Sigma, \varphi_g)$ *is a finite sequence* $\mathsf{E}_1, \ldots, \mathsf{E}_\ell$ *of actions of* Σ *such that:*

1. $\mathsf{W}_0 \bowtie \mathsf{E}_1 \bowtie \ldots \bowtie \mathsf{E}_\ell \models \varphi_g$*, and*
2. *For each* $1 \leq k \leq \ell$*,* E_k *is applicable in* $\mathsf{W}_0 \bowtie \mathsf{E}_1 \bowtie \ldots \bowtie \mathsf{E}_{\mathsf{k}-1}$.

Definition 22 (Plan Existence Problem). *Let* $n \geq 1$ *and* $\mathcal{T}$ *be a class of* DELPHIC*-planning tasks.* PLANEX*($\mathcal{T}$, n) is the following decision problem: "Given a* DELPHIC*-planning task* $\mathsf{T} \in \mathcal{T}$*, where* $|\mathcal{AG}| = n$*, does* T *have a solution?"*

From Lemma 1, we immediately get the following result:

Theorem 1. *Let* $T = (s_0, \mathcal{A}, \varphi_g)$ *be a DEL-planning task and let* $\mathsf{T} = (\mathsf{W}_0, \Sigma, \varphi_g)$ *be a* DELPHIC*-planning task such that* W_0 *is the solution of* s_0 *and* Σ *is the set of solutions of* $\mathcal{A}$*. Then,* $\alpha_1, \ldots, \alpha_\ell$ *is a plan for* PLANEX(T, n) *iff* $\mathsf{E}_1, \ldots, \mathsf{E}_\ell$ *is a plan for* PLANEX(T, n)*, where* E_i *is the solution of* α_i*, for each* $1 \leq i \leq \ell$.

4 Experimental Evaluation

In this section, we describe our experimental evaluation of the Answer Set Programming (ASP) encodings of DELPHIC and of the traditional Kripke semantics for DEL. Due to space constraints, we provide a brief overview of the encodings[2] (the full presentation can be found in the arXiv Appendix).

The aim of the evaluation is to compare the semantics of DELPHIC and the traditional Kripke-based one in terms of both time and space. We do so by testing the encodings on epistemic planning benchmarks collected from the literature[3] (*e.g., Collaboration and Communication*, *Grapevine* and *Selective Communication*). Time and space performances are respectively evaluated on the total solving time (given in seconds) and the grounding size (*i.e.,* the number of ground ASP atoms) provided by the ASP-solver *clingo* output statistics. We now describe the encodings (Sect. 4.1) and discuss the obtained results (Sect. 4.2).

[2] The full code and documentation of the ASP encodings are available at https://github.com/a-burigana/delphic_asp.

[3] Due to space limits, the description of the benchmarks is delegated to the arXiv Appendix. All benchmarks are available at https://github.com/a-burigana/delphic_asp.

4.1 ASP Encodings

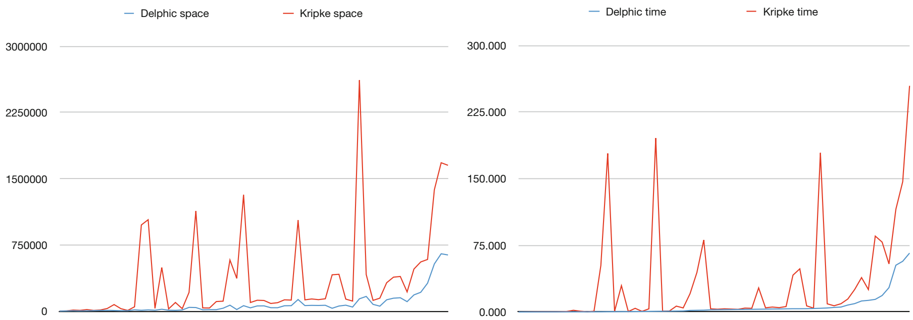
Since our goal is to achieve a fair comparison the two semantics, we implemented a baseline ASP encoding for both of them. Although optimizations for both encoding are possible, the baseline implementations are sufficient to show our claim. Towards the goal of a fair and transparent comparison, we opted for a declarative language such as ASP (notice that, as our goal is simply to compare the two baselines, the choice of an alternative declarative language would make little difference). In fact, while imperative approaches would render the comparison less clear, as one would need to delve into opaque implementation details, ASP allows to write the code that is transparent and easy to analyze. In fact, the two ASP encodings are very similar, since the representation of DELPHIC objects (possibility/eventuality spectrums) and DEL objects (MPKMs/MPEMs) closely mirror each other. The only difference is in the two update operators (*i.e.,* union update and product update). This homogeneity is instrumental to obtain a fair experimental comparison of the two encodings.

We now briefly describe our encodings, assuming that the reader is familiar with the basics concepts of ASP. The two encodings were developed by following the formal definitions of DELPHIC and DEL objects (possibility/eventuality spectrums and MPKMs/MPEMs) and update operators (union and product update) introduced in the previous sections. To increase the efficiency of the solving and grounding phases, the two encodings make use of the *multi-shot* solving approach provided by the ASP-solver *clingo*, which allows for a fine-grained control over grounding and solving of ASP programs. Specifically, this approach allows one to divide an ASP encoding into sub-programs, then handling grounding and solving of these sub-programs separately. In particular, this technique is useful to implement *incremental solving*, which, at each time step, allows to extend the ASP program in order to look for solutions of increasing size. Intuitively, every step mimics a Breadth-First Search over the planning state space: at each time step `t`, if a solution is not found (*i.e.,* there is no plan of length `t` that satisfies the goal), the ASP program is expanded to look for a longer plan. For a detailed introduction on multi-shot ASP, we refer the reader to [11,13].

Finally, to visually witness the compactness that possibility spectrums provide w.r.t. MPKMs (see Remark 1), we exploited the Python API offered by *clingo* to implement a graphical representation of the epistemic states visited by the planner. This provides an immediate way of concretely compare the size of output of the two encodings on a given domain instance. Due to space reasons, we report an example of graphical comparison in the arXiv Appendix.

4.2 Results

We ran our test on a 1.4 GHz Quad-Core Intel Core i5 machine with 8 GB of memory and with a macOS 12.6 operating system and using *clingo* version 5.6.2 with timeout (t.o.) of 10 min. The results are shown in Fig. 2. Space and time results are expressed in number of ASP atoms and in seconds, respectively. The comparison clearly shows that the DELPHIC encoding outperforms the one based



(a) Space results

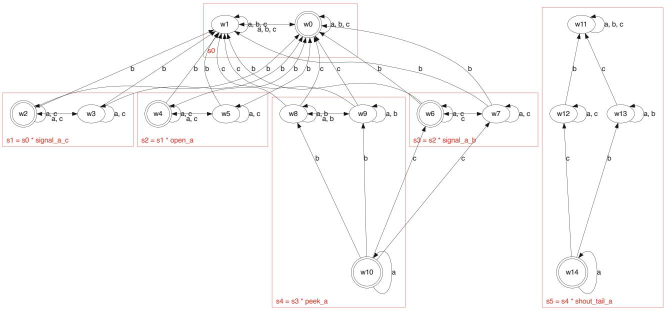
(b) Time results

Fig. 2. Results of the evaluation of the DELPHIC and Kripke encodings.

on the traditional Kripke semantics both in terms of space and time. As shown in Fig. 2.a, the number of ASP atoms produced by the DELPHIC encoding is smaller than the ones produced by the Kripke-based ones. The "spikes" witnessed in the latter case are found in presence of instances with longer solutions. This indicates that DELPHIC scales much better in terms of plan length than the traditional Kripke-semantics. In turn, this is positively reflected by the time results graph. In fact, observing space and time results together, we can see how the growth of the size of the epistemic states negatively affects the planning process in terms of time performances. This concretely shows that possibilities can be exploited to achieve more efficient planning tools, thus allowing epistemic planners to be able to deal with the full range of features offered by DEL.

We now analyze the results in detail. The central factor that contributes to the performance gains of DELPHIC is the fact that possibilities allow for a more efficient use of space during the computation of a solution. Specifically, this efficiency results from two key aspects. First, as shown in Remark 1, possibility spectrums are able to represent epistemic information in a more compact way. Working with compact objects contributes significantly to reducing the size of epistemic states after sequences of updates. Second, as shown in Example 6, possibilities naturally allow to reuse previously calculated information (*i.e.,* other possibilities that were calculated in previous states). We give a more concrete example of this property in Fig. 3, that shows a sequence of epistemic states (surrounded by rectangles) from a generalization of the Coin in the Box domain of Example 1. We clearly see how the possibilities w_0 and w_1 are *reused* in the epistemic states s_1, s_2, s_3 and s_4. The space efficiency provided by DELPHIC is clearly witnessed in Fig. 2.a. In presence of instances with longer solutions, DELPHIC outperforms the Kripke-based representation, as the latter requires a considerable amount of space to compute a solution (*i.e.,* the spikes of the graph).

The space efficiency of DELPHIC is directly reflected on time performances. Indeed, in Fig. 2.b are shown the same peaks in correspondence of instances with longer solutions. As a result, we can conclude that the DELPHIC framework



w0	-	looking_a, -looking_b, -looking_c, -opened, tail
w1	-	looking_a, -looking_b, -looking_c, -opened, -tail
w2	(w0, sig)	looking_a, -looking_b, looking_c, -opened, tail
w3	(w1, sig)	looking_a, -looking_b, looking_c, -opened, -tail
w4	(w2, sig)	looking_a, -looking_b, looking_c, opened, tail
w5	(w3, sig)	looking_a, -looking_b, looking_c, opened, -tail
w6	(w4, sig)	looking_a, -looking_b, looking_c, opened, tail
w7	(w5, tau)	looking_a, -looking_b, looking_c, opened, -tail
w8	(w0, sig)	looking_a, looking_b, -looking_c, -opened, tail
w9	(w1, sig)	looking_a, looking_b, -looking_c, -opened, -tail
w10	(w6, sig)	looking_a, looking_b, looking_c, opened, tail
w11	(w0, sig)	looking_a, -looking_b, -looking_c, -opened, tail
w12	(w6, sig)	looking_a, -looking_b, looking_c, opened, tail
w13	(w8, sig)	looking_a, looking_b, -looking_c, -opened, tail
w14	(w10, sig)	looking_a, looking_b, looking_c, opened, tail

Fig. 3. Reuse of previously calculated information in DELPHIC. This figure was obtained by running the terminal command `python delphic.py -i exp/CB/instance__pl_5.lp --print` (see https://github.com/a-burigana/delphic_asp for the complete documentation).

allows for a more scalable implementation both in terms of space and time performances. Finally, we point out that the analyzed performance gains are obtained in the *average case*, as there exist extreme (*worst*) cases where the two semantics produce epistemic states with the same structure. In fact, we recall that the DELPHIC framework is semantically equivalent to the Kripke-based one (Theorem 1). Thus, we can conclude that DELPHIC provides a practical and usable framework for DEL planning that can be exploited to tackle a wide range of concrete epistemic planning scenarios.

We close this section by noting that a similar, but less general result, was obtained by Fabiano et al. [10], where a possibility-based semantics is compared to the traditional Kripke-based one on a fragment of DEL called $m\mathcal{A}^*$ [4], that allows three kinds of actions, *i.e.*, *ontic*, *sensing* and *announcement* actions. Since DELPHIC is equivalent to the full DEL framework (see Theorem 1), our comparison indeed provides a generalization of the claim made by Fabiano et al.

5 Conclusions

We have introduced a novel epistemic planning framework, called DELPHIC, based on the formal notion of possibility, in place of the more traditional Kripke-based DEL representation. We have formally shown that these two frameworks are semantically equivalent. Possibilities provide a more compact representation of epistemic states, in particular by reusing common information across states. To show the benefits of possibilities, we have implemented DELPHIC and the Kripke-based approach in ASP, performing a comparative experimental evaluation with known benchmark domains. The results show that DELPHIC indeed outperforms the Kripke-based approach both in terms of space and time performances, and is thus a good candidate for practical DEL planning.

In the future, we plan to exploit the performance gains provided by the DELPHIC semantics in more competitive implementations based on C++. An interesting avenue of work is to deepen our analysis of possibility-based succinctness on fragments of DEL, where only a set of specific types of actions are allowed (*e.g.,* the language $m\mathcal{A}^*$ [4] and the framework by Kominis and Geffner [14]).

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