

Hidden Markov Models

1. Weather Prediction

Assume we need to predict Weather conditions of being either sunny, cloudy, or raining at a certain time point. However, many factors influence the Weather. Specifically, we only consider high and low pressure to affect the weather. Assume that low pressure causes cloudy and rainy Weather with the same probability of 0.4 while sunny Weather have 0.2 of being affected by low pressure. By contrast, high pressure causes sunny, rainy, and cloudy Weather with probability of 0.6,0.3,0.1 respectively. Note that high pressure can go down with probability of 0.7. Once low, it can go up with probability of 0.4

1. Represent the weather situation using a Hidden Markov Model
2. How this model can be used for predicting Weather? give an example of each scenario you propose.

2. Chocolate Factory

Imagine that you work at a Chocolate Factory. Your job is to keep an eye on the conveyor belt, watching the chocolates as they come out of the press one at a time. Suppose that the factory makes two types of chocolates: ones with almonds and ones without. For the first few problems, assume you can tell with 100% accuracy what the chocolate contains. In the control room, there is a lever that switches the almond control on and off. When the conveyor is turned on at the beginning of the day, there is a 50% chance that the almond lever is on, and a 50% chance that it is off. As soon as the conveyor belt is turned on, it starts making a piece of candy. Unfortunately, someone has let a monkey loose in the control room, and it has locked the door and started the conveyor belt. The lever cannot be moved while a piece of candy is being made. Between pieces, however, there is a 30% chance that the monkey switches the lever to the other position (i.e. turns almonds on if it was off, or off if it was on).

1. Draw a Markov Model that represents the situation and give the prior distribution on the states as well as the transition matrix.
2. Now assume that there is a coconut lever as well, so that there are four types of candy: Plain, Almond, Coconut, and Almond+Coconut. Again, there is to 50% chance of the lever being on at the beginning of the day, and the chance of the monkey switching the state of the second lever between candies is also 30%. Assume that the switching of the levers is independent of each other. Now Draw a model for production of all four types of chocolate.
3. What is the probability that the machine will produce, in order: Plain, Almond, Almond, Almond+Coconut?
4. Give an example of a set of hidden states that could be associated with the Markov Model you have defined earlier?

3. Word recognition



Imagine you work at an online Website and you are asked to develop a tool for decoding CAPTCHAs. How would you solve the problem of word recognition in the following cases:

1. assuming all characters are separated
2. assuming you are given a lexicon of words