(Description) Logics

for Information Modelling and Access

Enrico Franconi

`franconi@cs.man.ac.uk`

`http://www.cs.man.ac.uk/~franconi`

Department of Computer Science, University of Manchester

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Summary

- Description Logics
- The role of logics in Information Systems
- Conceptual Modelling and Query Management

Description Logics – the standard view

- Expressive decidable fragments of (first-order) classical logic
- Close correspondence with modal logics (e.g., $\mathcal{ALC}$ vs. $\mathbf{K}$)
- Sound and complete algorithms implemented in efficient reasoners
- Knowledge representation formalism derived by semantic networks and frames in Artificial Intelligence
- Close correspondence with well known database conceptual data models

Knowledge representation is about objects

Description logics describe classes of objects (concepts) and their inter-relationships (roles).

The $\mathcal{ALC}$ concept expression

$$\mathtt{Professor} \sqcap \exists \mathtt{TEACHES}.\, \mathtt{UG\text{-}Course} \sqcap \forall \mathtt{TEACHES}.\, \mathtt{CS\text{-}Course}$$

corresponds to the $\mathbf{K}$ formula

$$Professor \wedge \Diamond UG\text{-}Course \wedge \Box CS\text{-}Course$$

where the accessibility relation is interpreted as the *TEACHES* relation

Description Logics are multi-modal

The $\mathcal{ALC}$ concept expression

$\texttt{Professor} \sqcap \exists\texttt{TEACHES}.\,\texttt{UG-Course} \sqcap \exists\texttt{DEGREE}.\,\texttt{Bs}$

corresponds to the $\mathbf{K_m}$ formula (over the same object domain)

$Professor \wedge \Diamond_{TEACHES}\, UG\text{-}Course \wedge \Diamond_{DEGREE}\, Bs$

Modalities (as roles) may have different properties

The $\mathcal{ALC}$ concept expression

$$\texttt{Professor} \sqcap \exists \texttt{TEACHES}.\, \texttt{UG-Course} \sqcap$$
$$\exists \texttt{IS-PART}.\, (\texttt{Staff} \sqcap \exists \texttt{IS-LOCATED}.\, \texttt{Department})$$

corresponds to the $\mathbf{K_m} \cup \mathbf{K4_m}$ formula (over the same object domain)

$$Professor \wedge \Diamond_{TEACHES} UG\text{-}Course \wedge$$
$$\Diamond_{IS\text{-}PART}(Staff \wedge \Diamond_{IS\text{-}LOCATED} Department)$$

where *TEACHES* is a $\mathbf{K_m}$ modality and *IS-PART*, *IS-LOCATED* are $\mathbf{K4_m}$ modalities

Relational structures

- “Modal logics are not appropriate as a representational tool since they do not always capture the details of the models”
- Do we care?

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  Why?

Relational structures

- “Modal logics are not appropriate as a representational tool since they do not always capture the details of the models”
- Do we care?
- No: this is not a bug, it is a feature!
  Why?
- If a formula is satisfiable in a model, it is also satisfiable in a model with the indistinguishable property. Hence, reasoning is not affected.

Additional genuine modalities

- Time, space, belief, etc: combination of modal logics over *distinct* domains (i.e., the object and the modal domains)
- Example of $\mathcal{ALC} \cup \mathsf{LTL}$ concept expression:

  $\texttt{Professor} \sqcap \Diamond(\exists \texttt{TEACHES}.\, \texttt{UG-Course}) \sqcap \forall \Box \texttt{TEACHES}.\, \texttt{CS-Course}$
- Asymmetric extension

Global axioms

$$
\begin{aligned}
\exists \mathtt{TEACHES}.\,\mathtt{Course} &\sqsubseteq (\mathtt{Student} \sqcap \exists \mathtt{DEGREE}.\,\mathtt{Bs}) \sqcup \mathtt{Prof} \\
\mathtt{Prof} &\sqsubseteq \exists \mathtt{DEGREE}.\,\mathtt{Ms} \\
\exists \mathtt{DEGREE}.\,\mathtt{Ms} &\sqsubseteq \exists \mathtt{DEGREE}.\,\mathtt{Bs} \\
\mathtt{Ms} \sqcap \mathtt{Bs} &\sqsubseteq \bot
\end{aligned}
$$

- Axioms should be satisfied by each object in the domain
- Satisfiability and logical implication in $\mathcal{ALC}$ ($\mathbf{K_m}$) become EXPTIME-complete

Global axioms, II

- $\mathbf{K}_\mathbf{m}^{\mathcal{H}}$ extends $\mathbf{K}_\mathbf{m}$ with statements on inclusions between modalities
- Decision problems for $\mathbf{K}_\mathbf{m}^{\mathcal{H}}$ and $\mathbf{K4}_\mathbf{m}$ are in PSPACE
- The *universal modality* can be encoded in $\mathbf{K}_\mathbf{m}^{\mathcal{H}} \cup \mathbf{K4}_\mathbf{m}$, and axioms can be internalised:
  - Define new *transitive* modality U that *includes* all other modalities
  - Satisfiability of ϕ w.r.t. $\psi_1 \rightarrow \varphi_1, \ldots, \psi_n \rightarrow \varphi_n$ is equivalent to satisfiability of $\phi \wedge \Box_U((\psi_1 \rightarrow \varphi_1) \wedge \ldots \wedge (\psi_n \rightarrow \varphi_n))$
- Satisfiability and logical implication in $\mathbf{K}_\mathbf{m}^{\mathcal{H}} \cup \mathbf{K4}_\mathbf{m}$ are EXPTIME-complete
- FaCT implements $\mathbf{K}_\mathbf{m}^{\mathcal{H}} \cup \mathbf{K4}_\mathbf{m}$

n-ary Relations

- Relations between objects in the world may necessarily involve more than just two objects
- Full fledged relational structures are needed, beyond Kripke structures
- We want to maintain the modal logic flavour
- $\mathcal{DLR}$ properly extends $\mathcal{ALC}$ with n-ary relations

$\mathcal{DLR}$

$$R \rightarrow \top_n \mid RN \mid \neg R \mid R_1 \sqcap R_2 \mid R_1 \sqcup R_2 \mid U_i/n : C$$

$$C \rightarrow \top \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid C_1 \sqcup C_2 \mid \exists[U_i]R \mid \exists^{\leqslant k}[U_i]R$$

$\texttt{Works-for} \sqsubseteq \texttt{subj}/2 : \texttt{Employee} \sqcap \texttt{obj}/2 : \texttt{Project}$

$\texttt{Manager} \sqsubseteq \texttt{Employee} \sqcap \neg\exists[\texttt{subj}]\texttt{Works-for}$

$\mathcal{DLR}$ includes $\mathcal{ALCQI}$: if R is a binary relation (i.e., a *role*) with named attributes *first* and *second* then

$\exists R.\, C \equiv \exists[\textit{first}](R \sqcap (\textit{second}/2 : C))$

Reasoning in $\mathcal{DLR}$ is EXPTIME-complete

$\mathcal{DLR}$ syntax	$\mathcal{DLR}$ semantics	RD encoding
$\top_n$	$\top_n^{\mathcal{I}} \subseteq (\Delta^{\mathcal{I}})^n$	$\top_n$
RN	$\mathit{RN}^{\mathcal{I}} \subseteq \top_n^{\mathcal{I}}$	C_{RN}
$\neg R$	$\top_n^{\mathcal{I}} \setminus R^{\mathcal{I}}$	$\neg C_R$
$R_1 \sqcap R_2$	$R_1{}^{\mathcal{I}} \cap R_2{}^{\mathcal{I}}$	$C_{R_1} \sqcap C_{R_2}$
$U_i/n : C$	$\{\langle d_1, \ldots, d_n \rangle \in \top_n^{\mathcal{I}} \mid d_i \in C^{\mathcal{I}}\}$	$\top_n \sqcap \forall U_i .\, C$
$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$	$\top$
CN	$\mathit{CN}^{\mathcal{I}} \subseteq \top^{\mathcal{I}}$	CN
$\neg C$	$\top^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$\neg C$
$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	$C_1 \sqcap C_2$
$\exists[U_i]R$	$\{d \in \top^{\mathcal{I}} \mid \exists \langle d_1, \ldots, d_n \rangle \in R.\ d_i = d\}$	$\exists U_i^- .\, C_R$
$\exists^{\leqslant k}[U_i]R$	$\{d \in \top^{\mathcal{I}} \mid \sharp\{\langle d_1, \ldots, d_n \rangle \in R \mid d_i = d\} \leqslant k\}$	$\leqslant k U_i^- .\, C_R$

Encoding conceptual data models in $\mathcal{DLR}$

- Object-oriented data models (e.g., UML and ODMG)
- Semantic data models (e.g., EER and ORM)
- Frame-based ontology languages (e.g., OIL)

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- Theorems **prove** that a conceptual schema and its encoding as $\mathcal{DLR}$ inclusion dependencies constrain every database state in the same way – i.e., the models of the $\mathcal{DLR}$ theory correspond to the legal database states of the conceptual schema, and vice-versa.

Classical Integrity Constraints in $\mathcal{DLR}$

- arbitrary *boolean constructs*
- *unary inclusion* dependencies (e.g., referential integrity)
- special forms of *typed inclusion* dependencies
- *existence* and *exclusion* dependencies
- *unary functional* dependencies
- *view definitions*

Extensions of $\mathcal{DLR}$

$\mathcal{DLR}_{reg}$: regular expressions and recursive views (beyond FOL)

$\mathcal{DLR}_{\mathcal{US}}$: combination with temporal constructs to model temporal databases

$\mathcal{DLR}_{key}$: general key constraints

Queries under $\mathcal{DLR}$ constraints

- A query is an open FOL formula, whose predicates may be constrained by a $\mathcal{DLR}$ theory
- We consider only the conjunctive existential fragment (the conjunctive queries, or non-recursive datalog queries)
- Example:

$$\begin{aligned}\mathtt{Q}_1(\mathtt{x},\mathtt{y}) \text{ :- } & (\neg\mathtt{Professor})(\mathtt{x}) \wedge \mathtt{TEACHES}(\mathtt{x},\mathtt{y}) \wedge \\ & (\mathtt{UG\text{-}Course} \sqcup \mathtt{CS\text{-}Course})(\mathtt{x})\end{aligned}$$

Semantics of Evaluation and Containment

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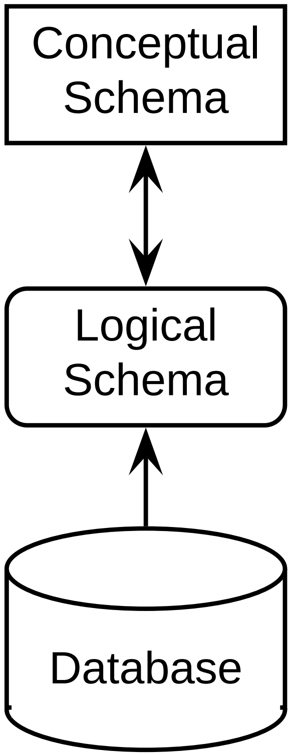
Containment of disjunctions of conjunctive queries under $\mathcal{DLR}$ ($\mathcal{DLR}_{\mathcal{US}}$) constraints is decidable in 2EXPTIME

The i•com tool for Intelligent Conceptual Modelling

- i•com is an advanced CASE tool which allows the user to design *multiple* extended Entity-Relationship schemas or UML class diagrams with inter- and intra-schema *constraints*.
- Complete logical reasoning is employed by the tool to:
  - verify the specification,
  - infer implicit facts,
  - devise stricter constraints,
  - and manifest any local inconsistency.

`http://www.cs.man.ac.uk/~franconi/icom/`

The role of logics in Information Systems



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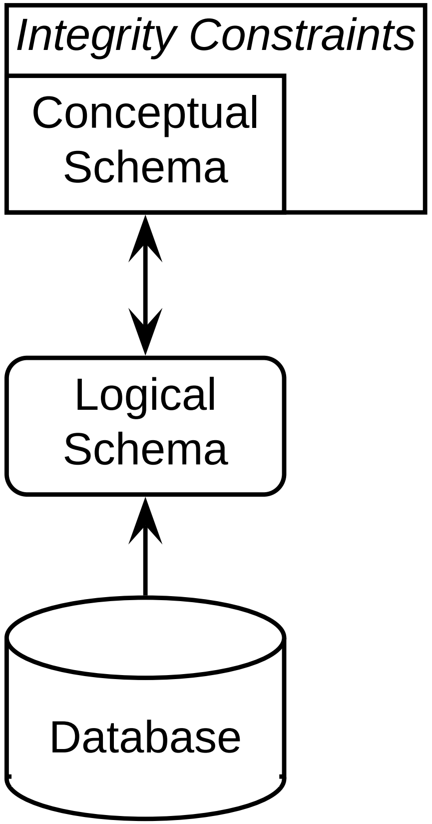

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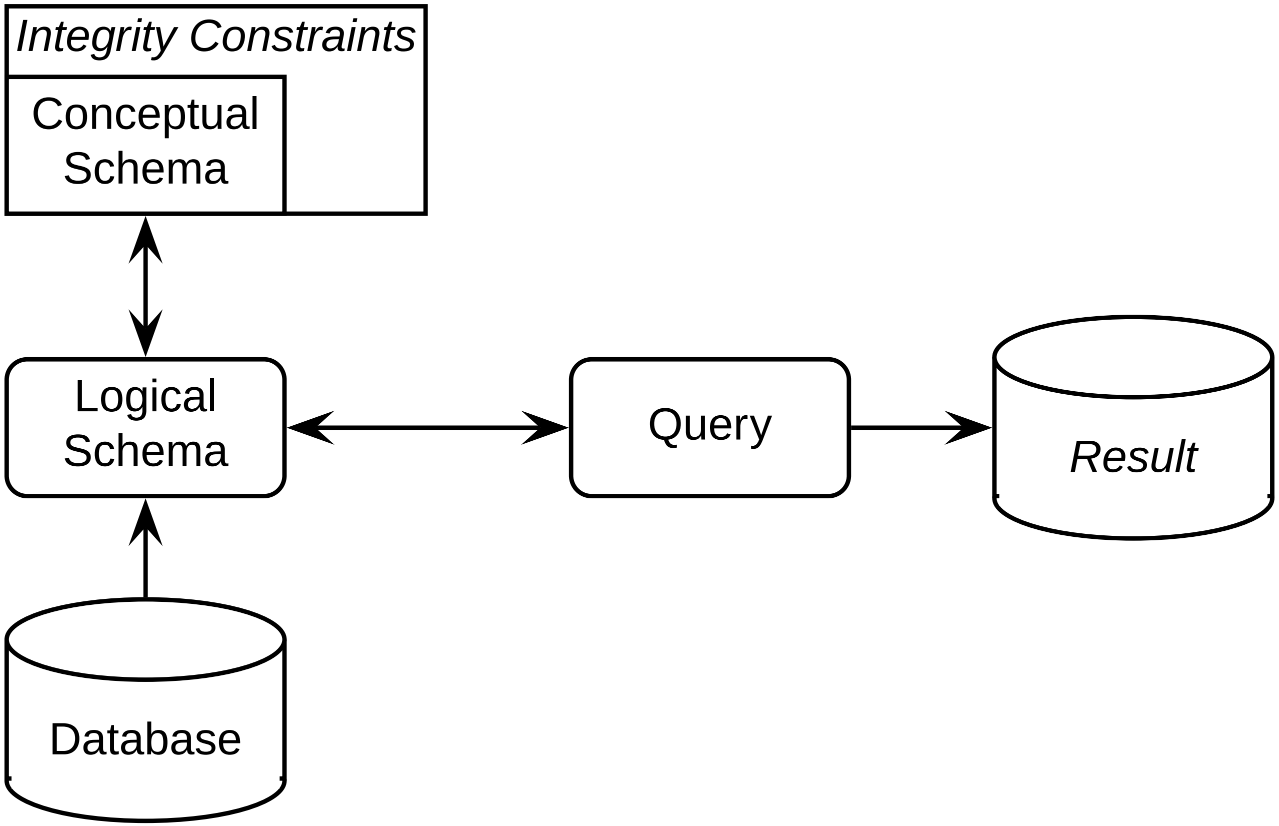

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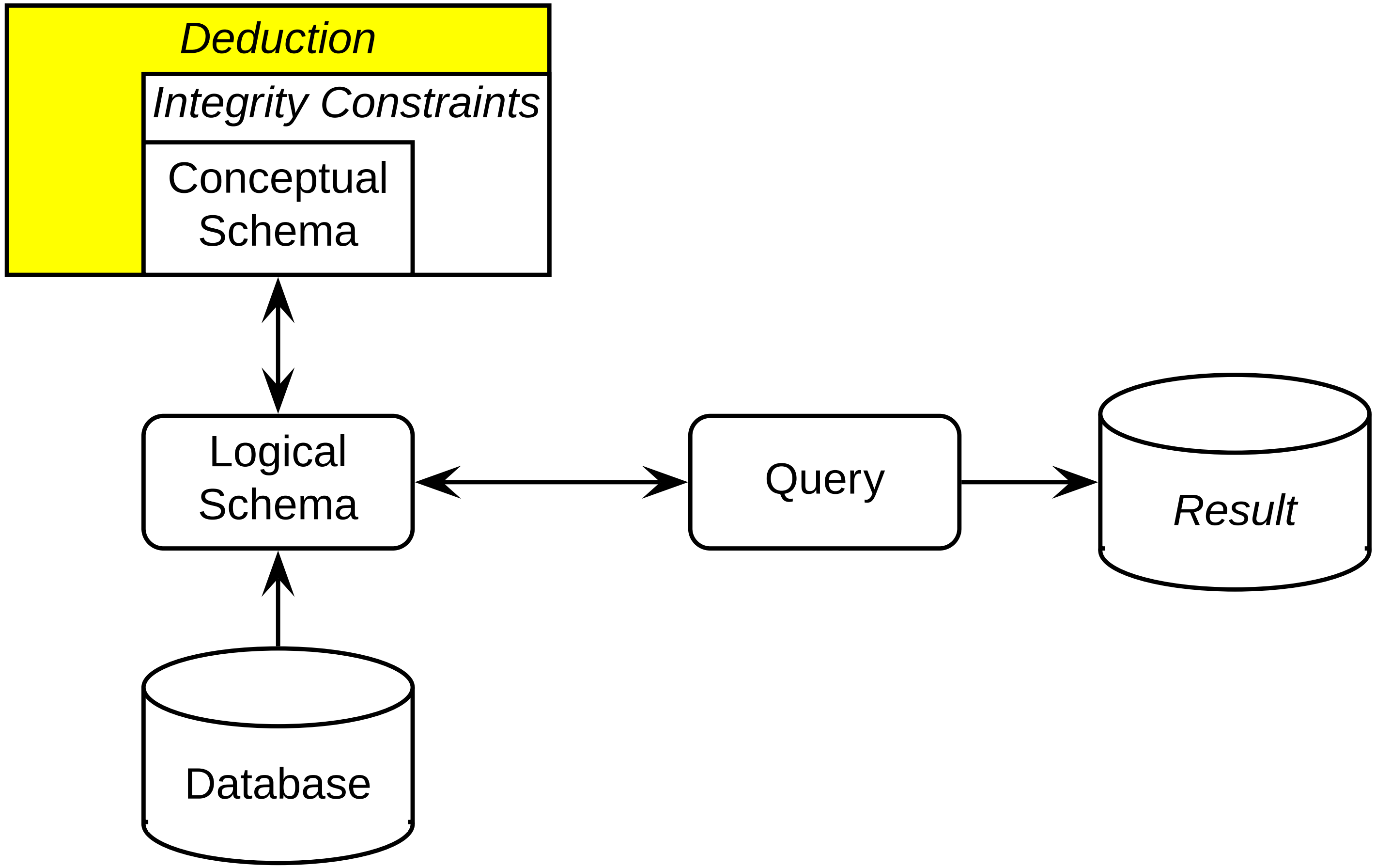

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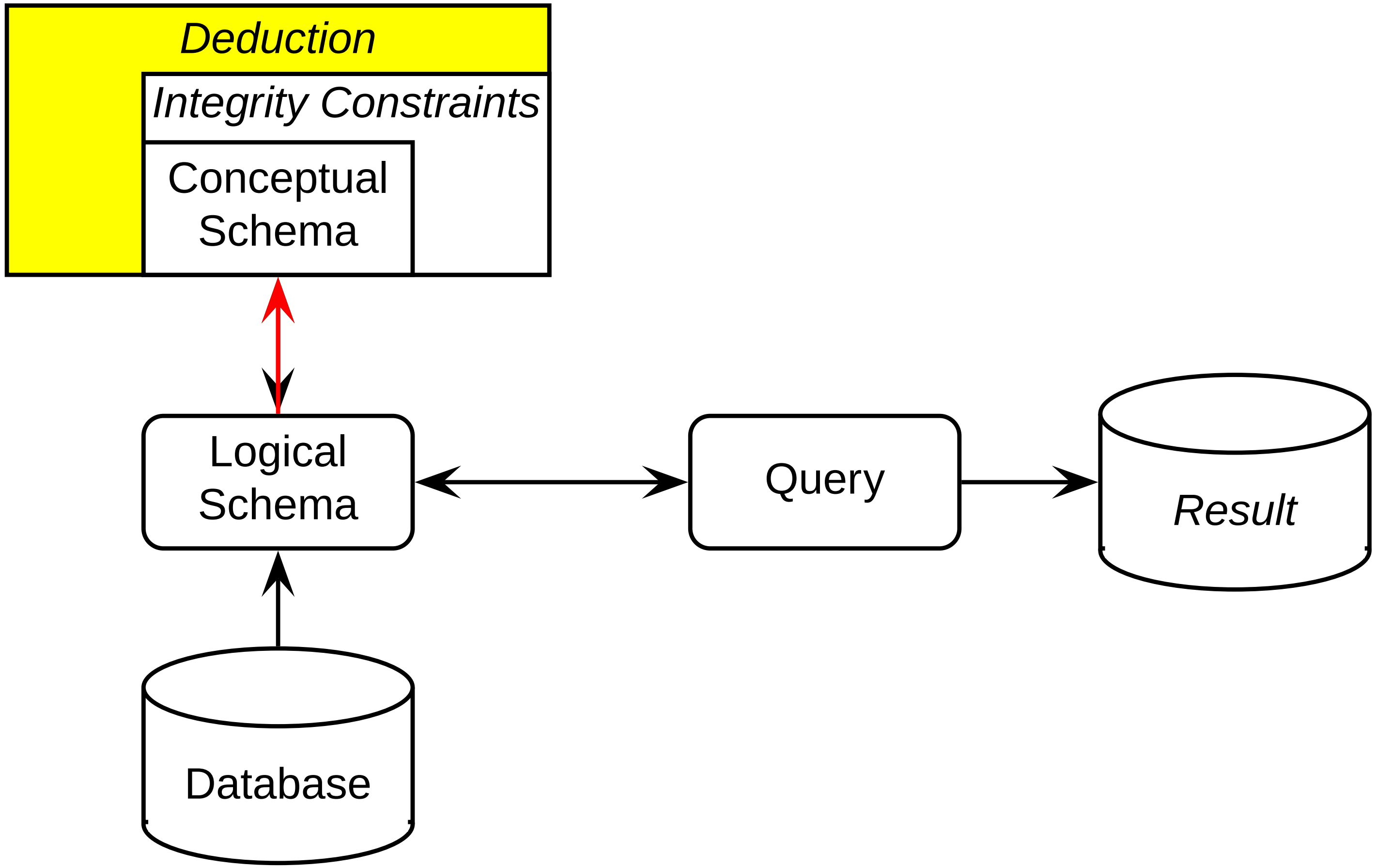

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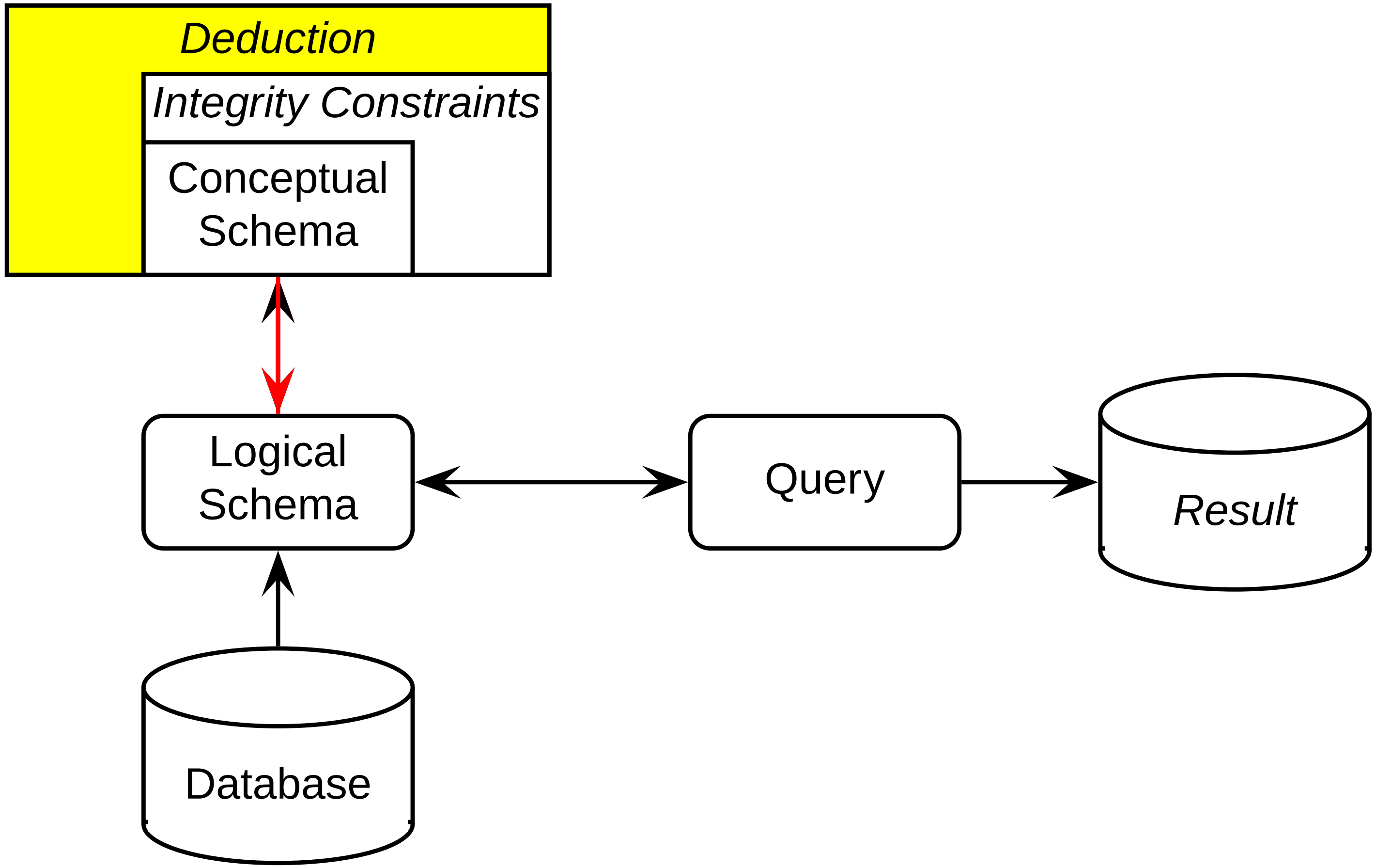

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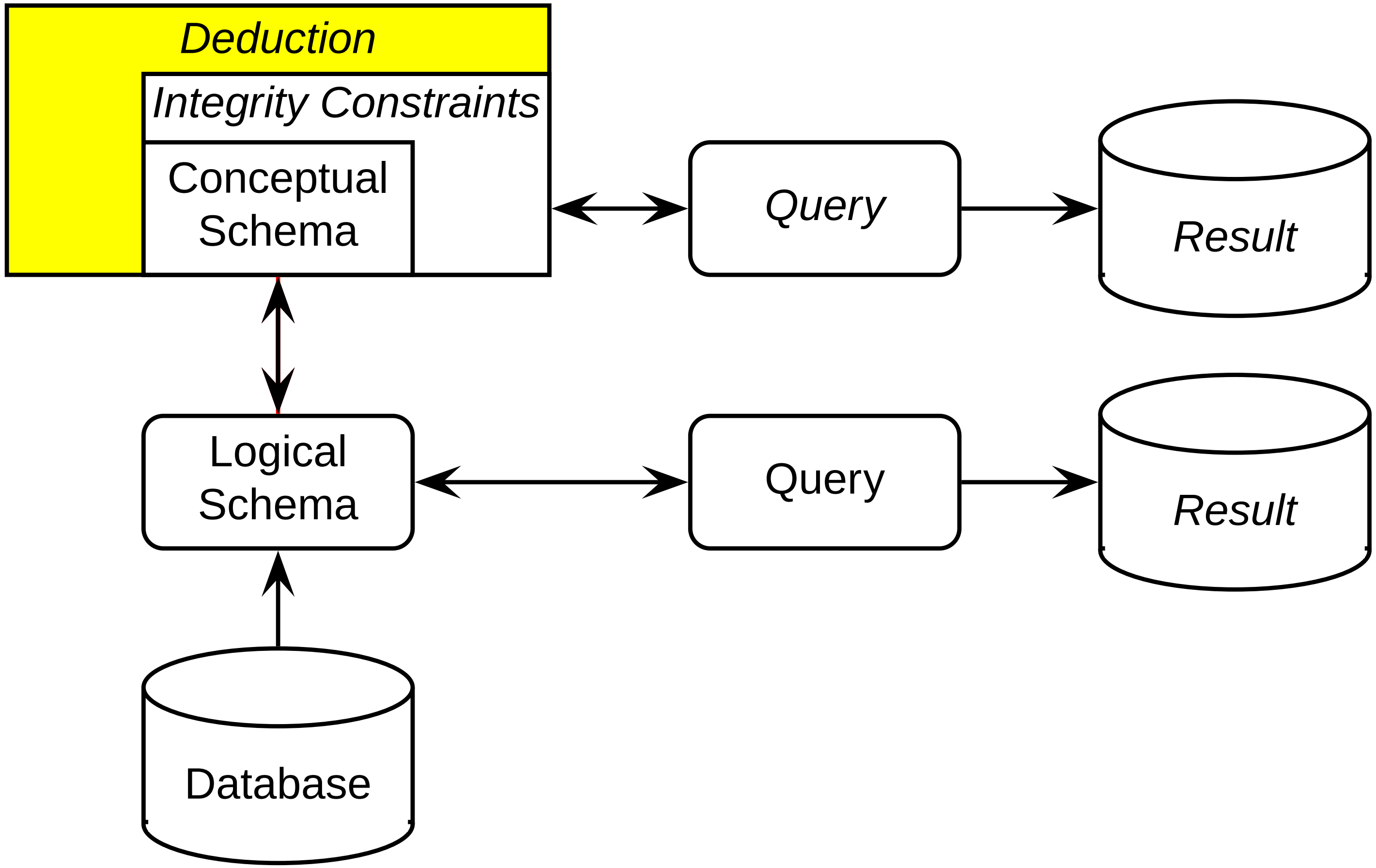

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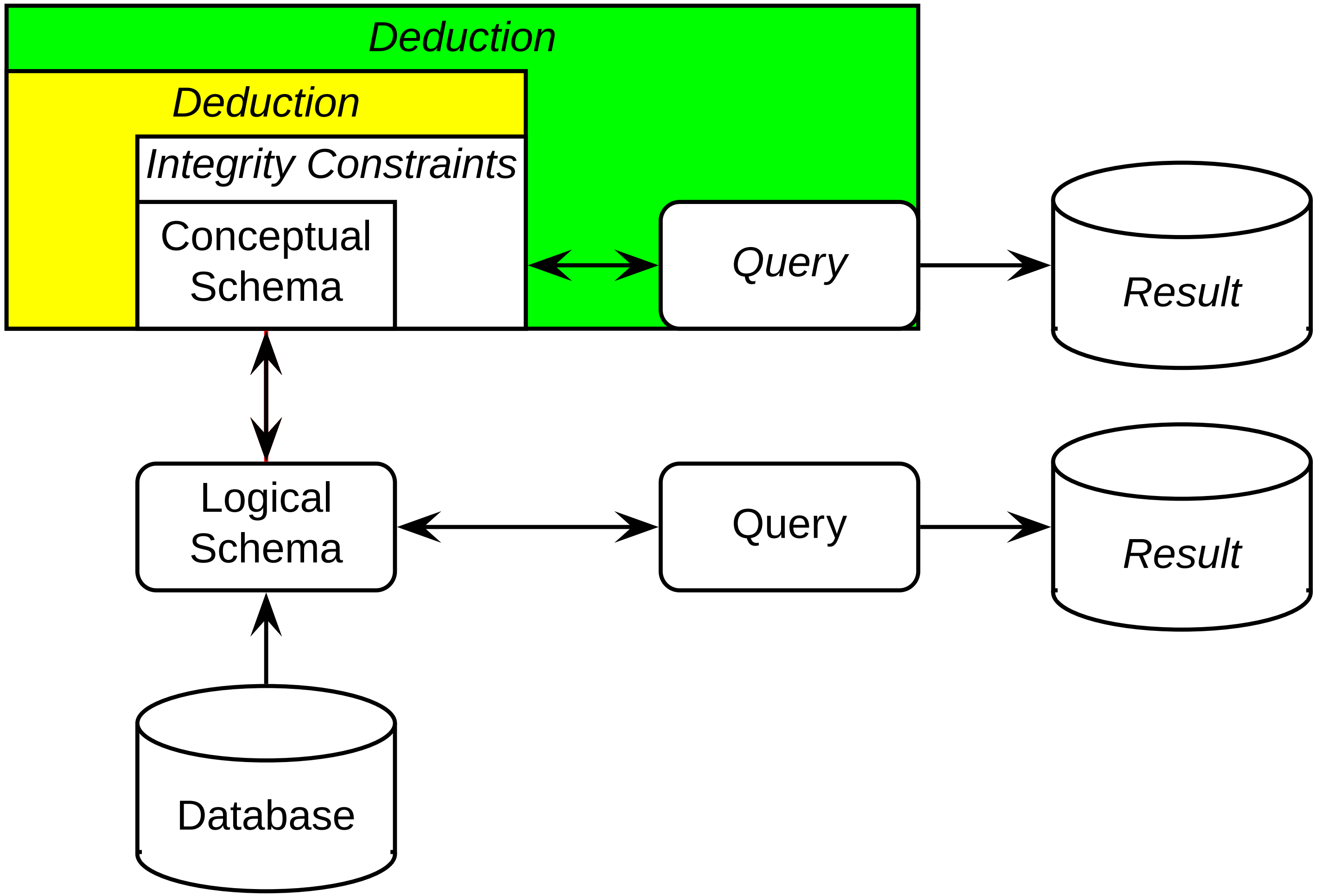

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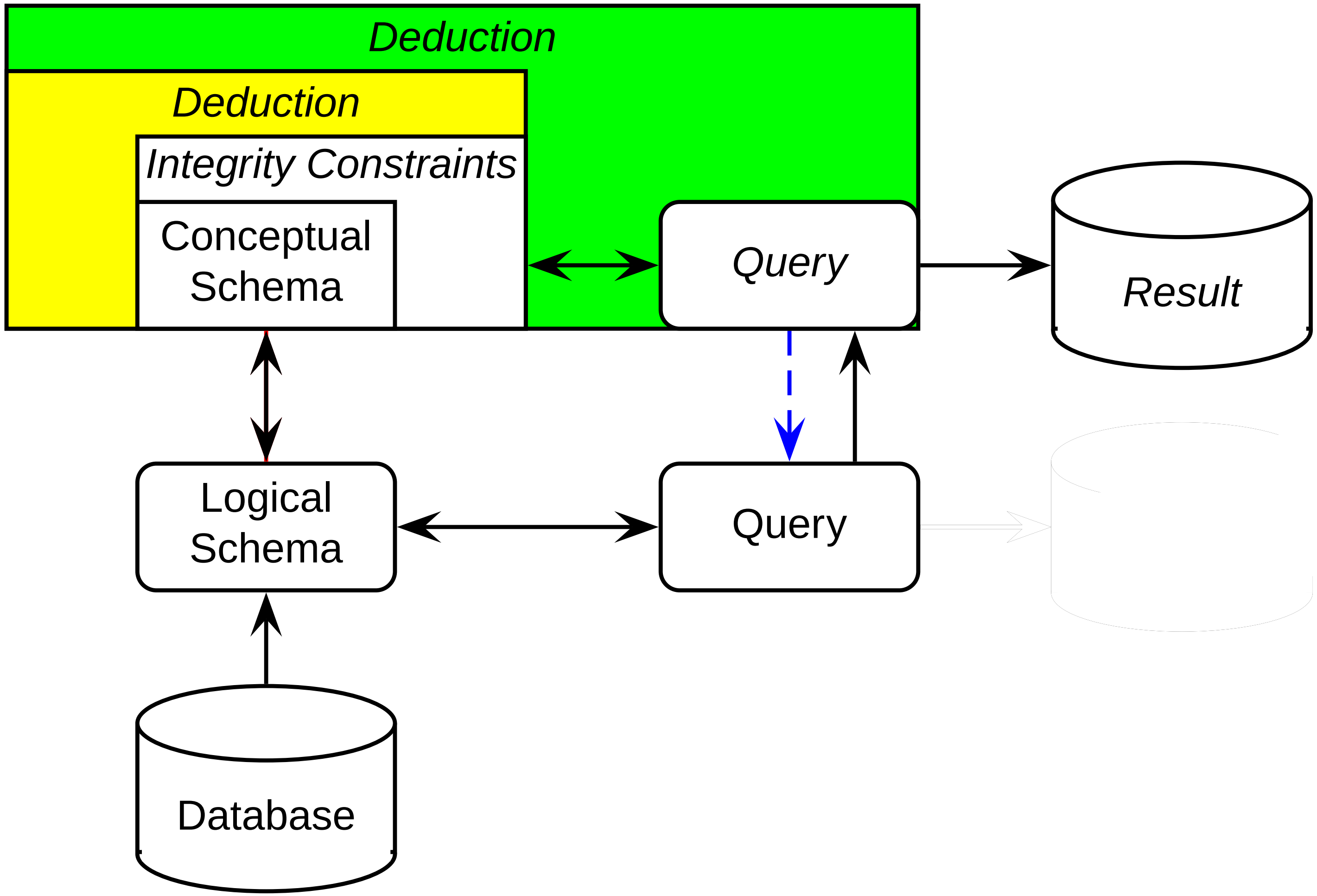
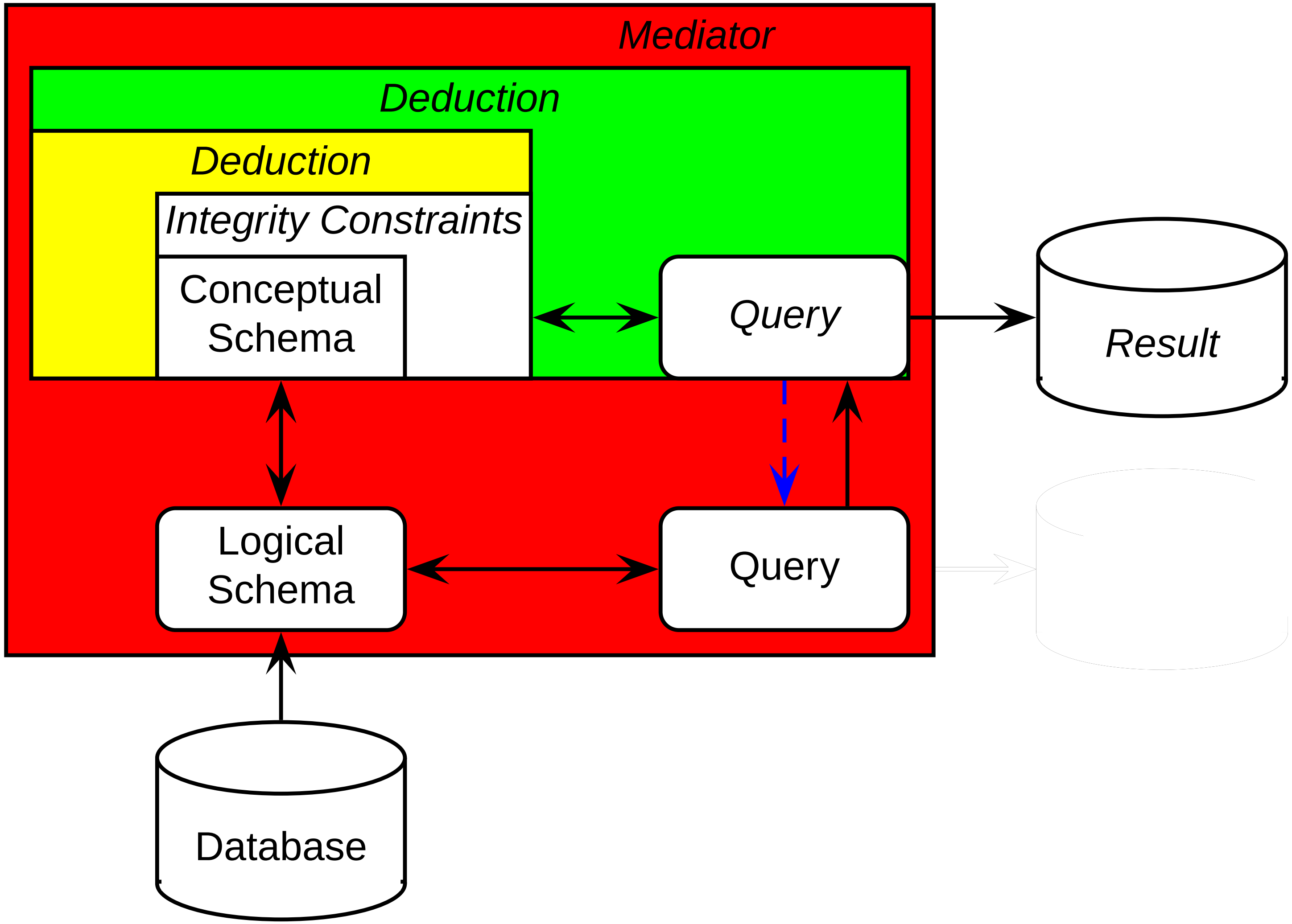


The role of logics in ~~Information Systems~~ a Mediator



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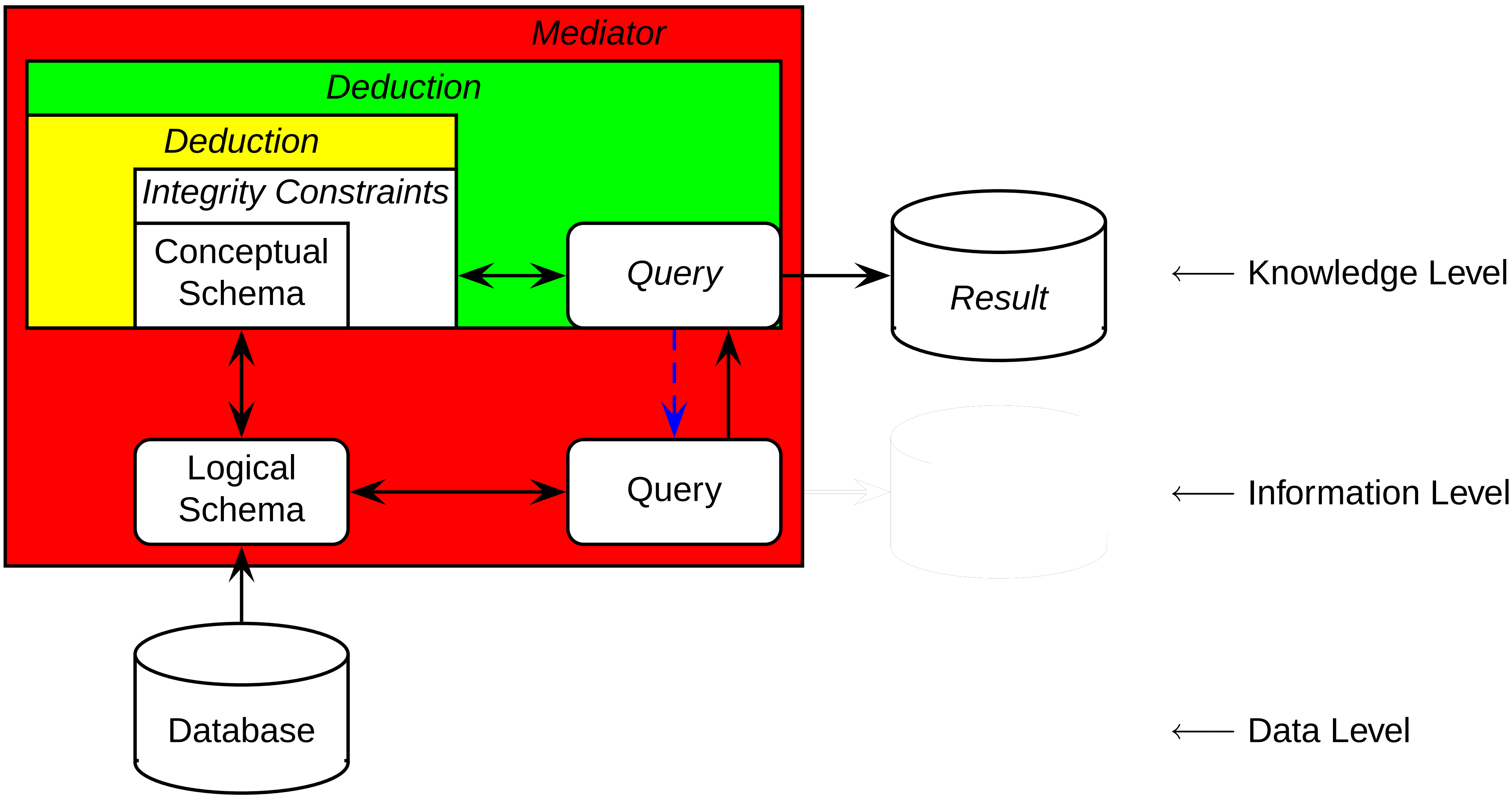


A Relational Database

$\texttt{CompanyEmployee}/2; \texttt{CompanyProject}/3$

CompanyEmployee	
name	project
john	esprit-dwq
...	...

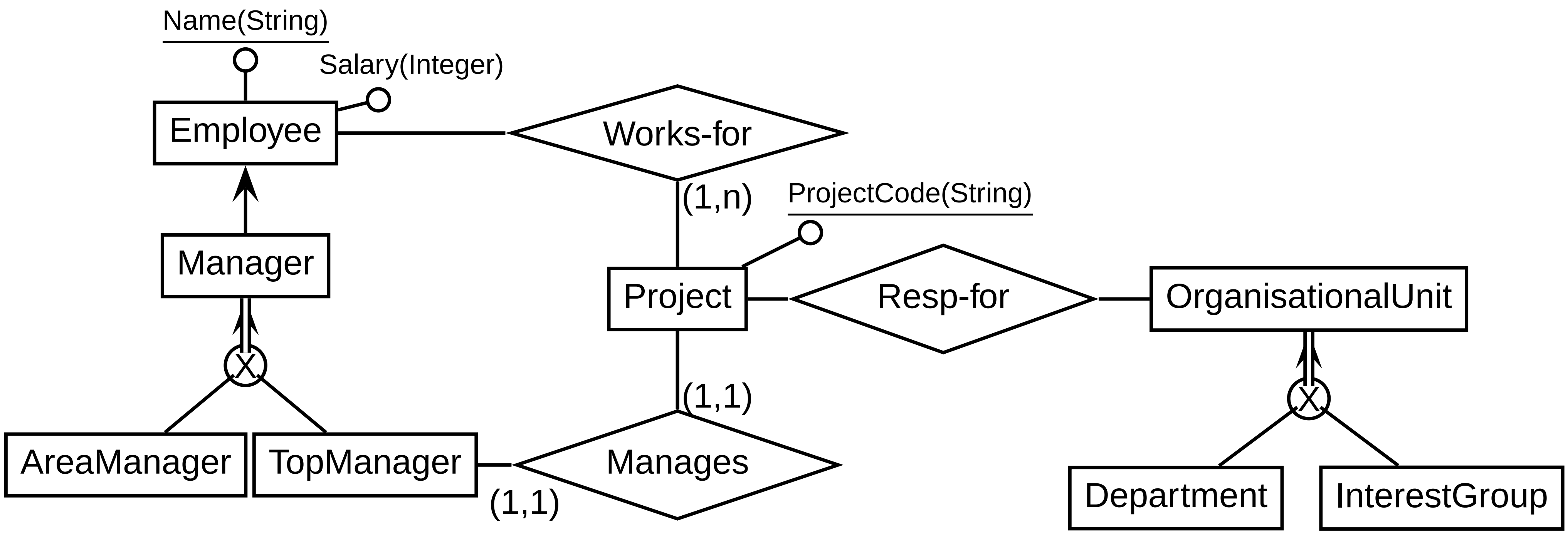
CompanyProject		
project	manager	department
esprit-dwq	enrico	cs-uman
...	...	...

Query = "Tell me the projects in which John works, and their managers and departments."

$$\texttt{Query} \equiv$$
$$\pi_{\texttt{project},\texttt{manager},\texttt{dept}.}\,\sigma_{\texttt{name}=\texttt{john}}\,(\texttt{CompanyEmployee} \bowtie_{\texttt{project}} \texttt{CompanyProject})$$

$$\texttt{Query}(\texttt{x},\texttt{y},\texttt{z}) \Leftrightarrow \texttt{CompanyEmployee}(\texttt{john},\texttt{x}) \wedge \texttt{CompanyProject}(\texttt{x},\texttt{y},\texttt{z})$$

Constraints from the Conceptual Schema



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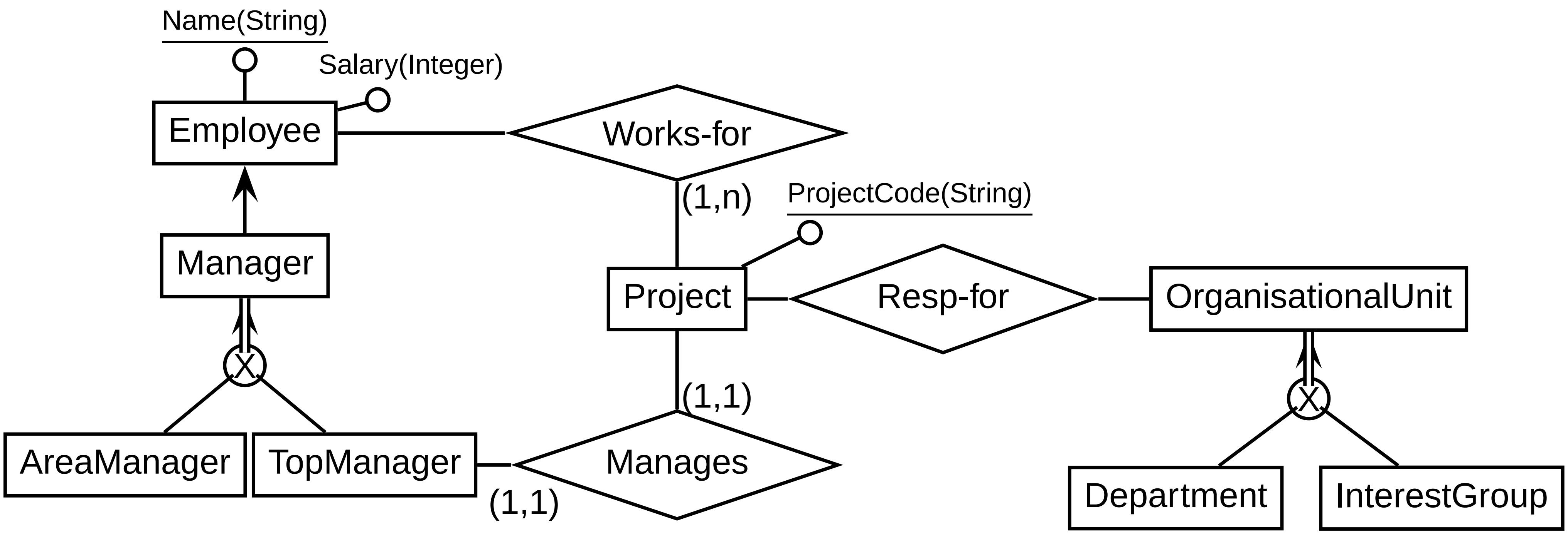


$\mathtt{Works\text{-}for} \sqsubseteq \mathtt{emp/2} : \mathtt{Employee} \sqcap \mathtt{act/2} : \mathtt{Project}$

$\mathtt{Manages} \sqsubseteq \mathtt{man/2} : \mathtt{TopManager} \sqcap \mathtt{prj/2} : \mathtt{Project}$

$\mathtt{Employee} \sqsubseteq \exists^{=1}[\mathtt{worker}](\mathtt{Name} \sqcap \mathtt{thename/2} : \mathtt{String}) \sqcap \exists^{=1}[\mathtt{payee}](\mathtt{Salary} \sqcap \mathtt{amount/2} : \mathtt{Integer})$

$\top \sqsubseteq \exists^{\leq 1}[\mathtt{thename}](\mathtt{Name} \sqcap \mathtt{worker/2} : \mathtt{Employee})$

$\mathtt{Manager} \sqsubseteq \mathtt{Employee} \sqcap (\mathtt{AreaManager} \sqcup \mathtt{TopManager})$

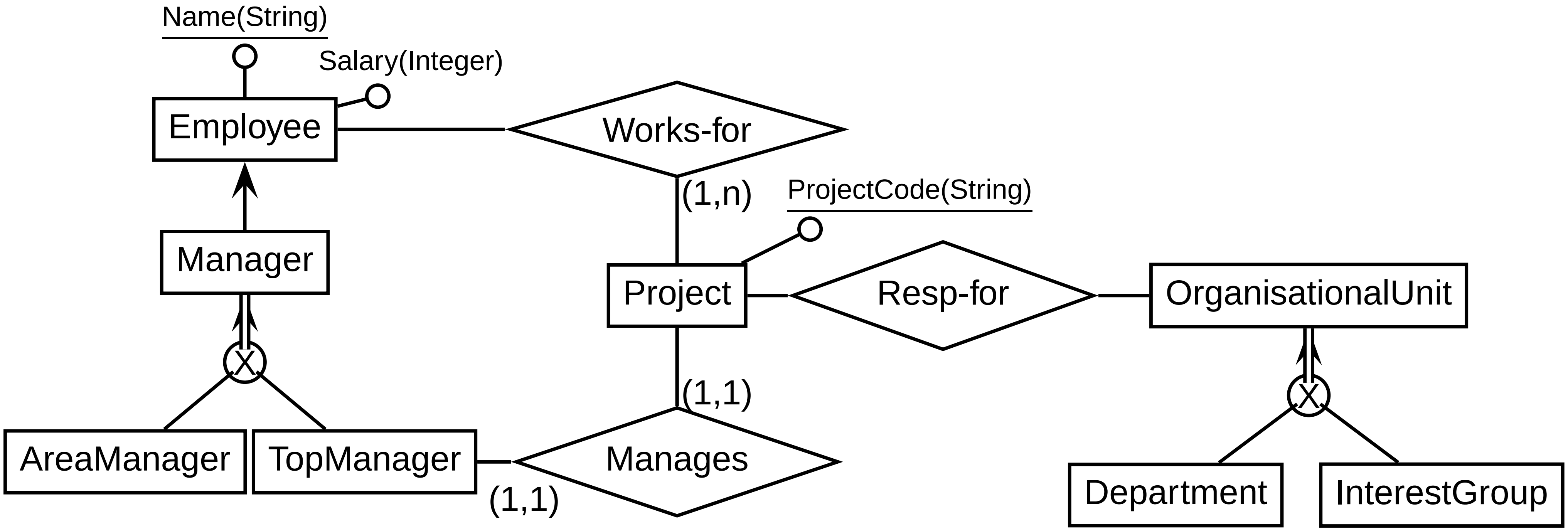
$\mathtt{AreaManager} \sqsubseteq \mathtt{Manager} \sqcap \neg\mathtt{TopManager}$

$\mathtt{TopManager} \sqsubseteq \mathtt{Manager} \sqcap \exists^{=1}[\mathtt{man}]\mathtt{Manages}$

$\mathtt{Project} \sqsubseteq \exists^{\geq 1}[\mathtt{act}]\mathtt{Works\text{-}for} \sqcap \exists^{=1}[\mathtt{prj}]\mathtt{Manages}$

...

Integrity Constraints and Logical Implication

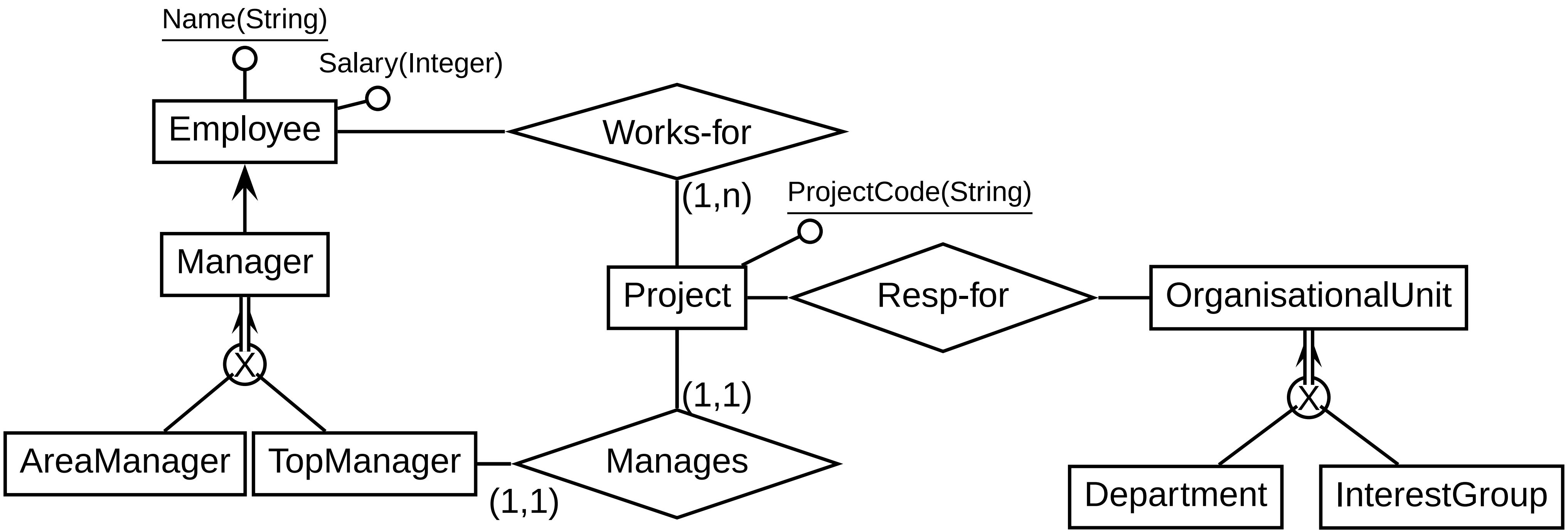


Managers are employees who do not work for a project (she/he just manages it):

$$\mathtt{Employee} \sqcap \neg(\exists^{\geq 1}[\mathtt{emp}]\mathtt{Works\text{-}for}) \sqsubseteq \mathtt{Manager}$$

$$\mathtt{Manager} \sqsubseteq \neg(\exists^{\geq 1}[\mathtt{emp}]\mathtt{Works\text{-}for})$$

Integrity Constraints and Logical Implication



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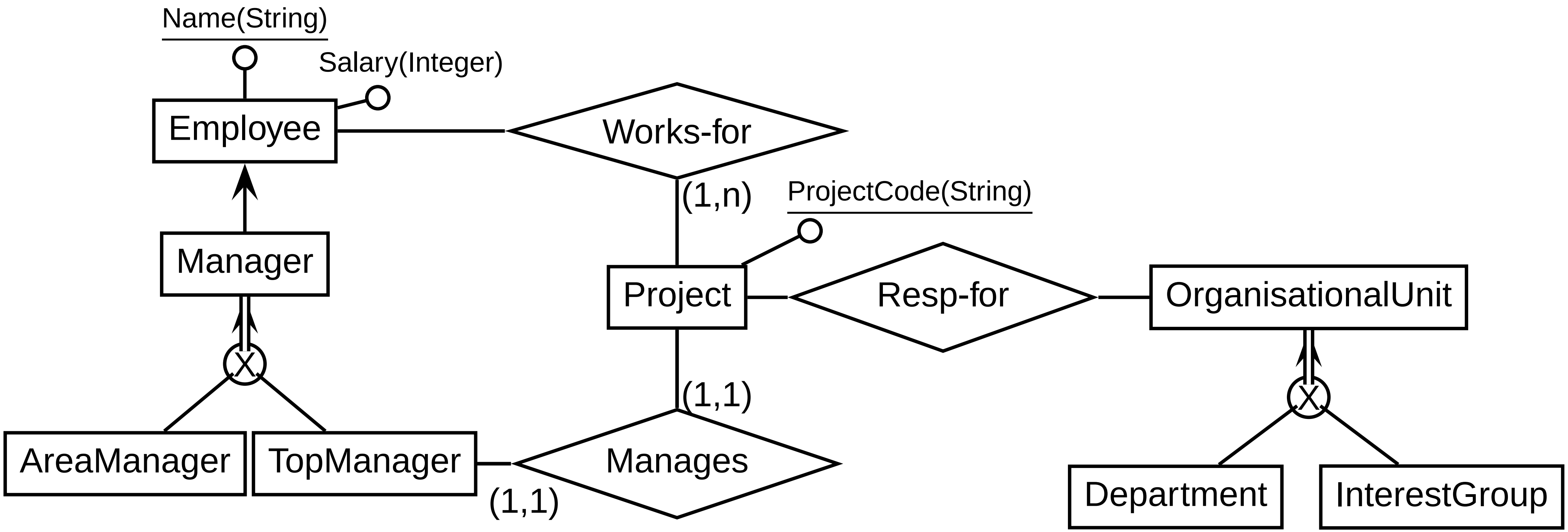
$\leadsto$ For every project, there is at least one employee who is not a manager:

$$\Sigma \models \texttt{Project} \sqsubseteq \exists^{\geq 1}[\texttt{act}](\texttt{Works-for} \sqcap \texttt{emp} : \neg\texttt{Manager})$$

Querying the Virtual Database (local-as-view)

$$\mathtt{Q}(x,y,z) \Leftarrow \mathtt{Project}(x) \wedge \mathtt{Works\text{-}for}(john,x) \wedge \mathtt{TopManager}(y) \wedge \mathtt{Manages}(y,x) \wedge \neg\mathtt{InterestGroup}(z) \wedge \mathtt{Resp\text{-}for}(z,x).$$



$$\mathtt{CompanyEmployee}(x,y) \Leftarrow \mathtt{Employee}(x) \wedge \mathtt{Project}(y) \wedge \mathtt{Works\text{-}for}(x,y).$$

$$\mathtt{CompanyProject}(x,y,z) \Leftarrow \mathtt{Project}(x) \wedge \mathtt{Manager}(y) \wedge \mathtt{Department}(z) \wedge \mathtt{Manages}(y,x) \wedge \mathtt{Resp\text{-}for}(z,x).$$

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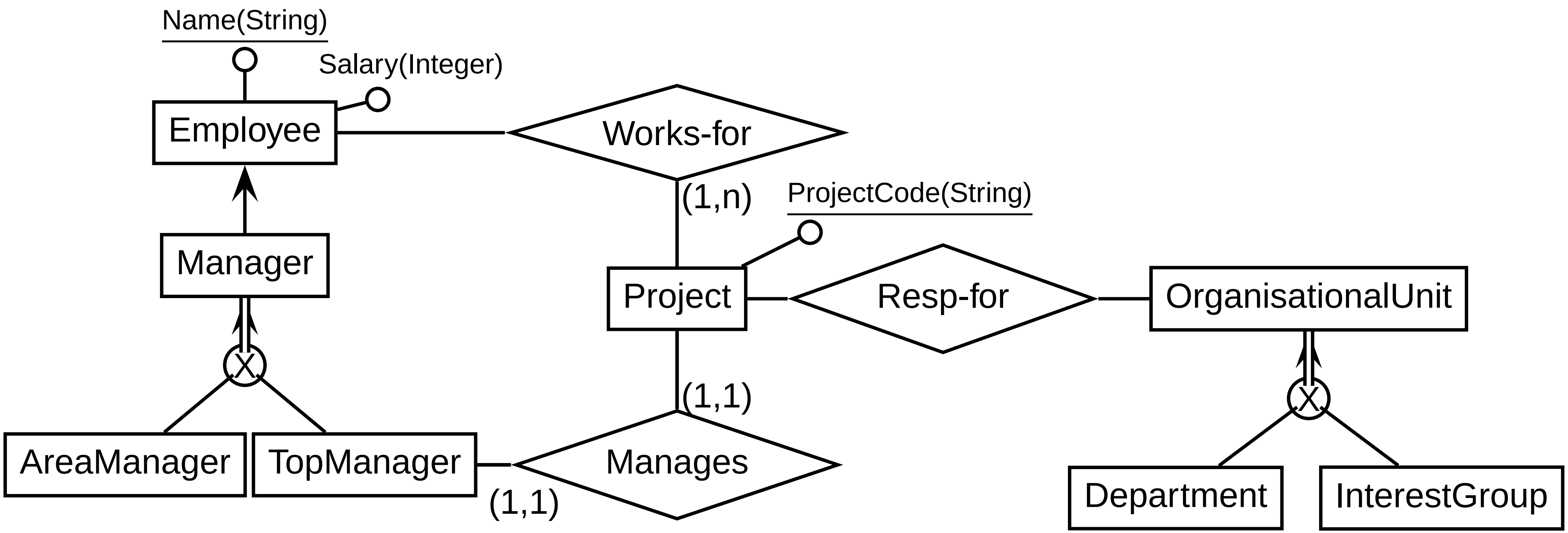

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$$\leadsto \mathtt{Q}(x,y,z) \Leftarrow \mathtt{CompanyEmployee}(john,x) \wedge \mathtt{CompanyProject}(x,y,z)$$

Querying the Virtual Database (global-as-view)

$$\texttt{Q}(x, y, z) \Leftarrow \texttt{Project}(x) \land \texttt{Works-for}(\texttt{john}, x) \land \texttt{TopManager}(y) \land \texttt{Manages}(y, x) \land$$
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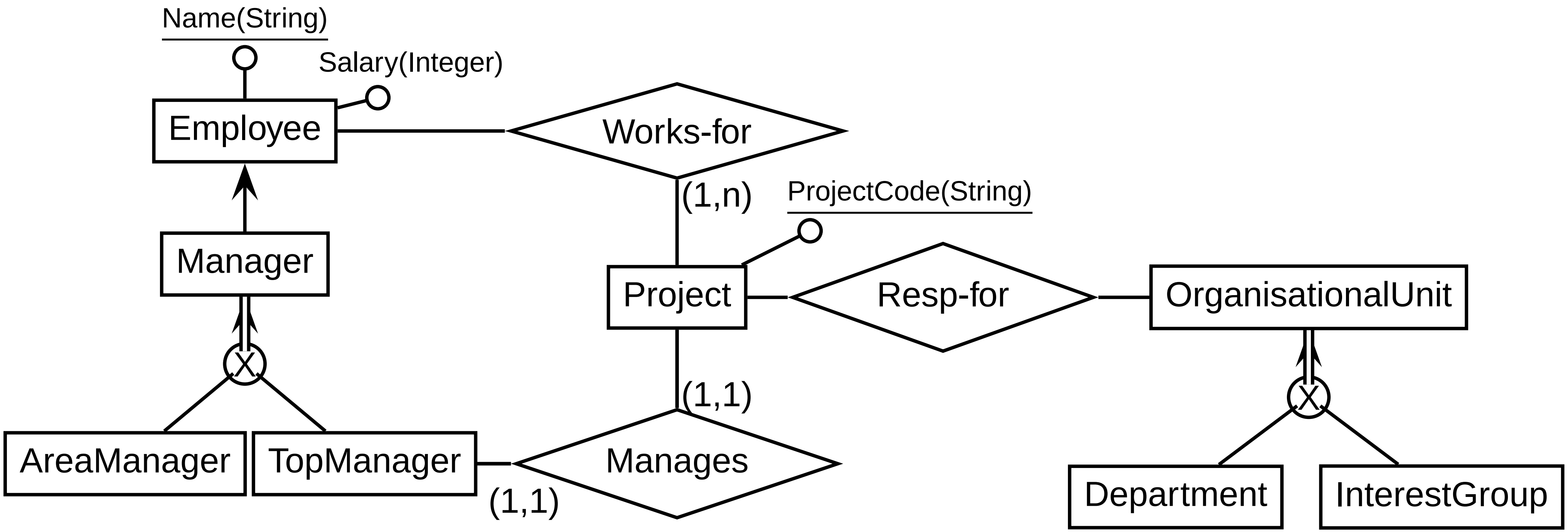

$\texttt{Project}(x) \Leftarrow \texttt{CompanyEmployee}(y, \mathbf{x}) \cup \texttt{CompanyProject}(\mathbf{x}, y, z)$

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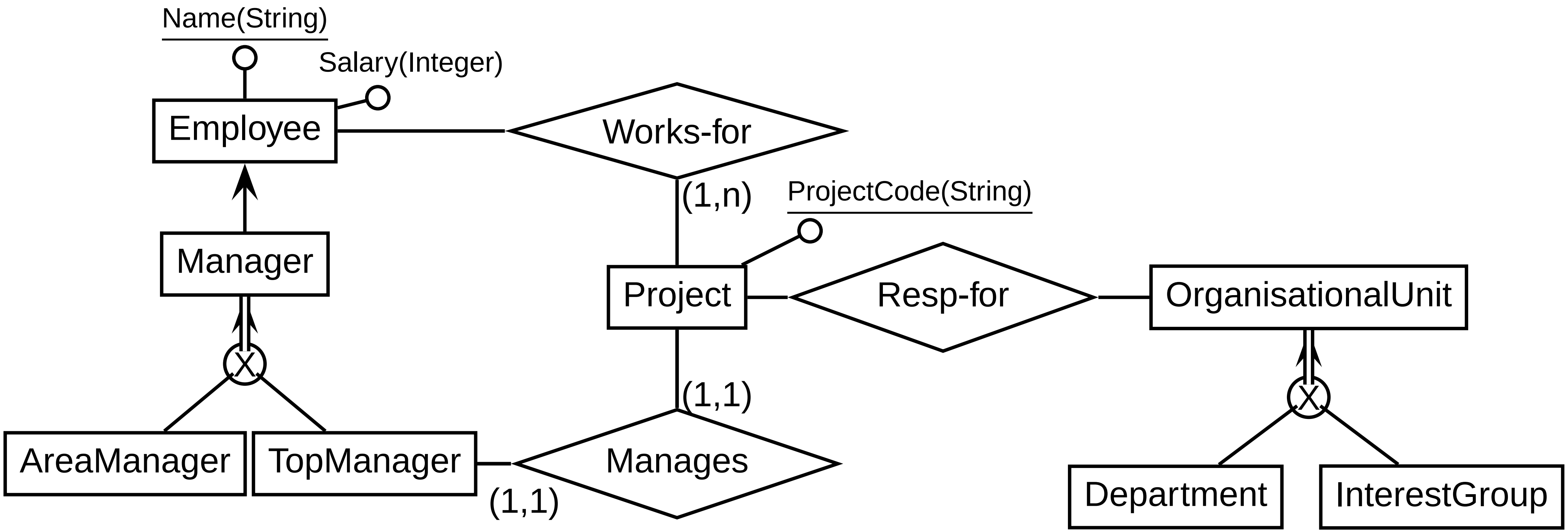

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Reasoning on Queries

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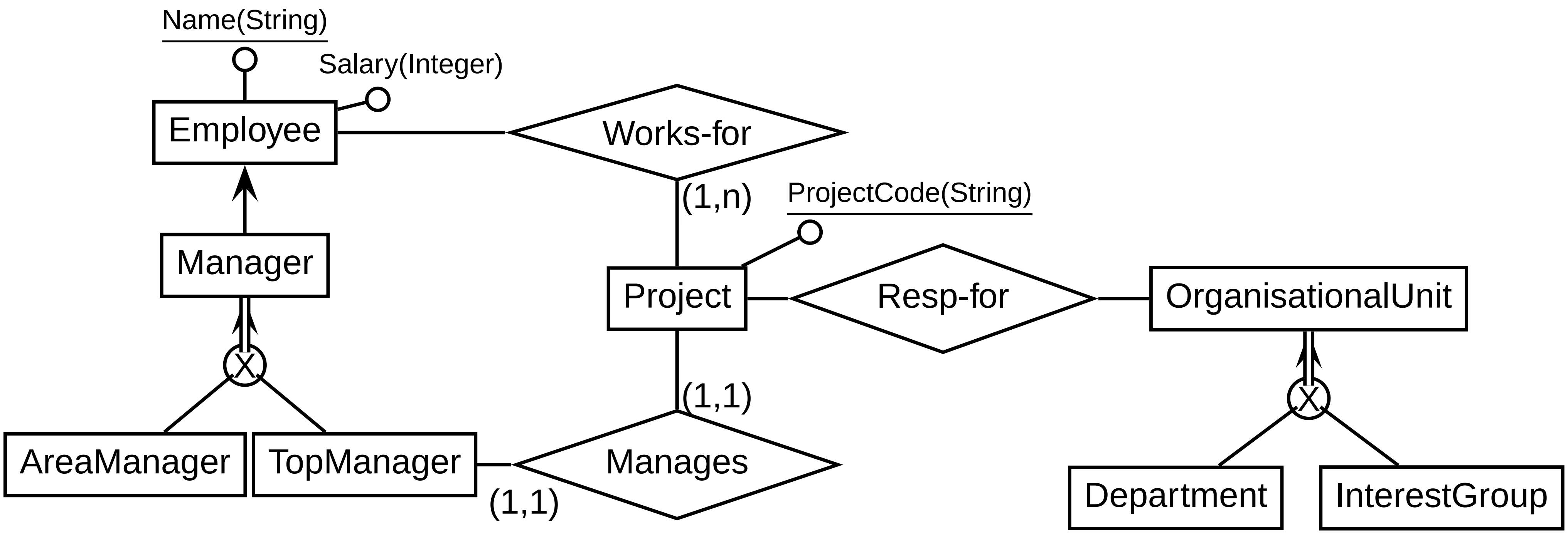
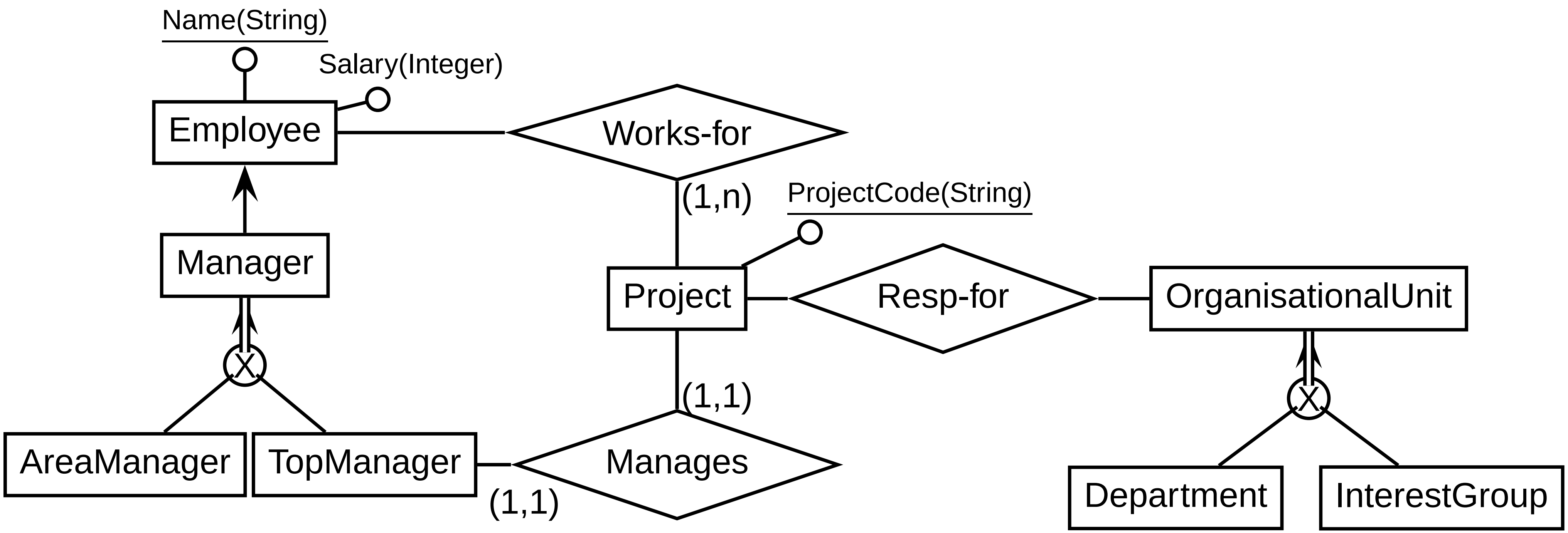


Reasoning on Queries

$$\mathtt{Q}(x, y) \Leftarrow \mathtt{Employee}(x) \wedge \mathtt{Works\text{-}for}(x, y) \wedge \mathtt{Manages}(x, y).$$



$\leadsto$ INCONSISTENT QUERY!