Description Logics and Logics

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Tense Logic: *(point ontology)*

- Tense logic is a propositional modal logic, interpreted over temporal structure $\mathcal{T} = (\mathcal{P}, <)$, where $\mathcal{P}$ is a set of time points and $<$ is a strict partial order on $\mathcal{P}$.

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$$\begin{aligned}\texttt{Mortal} \sqsubseteq\ & \texttt{LivingBeing} \sqcap \forall \texttt{LIVES-IN.Place} \sqcap \\ & (\texttt{LivingBeing}\, \mathcal{U}\, (\Box^{+} \neg \texttt{LivingBeing}))\end{aligned}$$

- Satisfiability in $\mathcal{ALC}_{\mathcal{US}}$ – the combination of tense logic with $\mathbf{K_m}$ – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (PSPACE-complete).

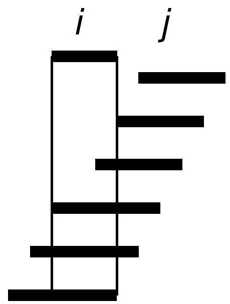
- Satisfiability in $\mathcal{ALCQI}_{\mathcal{US}}$ with ABox – the combination of tense logic with $\mathcal{ALCQI}$ with ABox – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (EXPTIME-complete).

$\mathcal{HS}$: Interval Temporal Propositional Modal Logic

- $\mathcal{HS}$ is a propositional modal logic interpreted over an interval set $\mathcal{T}^*_<$, defined as the the set of all closed intervals $[u, v] \doteq \{x \in \mathcal{P} \mid u \leq x \leq v, u \neq v\}$ in some temporal structure $\mathcal{T}$.

- $\mathcal{HS}$ extends propositional logic with modal formulæ $\langle R \rangle \phi$ and $[R]\phi$ – where R is a basic Allen's algebra temporal relation:

before (i, j)
meets (i, j)
overlaps (i, j)
starts (i, j)
during (i, j)
finishes (i, j)



- $\texttt{Mortal} \doteq \texttt{LivingBeing} \wedge \langle \text{after} \rangle . \neg \texttt{LivingBeing}$

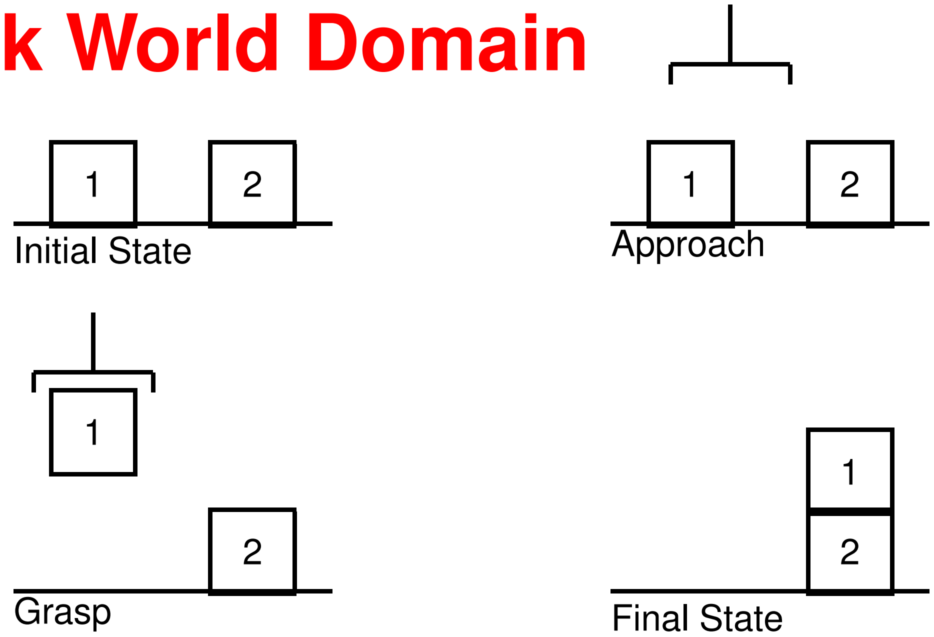
- Satisfiability $\mathcal{HS}$ is undecidable for the most interesting classes of temporal structures.

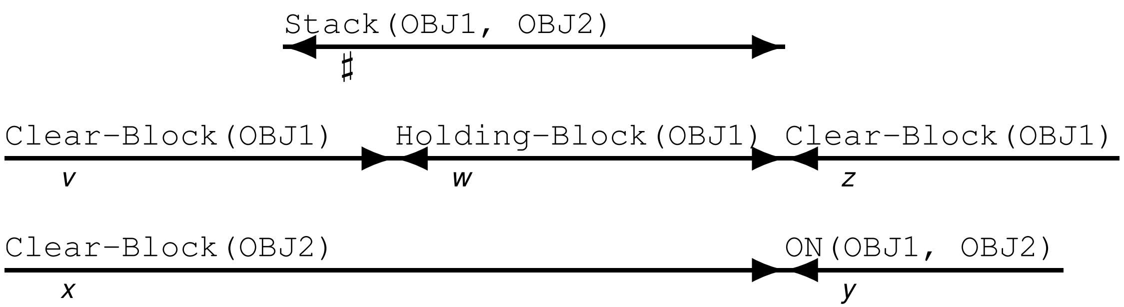
- Therefore, $\mathcal{HS} \cup \mathcal{ALC}$ is undecidable.

Decidable Interval Temporal Description Logics

- $\mathcal{HS}^*$:
  - No universal quantification, or restricted to homogeneous properties:
    $\Box(=, \text{starts}, \text{during}, \text{finishes}).\ \psi$
  - Allows for temporal variables:
    $\Diamond\ \vec{x}\ \mathsf{TN}(\vec{x}).\ \psi$
    $\psi @ x$
- Global roles – denoting temporal *independent* properties.
- Logical implication in the combined language $\mathcal{HS}^* \cup \mathcal{ALC}$ is decidable (PSPACE-hard); satisfiability is PSPACE-complete.
- Logical implication in $\mathcal{HS}^* \cup \mathcal{F}$ is NP-complete.
- Useful for event representation and plan recognition.

The Block World Domain





$$
\begin{aligned}
\texttt{Stack} \doteq\ & \Diamond(x\ y\ z\ v\ w)\ (\sharp \text{ finishes } x)(\sharp \text{ meets } y)(\sharp \text{ meets } z)(v \text{ overlaps } \sharp)(w \text{ finishes } \sharp)(v \text{ meets } w).\\
& ((\star\texttt{OBJECT2} : \texttt{Clear-Block})@x \sqcap \\
& \ (\star\texttt{OBJECT1} \circ \texttt{ON} = \star\texttt{OBJECT2})@\texttt{y} \sqcap \\
& \ (\star\texttt{OBJECT1} : \texttt{Clear-Block})@\texttt{v} \sqcap \\
& \ (\star\texttt{OBJECT1} : \texttt{Holding-Block})@\texttt{w} \sqcap \\
& \ (\star\texttt{OBJECT1} : \texttt{Clear-Block})@\texttt{z})
\end{aligned}
$$

$\ddot{\mathcal{L}}^n$ FOL fragments

- $\ddot{\mathcal{L}}^n$ is the set of function-free FOL formulas with equality and constants, with only unary and binary predicates, and which can be expressed using at most n variable symbols.
- Satisfiability of $\ddot{\mathcal{L}}^3$ formulas is undecidable.
- Satisfiability of $\ddot{\mathcal{L}}^2$ formulas is NEXPTIME-complete.

The $\mathcal{DL}$ description logic

$\mathcal{ALCI}$ + propositional calculus on roles,

+ the concept $(R \subseteq S)$.

- The $\mathcal{DL}$ description logic and $\ddot{\mathcal{L}}^3$ are equally expressive.
- The $\mathcal{DL}^-$ description logic (i.e., $\mathcal{DL}$ without the composition operator) and $\ddot{\mathcal{L}}^2$ are equally expressive.
- Open problem: relation between $\mathcal{DL}$ including cardinalities and $\ddot{\mathcal{C}}^n$– adding counting quantifiers to $\ddot{\mathcal{L}}^n$.

Guarded Fragments of FOL

The *guarded fragment* GF of FOL is defined as:

1. Every relational atomic formula is in GF
2. GF is propositionally closed
3. If $\mathbf{x}$, $\mathbf{y}$ are tuples of variables, $\alpha(\mathbf{x}, \mathbf{y})$ is atomic, and $\psi(\mathbf{x}, \mathbf{y})$ is a formula in GF, such that free$(\psi) \subseteq$ free$(\alpha) = \{\mathbf{x}, \mathbf{y}\}$, then the following formulae are in GF:

   $\exists \mathbf{y}.\ \alpha(\mathbf{x}, \mathbf{y}) \wedge \psi(\mathbf{x}, \mathbf{y})$

   $\forall \mathbf{y}.\ \alpha(\mathbf{x}, \mathbf{y}) \rightarrow \psi(\mathbf{x}, \mathbf{y})$

The guarded fragment contains the modal fragment of FOL (and Description Logics); a weaker definition (LGF) is needed to include temporal logics.

Properties of GF

- GF has the finite model property
- GF and LGF have the tree model property
- Many important model theoretic properties which hold for FOL and the modal fragment, do hold also for GF and LGF
- Satisfiability is decidable for GF and LGF (deterministic double exponential time complete)
- Bounded-variable or bounded-arity fragments of GF and LGF (which include Description Logics) are in EXPTIME.
- GF with fix-points is decidable.