Description Logics and Logics

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Summary

- Description Logics
- $\mathcal{K}_{(m)}$, multi-modal Normal Modal Logic $\mathcal{K}$
- Propositional Dynamic Modal Logics
- Propositional μ-calculus
- Propositional Temporal Modal Logics
- $\ddot{\mathcal{L}}^n$ FOL fragments
- Guarded Fragment of FOL

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What is a Description Logic

A logical system, based on **objects** (individuals), **classes** (concepts), and **relationships** (roles), constituted by:

- a *description language*, which specifies how to construct concept and relationship expressions,
- a *knowledge base language*, which specifies properties of objects, concepts, and relationships,
- a set of (decidable) *reasoning services* over a knowledge base, with sound and complete procedures.

An example

Σ:

TBox

$\exists\texttt{TEACHES}.\texttt{Course} \sqsubseteq \neg\texttt{Undergrad} \sqcup \texttt{Prof}$

ABox

$\texttt{TEACHES}(\texttt{mary}, \texttt{cs415}), \texttt{Course}(\texttt{cs415}),$
$\texttt{Undergrad}(\texttt{mary})$

$\Sigma \models \texttt{Prof}(\texttt{mary})$

Description Logics: syntax

$C, D \quad \rightarrow \quad A \mid$	A	(primitive concept)
$\neg C \mid$	(**not** C)	(complement)
$C \sqcap D \mid$	(**and** C D $\ldots$)	(conjunction)
$C \sqcup D \mid$	(**or** C D $\ldots$)	(disjunction)
$\forall R.C \mid$	(**all** R C)	(universal quant.)
$\exists R.C \mid$	(**some** R C)	(existential quant.)

TBox: $(C \dot{\sqsubseteq} D)$, $(R \dot{\sqsubseteq} S)$
ABox: $C(a)$, $R(a, b)$

$\mathcal{ALC}$

R, S	$\rightarrow$	$P \mid$	P	(primitive role)
		$R \sqcup S \mid$	(`or` R S)	(disjunction)
		$R \circ S \mid$	(`compose` R S)	(role chain)
		$R^* \mid$	(`trans` R)	(transitive closure)
		$\mathsf{id}(C) \mid$	(`self` C)	(role identity)

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R^{-1} | (`inverse` R) (inverse role)

$\mathcal{I}$

$\geq n\, R.\, C$ |	(`atleast` n R C)	(minimum cardin.)
$\leq n\, R.\, C$ |	(`atmost` n R C)	(maximum cardin.)

$\mathcal{Q}$

		$f : C$ |	(`in` f C)	(selection)
		$f \uparrow$ |	(`undefined` f)	(undefinedness)

		f	f	(feature)
f, g	$\rightarrow$	p |	p	(primitive feature)
		$f \circ g$	(`compose` f g)	(feature chain)

$$\mathcal{F}$$

$\{ a, b \ldots \}$ (**oneof** a b ...) (enumeration)

$\mathcal{O}$

Description Logics: semantics

$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$	$A(\gamma)$
$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$\neg F_C(\gamma)$
$(C \sqcap D)^{\mathcal{I}} = C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$F_C(\gamma) \wedge F_D(\gamma)$
$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$	$F_C(\gamma) \vee F_D(\gamma)$
$(\forall R.\, C)^{\mathcal{I}} = \{i \in \Delta^{\mathcal{I}} \mid \forall j.\, R^{\mathcal{I}}(i,j) \Rightarrow C^{\mathcal{I}}(j)\}$	$\forall x.\, F_R(\gamma, x) \Rightarrow F_C(x)$
$(\exists R.\, C)^{\mathcal{I}} = \{i \in \Delta^{\mathcal{I}} \mid \exists j.\, R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\}$	$\exists x.\, F_R(\gamma, x) \wedge F_C(x)$

$(\geq n\, R.\, C)^{\mathcal{I}}$	$= \{i \in \Delta^{\mathcal{I}} \mid \sharp\{j \mid R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\} \geq n\}$	$\exists^n x.\, F_R(\gamma, x) \wedge F_C(x)$
$(\leq n\, R.\, C)^{\mathcal{I}}$	$= \{i \in \Delta^{\mathcal{I}} \mid \sharp\{j \mid R^{\mathcal{I}}(i,j) \wedge C^{\mathcal{I}}(j)\} \leq n\}$	$\neg\exists^{n+1} x.\, F_R(\gamma, x) \wedge F_C(x)$
$(f : C)^{\mathcal{I}}$	$= \{i \in \mathrm{dom}\, f^{\mathcal{I}} \mid C^{\mathcal{I}}(f^{\mathcal{I}}(i))\}$	$\exists x.\, F_f(\gamma, x) \wedge F_C(x)$
$(f\!\uparrow)^{\mathcal{I}}$	$= \Delta^{\mathcal{I}} \setminus \mathrm{dom}\, f^{\mathcal{I}}$	$\neg\exists x.\, F_f(\gamma, x)$
$\{\, a, b \ldots \}^{\mathcal{I}}$	$= \{a^{\mathcal{I}}, b^{\mathcal{I}} \ldots\}$	$\gamma = a \vee \gamma = b \vee \ldots$
$P^{\mathcal{I}}$	$\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$	$P(\alpha, \beta)$
$(R \sqcup S)^{\mathcal{I}}$	$= R^{\mathcal{I}} \cup S^{\mathcal{I}}$	$F_R(\alpha, \beta) \vee F_S(\alpha, \beta)$
$(R \circ S)^{\mathcal{I}}$	$= R^{\mathcal{I}} \circ S^{\mathcal{I}}$	$\exists x.\, F_R(\alpha, x) \wedge F_S(x, \beta)$
$(R^*)^{\mathcal{I}}$	$= (R^{\mathcal{I}})^*$	$\ldots$
$(\mathsf{id}(C))^{\mathcal{I}}$	$= \{(i,i) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid C^{\mathcal{I}}(i)\}$	$F_C(\alpha) \wedge \alpha = \beta$
$(R^{-1})^{\mathcal{I}}$	$= \{(i,j) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid R^{\mathcal{I}}(j,i)\}$	$F_R(\beta, \alpha)$
$f^{\mathcal{I}}$	$= \{(i,j) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid f^{\mathcal{I}}(i) = j\}$	$F_f(\alpha, \beta)$
$p^{\mathcal{I}}$	$= \Delta^{\mathcal{I}} \xrightarrow{partial} \Delta^{\mathcal{I}}$	$p(\alpha) = \beta$
$(f \circ g)^{\mathcal{I}}$	$= f^{\mathcal{I}} \circ g^{\mathcal{I}}$	$\exists x.\, F_f(\alpha) = x \wedge F_g(x) = \beta$

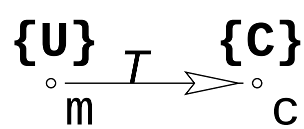
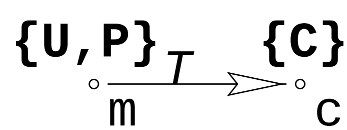
Another Example

$$
\begin{array}{rcl}
\exists\texttt{TEACHES.Course} & \dot{\sqsubseteq} & (\texttt{Student} \sqcap \exists\texttt{DEGREE.Bs}) \sqcup \texttt{Prof} \\
\texttt{Prof} & \dot{\sqsubseteq} & \exists\texttt{DEGREE.Ms} \\
\exists\texttt{DEGREE.Ms} & \dot{\sqsubseteq} & \exists\texttt{DEGREE.Bs} \\
\texttt{Ms} \sqcap \texttt{Bs} & \dot{\sqsubseteq} & \bot
\end{array}
$$

TEACHES(paul,cs415), Course(cs415),
$\Big(\exists\texttt{FRIEND.}(\{\texttt{ paul }\} \sqcap\ \leq 1\,\texttt{DEGREE})\Big)(\texttt{john})$

$\Sigma \models$ Student(paul)

$\mathcal{ALC}$ and $\mathcal{K}_{(m)}$

$\mathcal{ALC}$	$\mathcal{K}_{(m)}$
$C^{\mathcal{I}}$ is a set of individuals	$\alpha_C^{\mathcal{I}}$ is a set of worlds
$R^{\mathcal{I}}$ is a set of pairs of individuals	R is an accessibility relation
A	P_A
$C \sqcap D$	$\alpha_C \land \alpha_D$
$C \sqcup D$	$\alpha_C \lor \alpha_D$
$\neg C$	$\neg \alpha_C$
$\forall R. C$	$\Box_R \ \alpha_C$
$\exists R. C$	$\Diamond_R \ \alpha_C$
$o \in C^{\mathcal{I}}$	$\mathcal{I}, o \models \alpha_C$
$\exists \mathtt{T}.\mathtt{C} \ \sqsubseteq \ \neg \mathtt{U} \sqcup \mathtt{P}$	$\Diamond_{\mathtt{T}} \mathtt{C} \rightarrow \neg \mathtt{U} \lor \mathtt{P}$
$\mathtt{U}(\mathtt{m}), \mathtt{T}(\mathtt{m}, \mathtt{c}), \mathtt{C}(\mathtt{c})$	
$\Sigma \models \mathtt{P}(\mathtt{m})$	

Some Results on satisfiability

$\mathcal{ALC}$	$\Longleftrightarrow$	$\mathcal{K}_{(m)}$
$\cap$		
$\mathcal{ALC}^{+}$	$\Longleftrightarrow$	$\mathcal{K}_{(m)} \cup S4$
$\cap$		
$\ldots$		
$\mathcal{ALC}$	$\Longleftrightarrow$	$\mathcal{K}_{(m)}$
$\cap$		
$\mathcal{ALCO}$	$\Longleftrightarrow$	$\mathcal{K}_{(m)}$ with nominals
$\mathcal{ALC}$	$\Longleftrightarrow$	$\mathcal{K}_{(m)}$
$\cap$		
$\mathcal{ALC}$ with axioms	$\Longleftrightarrow$	$\mathcal{K}_{(m)}$ theories

- $\mathcal{ALC}$: PSPACE-complete
- $\mathcal{ALC}^{+}$: PSPACE-complete
- $\mathcal{ALCO}$: PSPACE-complete
- $\mathcal{ALC}$ with axioms: EXPTIME-complete

$\mathcal{ALC}_{reg}$ **and** $\mathcal{PDL}$

- The domain of the interpretation is to be read as a set of program states.
- Concepts are to be interpreted as the set of states in which they hold.
- Roles are to be interpreted as *nondeterministic programs*.

$\mathcal{ALC}_{reg}$ and $\mathcal{PDL}$

- $\forall R. C$ as $[\mathbf{R}]\mathbf{C}$: "whenever program R terminates, proposition C holds on termination".
- $\mathbf{R_1} \circ \mathbf{R_2}$ as "run R_1 and R_2 consecutively".
- $\mathbf{R_1} \sqcup \mathbf{R_2}$ as "nondeterministically do R_1 or R_2".
- $\mathbf{R}^\star$ as "repeat program R a nondeterministically chosen number of times ≥ 0".
- $\mathsf{id}(\mathbf{C})$ as "proceed without changing the program state iff proposition C holds".

- $\mathbf{R^{-1}}$ as "run R in reverse" ($\mathcal{ALCI}_{reg}$ and $\mathcal{CPDL}$).

Internalization of axioms

$$\psi \models \varphi \quad \rightsquigarrow \quad \models [\nu]\, \psi \Rightarrow \varphi$$

$$C \mathrel{\dot{\sqsubseteq}} D \models \varphi \quad \rightsquigarrow \quad \models [\nu]\, (\alpha_C \Rightarrow \alpha_D) \Rightarrow \alpha_\varphi$$

$$\nu \doteq (R_1 \vee {R_1}^{-1} \vee \ldots \vee R_n \vee {R_n}^{-1})^\star$$

- Reasoning with theories is reduced to satisfiability of single formulas
- The complexity does not change

Results: satisfiability

$$
\begin{array}{ccl}
\mathcal{ALC}_{reg} & \Longleftrightarrow & \mathcal{PDL} \\
\Uparrow & & \\
\mathcal{ALCI}_{reg} & \Longleftrightarrow & \mathcal{CPDL} \\
\Uparrow & & \\
\mathcal{ALCFI}_{reg}{}^{-} & \Longleftrightarrow & \mathcal{DCPDL} \\
\Uparrow & & \\
\mathcal{ALCFI}_{reg} & & \\
\Uparrow & & \\
\mathcal{ALCQI}_{reg}{}^{-} & \Longleftrightarrow & \mathcal{CPDL} + \text{graded modalities} \\
\Uparrow & & \\
\mathcal{ALCQI}_{reg} & &
\end{array}
$$

- Satisfiability is EXPTIME-complete (subsumption and logical implication *wrt a free TBox* can be reduced to satisfiability).
- $\mathcal{CPDL}$ has the finite model property, $\mathcal{DCPDL}$ not.

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Results: satisfiability with individuals

$$\mathcal{ALC}_{reg} \iff \mathcal{PDL}$$

$$\Uparrow$$

$$\mathcal{ALCQO}_{reg} \iff \mathcal{PDL} + \text{graded modalities and nominals}$$

$$\mathcal{ALCI}_{reg} \iff \mathcal{CPDL}$$

$$\Uparrow$$

$$\mathcal{ALCIO}_{reg} \iff \mathcal{CPDL} + \text{nominals}$$

$$\mathcal{ALCI}_{reg} \iff \mathcal{CPDL}$$

$$\Uparrow$$

$$\mathcal{ALCQI}_{reg} + \text{ABox}$$

Satisfiability is EXPTIME-complete.

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So what?

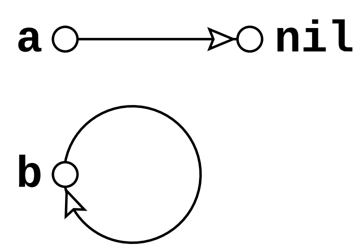
- DL extend modal logic in interesting ways:
  - Reasoning in DL is always reasoning with theories.
  - Nominals.
  - Graded modalities.
- Results:
  - $\mathcal{ALC}$ theories are EXPTIME-complete.
  - $\mathcal{ALCQI}_{reg}$ theory reduces to $\mathcal{ALC}_{reg}$ theory ($\mathcal{DCPDL}$ to $\mathcal{PDL}$).
  - $\mathcal{ALCIO}_{reg}$ theory reduces to $\mathcal{ALC}_{reg}$ theory ($\mathcal{CPDL}$ + nominals to $\mathcal{PDL}$).
  - $\mathcal{ALCQI}_{reg}$ + Abox theory reduces to $\mathcal{ALC}_{reg}$ theory.

Inductive Definitions

- An $\texttt{Empty-List}$ is a $\texttt{List}$.
- A $\texttt{Node}$, that has exactly one $\texttt{SUCCESSOR}$ that is a $\texttt{List}$, is a $\texttt{List}$.
- Nothing else is a $\texttt{LIST}$.

$$\texttt{Node} \doteq \neg\texttt{Empty-List}$$
$$\texttt{List} \doteq \texttt{Empty-List} \sqcup (\texttt{Node} \sqcap \leq 1\,\texttt{SUCCESSOR} \sqcap \exists\texttt{SUCCESSOR.List})$$

$$\Delta = \{\texttt{a}, \texttt{b}, \texttt{nil}\}$$
$$\texttt{Node}^{\mathcal{I}} = \{\texttt{a}, \texttt{b}\}$$
$$\texttt{Empty-List}^{\mathcal{I}} = \{\texttt{nil}\}$$
$$\texttt{SUCCESSOR}^{\mathcal{I}} = \{\langle\texttt{a}, \texttt{nil}\rangle, \langle\texttt{b}, \texttt{b}\rangle\}$$



With descriptive semantics:
$\texttt{List}^{\mathcal{I}} = \{\texttt{a}, \texttt{nil}\}$; $\texttt{List}^{\mathcal{I}} = \{\texttt{a}, \texttt{b}, \texttt{nil}\}$

With least fixpoint semantics:
$\texttt{List}^{\mathcal{I}} = \{\texttt{a}, \texttt{nil}\}$

Propositional μ-calculus

$\texttt{Node} \doteq \neg\texttt{Empty-List}$

$\texttt{List} \doteq \mu X.(\texttt{Empty-List} \sqcup$
$(\texttt{Node} \sqcap \leq 1\, \texttt{SUCCESSOR} \sqcap \exists \texttt{SUCCESSOR}.X))$

$$C \;\rightarrow\; \ldots \mid \mu X.C \mid \nu X.C \mid X$$

$\mu\mathcal{ALC} \quad \Longleftrightarrow \quad$ Propositional μ-calculus

$\Uparrow$

$\mu\mathcal{ALCQI}$

- Satisfiability is EXPTIME-complete.
- Can express *well-foundedness* of relations, useful to describe finite structures.
- Open problems: individuals.

Tense Logic

(point ontology)

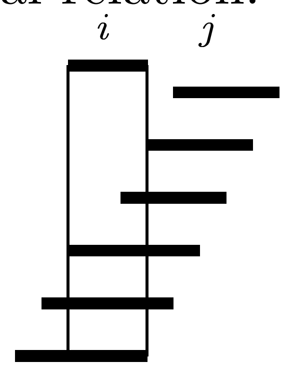
- Tense logic is a propositional modal logic, interpreted over temporal structure $\mathcal{T} = (\mathcal{P}, <)$, where $\mathcal{P}$ is a set of time points and $<$ is a strict partial order on $\mathcal{P}$.
- $\texttt{Mortal} \sqsubseteq \texttt{LivingBeing} \sqcap \forall\texttt{LIVES-IN.Place} \sqcap (\texttt{LivingBeing}\ \mathcal{U}\ (\Box^{+}\neg\texttt{LivingBeing}))$
- Satisfiability in $\mathcal{ALCT}$ – the combination of tense logic with $\mathcal{K}_{(m)}$ – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (PSPACE-complete).
- Satisfiability in $\mathcal{ALCQIT}_{reg}$ with ABox – the combination of tense logic with $\mathcal{ALCQI}_{reg}$ with ABox – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (EXPTIME-complete).

$\mathcal{HS}$: Interval Temporal Propositional Modal Logic

- $\mathcal{HS}$ is a propositional modal logic interpreted over an interval set $\mathcal{T}^*_<$, defined as the the set of all closed intervals $[u, v] \doteq \{x \in \mathcal{P} \mid u \leq x \leq v, u \neq v\}$ in some temporal structure $\mathcal{T}$.

- $\mathcal{HS}$ extends propositional logic with modal formulæ $\langle R \rangle \phi$ and $[R]\phi$ – where R is a basic Allen's algebra temporal relation:

before (i, j)
meets (i, j)
overlaps (i, j)
starts (i, j)
during (i, j)
finishes (i, j)



- $\texttt{Mortal} \doteq \texttt{LivingBeing} \wedge \langle \mathsf{after} \rangle . \neg \texttt{LivingBeing}$

- Satisfiability $\mathcal{HS}$ is undecidable for the most interesting classes of temporal structures.

- Therefore, $\mathcal{HS} \cup \mathcal{K}_{(m)}$ is undecidable.

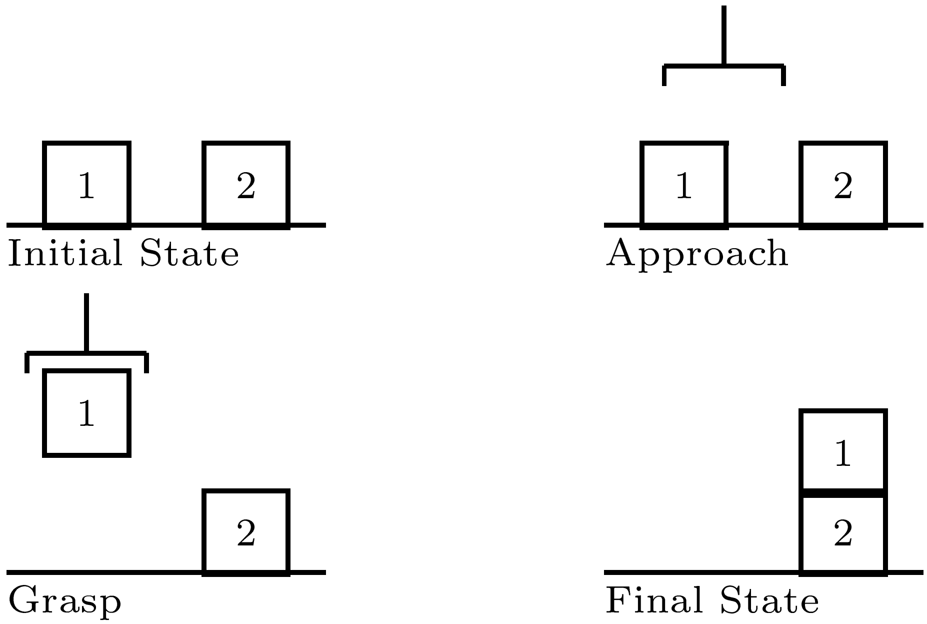
Decidable Interval Temporal Description Logics

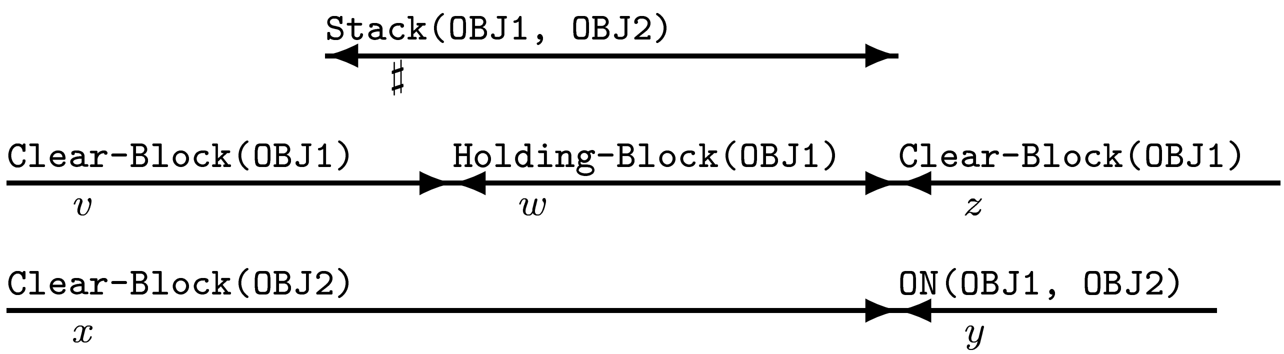
The combination of $\mathcal{ALCF}$ and $\mathcal{HS}^*$:

- $\mathcal{HS}^*$:
  - No universal quantification, or restricted to homogeneous properties: $\Box(=, \mathsf{starts}, \mathsf{during}, \mathsf{finishes}).\ \psi$
  - Allows for temporal variables: $\Diamond\ \overrightarrow{x}\ \mathsf{TN}(\overrightarrow{x}).\,\psi$ $\psi@x$
- Global roles – denoting temporal *independent* properties.

- Logical implication in the combined language is decidable (PSPACE-hard); satisfiability is PSPACE-complete.
- Logical implication in $\mathcal{TL}$-$\mathcal{F}$ is NP-complete.
- Useful for event representation and plan recognition.

The Block World Domain





$$\texttt{Stack} \doteq$$

$$\diamond(x\ y\ z\ v\ w)\ (\sharp \text{ finishes } x)(\sharp \text{ meets } y)(\sharp \text{ meets } z)$$
$$(v \text{ overlaps } \sharp)(w \text{ finishes } \sharp)(v \text{ meets } w).$$
$$\Big((\star\texttt{OBJECT2} : \texttt{Clear-Block})@x \sqcap$$
$$(\star\texttt{OBJECT1}\circ\texttt{ON} \stackrel{\downarrow}{=} \star\texttt{OBJECT2})@\texttt{y} \sqcap$$
$$(\star\texttt{OBJECT1} : \texttt{Clear-Block})@\texttt{v} \sqcap$$
$$(\star\texttt{OBJECT1} : \texttt{Holding-Block})@\texttt{w} \sqcap$$
$$(\star\texttt{OBJECT1} : \texttt{Clear-Block})@\texttt{z}\Big)$$

$\ddot{\mathcal{L}}^n$ FOL fragments

- $\ddot{\mathcal{L}}^n$ is the set of function-free FOL formulas with equality and constants, with only unary and binary predicates, and which can be expressed using at most n variable symbols.
- Satisfiability of $\ddot{\mathcal{L}}^3$ formulas is undecidable.
- Satisfiability of $\ddot{\mathcal{L}}^2$ formulas is NEXPTIME-complete.

The $\mathcal{DL}$ description logic

$\mathcal{ALCQI}_{reg}$	–	the transitive closure operator,
	–	number restriction operators,
	+	propositional calculus on roles,
	+	the concept $(R \subseteq S)$.

- The $\mathcal{DL}$ description logic and $\ddot{\mathcal{L}}^3$ are equally expressive.
- The $\mathcal{DL}^-$ description logic (i.e., $\mathcal{DL}$ without the composition operator) and $\ddot{\mathcal{L}}^2$ are equally expressive.
- Open problem: relation between $\mathcal{DL}$ including cardinalities and $\ddot{\mathcal{C}}^n$– adding counting quantifiers to $\ddot{\mathcal{L}}^n$.

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Guarded Fragments of FOL

The *guarded fragment* GF of FOL is defined as:

1. Every relational atomic formula is in GF
2. GF is propositionally closed
3. If $\mathbf{x}$, $\mathbf{y}$ are tuples of variables, $\alpha(\mathbf{x}, \mathbf{y})$ is atomic, and $\psi(\mathbf{x}, \mathbf{y})$ is a formula in GF, such that $\mathsf{free}(\psi) \subseteq \mathsf{free}(\alpha) = \{\mathbf{x}, \mathbf{y}\}$, then the following formulae are in GF:

   $\exists \mathbf{y}.\ \alpha(\mathbf{x}, \mathbf{y}) \land \psi(\mathbf{x}, \mathbf{y})$

   $\forall \mathbf{y}.\ \alpha(\mathbf{x}, \mathbf{y}) \rightarrow \psi(\mathbf{x}, \mathbf{y})$

The guarded fragment contains the modal fragment of FOL (and Description Logics); a weaker definition (LGF) is needed to include temporal logics.

Properties of GF

- GF has the finite model property
- GF and LGF have the tree model property
- Many important model theoretic properties which hold for FOL and the modal fragment, do hold also for GF and LGF
- Satisfiability is decidable for GF and LGF (deterministic double exponential time complete)
- Bounded-variable or bounded-arity fragments of GF and LGF (which include Description Logics) are in EXPTIME.
- GF with fix-points is decidable.

That's not all, folks...

- Defaults and non monotonic logic
- Other modal extensions
- Autoepistemic logic
- Formal ontology
- Decision procedures
- Database theory
- ...

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