Description Logics with n-ary Relations

Enrico Franconi and Ian Horrocks
Dept. of Computer Science, Univ. of Manchester
Manchester M13 9PL, UK
{franconi | horrocks}@cs.man.ac.uk
http : //www.cs.man.ac.uk/{∼franconi | ∼horrocks}

In this abstract we will briefly mention the research work which has been done at the University of Manchester on recent advances in Description Logics (DLs), namely the $\mathcal{DLR}$ generalisation of DLs towards n-ary relations. $\mathcal{DLR}$ was first introduced by [DL99] as a means for encoding expressive semantic data models for information systems such as extended Entity Relationship. It turns out that $\mathcal{DLR}$ elegantly generalises standard DLs, by allowing formulae referring to n-ary relations, not just to binary relations (i.e., roles). Still, the flavour of the obtained language is a modal logic, in the sense that formulae state properties holding in classes of worlds, determined by means of possibly complex paths constructed from n-ary relations. As a special case of the language, we again obtain a standard DL or a standard normal modal logic. We present here the first order fragment of the language introduced in [DL99]. We have chosen to limit the expressivity to first order since we are looking for a language implementable with the current technology. In particular, we refer to the current academic implementations of expressive DLs, namely the systems FaCT [Hor98] and iFaCT (FaCT extended with inverse relations). It has been recently demonstrated [HPS99] that the logic we are considering here allows for the implementation of sound and complete reasoning algorithms that behave quite well both in realistic applications and systematic tests.

The semantics of $\mathcal{DLR}$ rely on a model theoretic *interpretation* $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, consisting of a set $\Delta^{\mathcal{I}}$, called the *domain*, and an interpretation function $\cdot^{\mathcal{I}}$ that assigns to each concept C a subset $C^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$ and to each relationship $\boldsymbol{R}$ of arity n a subset $\boldsymbol{R}^{\mathcal{I}}$ of $(\Delta^{\mathcal{I}})^n$, such that the conditions in column 2 of Table 1 are satisfied (where A is an atomic concept, $\boldsymbol{P}$ is an atomic relationship, C and D are concepts, R and S are relationships, $\$i$ denotes the ith component of an n-ary relationship, and k is a non-negative integer).

It turns out that first order $\mathcal{DLR}$ theories can be encoded into an expressive DL theory, equipped

$\mathcal{DLR}$ syntax	$\mathcal{DLR}$ semantics	DL encoding
$\top_{\mathcal{C}}$	$\top_{\mathcal{C}}^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$	$\top_{\mathcal{C}} \quad (\top_{\mathcal{C}} \sqsubseteq \top)$
A	$A^{\mathcal{I}} \subseteq \top_{\mathcal{C}}^{\mathcal{I}}$	$A \quad (A \sqsubseteq \top_{\mathcal{C}})$
$\neg C$	$\top_{\mathcal{C}}^{\mathcal{I}} \setminus C^{\mathcal{I}}$	$\neg C$
$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$	$C \sqcap D$
$\exists[\$i]\boldsymbol{R}$	$\{d \in \top_{\mathcal{C}}^{\mathcal{I}} \mid \exists\langle d_1, \ldots, d_n\rangle \in \boldsymbol{R}.d_i = d\} \leqslant k\}$	$\exists U_i^-.C_R$
$\leqslant k[\$i]\boldsymbol{R}$	$\{d \in \top_{\mathcal{C}}^{\mathcal{I}} \mid \sharp\{\langle d_1, \ldots, d_n\rangle \in \boldsymbol{R} \mid d_i = d\} \leqslant k\}$	$\leqslant k U_i^-.C_R$
$\top_n$	$\top_n^{\mathcal{I}} \subseteq (\Delta^{\mathcal{I}})^n$	$\top_n \quad (\top_n \sqsubseteq \top)$
$\boldsymbol{P}$	$\boldsymbol{P}^{\mathcal{I}} \subseteq \top_n^{\mathcal{I}}$	$C_P \quad (C_P \sqsubseteq \top_n)$
$\neg\boldsymbol{R}$	$\top_n^{\mathcal{I}} \setminus \boldsymbol{R}^{\mathcal{I}}$	$\neg C_R$
$\boldsymbol{R} \sqcap \boldsymbol{S}$	$\boldsymbol{R}^{\mathcal{I}} \cap \boldsymbol{S}^{\mathcal{I}}$	$C_R \sqcap C_S$
$\$i_n : C$	$\{\langle d_1, \ldots, d_n\rangle \in \top_n^{\mathcal{I}} \mid d_i \in C^{\mathcal{I}}\}$	$\top_n \sqcap \forall U_i.C$

Table 1: $\mathcal{DLR}$ concepts and relationships, their semantics and their DL encodings

with generalised axioms, inverse relationships, and graded modalities (i.e., qualified number restrictions) [DL99]. A $\mathcal{DLR}$ theory consists of a finite set of axioms of the form $C \sqsubseteq D$ and $R \sqsubseteq S$, where C and D are concepts, and R and S are relationships of equal arity. Such a theory can be encoded in an equisatisfiable DL theory using the encodings described in column 3 of Figure 1 (where C_R is a concept corresponding to the $\mathcal{DLR}$ relationship R, and U_i is a functional role representing the ith component of a $\mathcal{DLR}$ relationship). Note that $\mathcal{DLR}$ relations are reified as DL concepts, and the components of the relationship represented as functional roles. While the encoding obviously relies on the ability of the DL to reason with inverse roles, another less obvious consequence is that this DL does not have the finite model property. New techniques have been studied which extend the algorithm of the FaCT system with blocking strategies that allow complete reasoning with such an expressive DL [HS99].

The $\mathcal{DLR}$ Description Logic was designed such that it is able to capture database schemata expressed in the most interesting Semantic Data Models and Object-Oriented Data Models [DL99, FS99]. For that purpose, the first order fragment suffices. The most common conceptual data model for database and information system design is the Entity-Relationship (ER) semantic data model. [DL99] refers to an extended version of the basic ER model, called EER, including taxonomic relationships, arbitrary boolean constructs, and entity definitions by means of both necessary and sufficient conditions and, more generally, by means of generalised axioms. It is shown how a schema expressed in an EER conceptual data model can be expressed in a $\mathcal{DLR}$ theory – whose models correspond with legal database states of the EER schemata – allowing for reasoning services such as satisfiability of a schema or the computation of a logically implied formula.

Various experiments have been done in the context of the European ESPRIT-4 Long Term Research project "Foundations of Data Warehouse Quality" (DWQ), using EER schemata as a source for testing the efficiency of reasoning with $\mathcal{DLR}$ theories in realistic domains. Previous approaches [CDR98] based on the translation of first order $\mathcal{DLR}$ theories into the basic $\mathcal{ALC}$ DL (in correspondence with multi-modal $\mathcal{K}$) introduce very large numbers of axioms, and have proved impractical. Using iFaCT and a slightly simplified version of the above encoding, promising initial results have been obtained with a $\mathcal{DLR}$ theory representing (fragments of) extended real world database schemata. Work is in progress to extend the algorithm implemented in iFaCT in order to allow a complete encoding of $\mathcal{DLR}$ [HST99]).

References

[CDR98] D. Calvanese, G. De Giacomo, and R. Rosati. A note on encoding inverse roles and functional restrictions in $\mathcal{ALC}$ knowledge bases. In E. Franconi, G. De Giacomo, R. M. MacGregor, W. Nutt, C. A. Welty, and F. Sebastiani, editors, *Collected Papers from the International Description Logics Workshop (DL'98)*, pages 69–71. CEUR, May 1998.

[DL99] G. De Giacomo and M. Lenzerini. Description logic framework for information integration. In *Proceedings of the 6th International Conference on the Principles of Knowledge Representation and Reasoning (KR-98)*, pages 2–13. Morgan Kaufmann, 1999.

[FS99] Enrico Franconi and Ulrike Sattler. A data warehouse conceptual data model for multidimensional aggregation: a preliminary report. *AI*IA Italian Association for Artificial Intelligence journal*, 1999. To appear.

[Hor98] I. Horrocks. Using an expressive description logic: FaCT or fiction? In A. G. Cohn, L. Schubert, and S. C. Shapiro, editors, *Principles of Knowledge Representation and Reasoning: Proceedings of the Sixth International Conference (KR'98)*, pages 636–647. Morgan Kaufmann Publishers, San Francisco, CA, June 1998.

[HPS99] I. Horrocks and P. F. Patel-Schneider. Optimising description logic subsumption. *Journal of Logic and Computation*, 1999. To appear.

[HS99] I. Horrocks and U. Sattler. A description logic with transitive and inverse roles and role hierarchies. *Journal of Logic and Computation*, 1999. To appear.

[HST99] I. Horrocks, U. Sattler, and S. Tobies. Practical reasoning for expressive description logics. Submitted, 1999.