Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2009/2010
Final exam – 5/2/2010 – Part 2

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE (or believed to be so, under certain assumptions, which you should state). You must give an explanation of your answer to receive full credit.

(a) $\mathrm{P}^{\mathrm{coNP}} = \mathrm{coNP}^{\mathrm{P}}$.

(b) Let L_1 and L_2 be languages. If $L_1 <_{poly} L_2$ and $L_2 \in$ NPSPACE, then $L_1 \in$ PSPACE.

(c) $\textsc{Size}(n^{O(1)}) \subseteq \mathrm{P}$.

Problem 2.2 [6 points] For a string $x \in \{0,1\}^*$, let x_{even} denote the *even part* of x, defined as the string constituted by the sequence of symbols of x that appear in x in even positions. Similarly, let x_{odd} denote the *odd part* of x, defined analogously. Let L be a language over $\Sigma = \{0,1\}$, and let

$$L' = \{\ w \in \Sigma^* \mid w \text{ has a substring } x \text{ such that either } x_{even} \in L \text{ or } x_{odd} \in L\}.$$

Show that if L is in NP, then also L' is in NP.

Problem 2.3 [6 points] Consider the proof of Cook's theorem that CSAT is NP-hard. Describe how in that proof the computation of a non-deterministic TM with running time $p(n)$, where $p(n)$ is a polynomial in n, is represented using propositional variables. Consider then *only* the conditions holding between such propositional variables that do *not* depend on the actual transitions of the TM, and provide the CNF-formulas that encode such conditions. How many clauses of which length are necessary to encode such conditions?

Problem 2.4 [6 points]

(a) Give the definition of when a language L belongs to $\textsc{Depth}(d(n))$.

(b) Provide a sketch of the proof that for every language L, we have that $L \in \textsc{Depth}(O(n))$. What other result can be proved by using essentially the same argument?

Problem 2.5 [6 points]

(a) Provide the definition of a Binary Decision Diagram (BDD) for n boolean variables. Consider the function $f_v(x_1, \ldots, x_n)$ associated to a node v of a BDD. Describe how, given binary values $a_1, \ldots, a_n$, the value $f_v(a_1, \ldots, a_n)$ can be computed. Define the complexity measures for BDDs.

(b) Construct a BDD that computes the minimum and maximum bits for 3 inputs x_1, x_2, x_3.

(c) Give the values of the complexity measures for the BDD you have constructed.

(d) Illustrate how the maximum bit is computed on the example of $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, and on the example of $x_1 = 0$, $x_2 = 0$, $x_3 = 0$.