Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2008/2009
Final exam – 30/1/2009 – Part 2

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE (or believed to be so, under certain assumptions, which you should state). You must give an explanation of your answer to receive full credit.

(a) Let L_1 and L_2 be languages. If $L_1 <_{poly} L_2$ and $L_2 \in$ NPSpace, then $L_1 \in$ PSpace.

(b) LogSpace $\subseteq$ Depth$((\log n)^2)$.

(c) There is a language L such that $L \notin$ Size$(O(2^n))$.

Problem 2.2 [6 points] For a language L, let

$$L' = \{\ w\#0^n \mid w \text{ is a substring of some } x \text{ in } L \text{ with } |x| = n\ \}.$$

Show that if L is in NP, then also L' is in NP.

Problem 2.3 [6 points] Consider the proof of Savitch's theorem that NPSpace = PSpace. Describe the main idea at the basis of the proof, and how this idea is exploited to simulate the computation of a non-deterministic TM N with space bound $t(n)$ by a deterministic TM D. What is the space bound of D?

Problem 2.4 [6 points] Let $\Sigma = \{0, 1\}$,

- $f_1 : \Sigma^* \to \Sigma^*$ be a function computable by a TM A_1 in space $s_1(n)$,
- $f_2 : \Sigma^* \to \Sigma^*$ be a function computable by a TM A_2 in space $s_2(n)$,
- $\ell_1(n) = \max_{x \in \Sigma^n} |f_1(x)|$, and
- $f(x) = f_2(f_1(x))$.

By exploiting the technique of *emulative composition*, describe how f can be computed in space

$$s(n) = s_1(n) + s_2(\ell_1(n)) + O(\log(\ell_1(n))) + \delta'(n),$$

where $\delta'(n) = O(\log(s_1(n) + s_2(\ell_1(n))))$. In your explanation you should justify the various components of the space bound $s(n)$.

Problem 2.5 [6 points]

(a) Provide the definition of a circuit with n boolean input variables. Define the complexity measures for circuits.

(b) Construct a circuit with 3 inputs x_1, x_2, x_3 that computes the even and odd parity bits for its inputs. An even (resp., odd) parity bit is set to 1 if the number of ones in a given set of bits is odd (resp., even), thus making the total number of ones, including the parity bit, even (resp., odd). Give the values of the complexity measures for the circuit you have constructed.

(c) Describe how a language L over $\{0, 1\}$ can be put into relationship with circuits that compute boolean functions.