Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2007/2008
Final exam – 30/6/2008 – Part 1

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] You must give a brief explanation of your answer to receive full credit.

(a) Decide whether the following statement is TRUE or FALSE: The complement of a recursive language is non-recursive.

(b) Decide whether the following statement is TRUE or FALSE: There exist two languages L_1 and L_2 such that there exists a reduction from L_1 to L_2, but there is no reduction from L_2 to L_1.

(c) What is the complexity of transforming a 2-tape deterministic TM into a 1-tape deterministic TM?

(d) Determine whether the following problem is decidable: Given a pair $\langle M, w\rangle$ constituted by the encoding $\mathcal{E}(M)$ of a TM M followed by an input string w, decide whether for each pair of states q, q' of M, M contains a transition $\delta(q, a) = (q', a', d)$ (for some alphabet symbol a appearing in w, tape alphabet symbol a', and direction d).

Problem 1.2 [6 points] Construct a TM M that accepts the language $L = \{w\texttt{\#}n \mid n$ is a number represented in binary with the least significant digit on the *left*, and $w \in \{\texttt{a}, \texttt{b}\}^*$ with $|w|_\texttt{b} > n\}$, where $|w|_\texttt{b}$ denotes the number of occurrences of `b` in w.

E.g.: `abbab#01` $\in L$, `abab#01` $\notin L$, `#0` $\notin L$.

Show the sequence of IDs of M on the input strings "`abbab#01`" and "`abab#01`".

Problem 1.3 [6 points] For a Turing Machine M, let $\mathcal{E}(M)$ denote the encoding of M, and $\langle M, w\rangle$ denote the encoding $\mathcal{E}(M)$ of M together with an input string w. Consider the language $L = \{\langle M, w\rangle \mid M$, when started on the input string w, eventually erases the whole tape. Show that L is recursively enumerable.

[*Hint*: Make use of a universal TM.]

Problem 1.4 [6 points]

- Let f and g be primitive recursive functions. Show that the following predicate p is primitive recursive:

$$p(x) = \begin{cases} 1 & \text{if } f(i) = g(j), \text{ for some } 0 \le i, j \le x \\ 0 & \text{otherwise} \end{cases}$$

- Show that the following function f is primitive recursive:

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \text{ or } x = 1 \\ f(x-1) + 3 \cdot f(x-2) & \text{if } x \ge 2 \end{cases}$$

Problem 1.5 [6 points] Let g and h be partial number-theoretic functions with n and $n + 2$ variables, respectively.

(a) Provide the definition of the $n + 1$-variable function f obtained from g and h by *primitive recursion*.

(b) State the conditions under which $f(x_1, \ldots, x_n, y)$ is defined, where y is the recursive variable.