Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2006/2007
Final Exam – 1/2/2007 – Part 1

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [4.5 points] Decide, if possible, which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $(L_1^* \cdot L_2^*)^+ = (L_1^+ \cdot L_2^+)^*$.

(b) If L_1 and L_2 are both non-regular, then $L_1 \cup L_2$ could be regular.

(c) For all languages L_1 and L_2, if $L_1 \subseteq L_2$, then $L_1^* \subseteq L_2^*$.

Problem 1.2 [1.5 points] Simplify the regular expression E as much as possible, where

$$E = (((a+b)^* \cdot ((b \cdot \emptyset) + \varepsilon))^* + (b+a)^*) + \varepsilon$$

Motivate each simplification step you have applied by an algebraic law for regular expressions.

Problem 1.3 [6 points] Given a regular expression E, we are interested in an NFA A such that $\mathcal{L}(A) = \mathcal{L}(E)$.

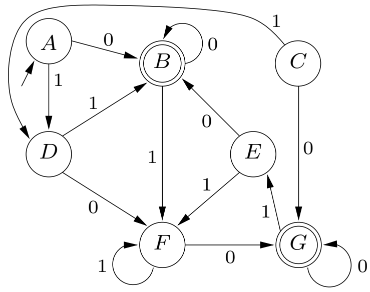
(a) Describe an algorithm to construct such an NFA.

(a) Illustrate the steps of the algorithm you have chosen to construct E, on the example of the regular expression $E = ((a \cdot b) + (b+c)^*)^*$

Problem 1.4 [6 points] Given an DFA $D = (Q, \Sigma, \delta, q_0, F)$, we are interested in the DFA $D_m = (Q_m, \Sigma, \delta_m, q_{0m}, F_m)$ with minimal number of states such that $\mathcal{L}(D_m) = \mathcal{L}(D)$.

(a) Describe how such a minimal DFA can be defined, and provide an algorithm for its construction.

(b) Illustrate the steps of the algorithm you have followed to construct D_m in the case of the DFA D over $\Sigma = \{0, 1\}$ specified by the given transition diagram.



Problem 1.5 [6 points] Show that the following language L is not regular:

$$L = \{\ xy \mid x, y \in \{0,1\}^* \text{ and } \#_0(x) \geq \#_0(y)\ \}$$

where $\#_0(w)$ denotes the number of 0's in a string w.

[*Hint*: Use the pumping lemma, and consider e.g., the string $0^n 1^{2n} 0^n$, for a suitable value of n.]

Problem 1.6 [6 points] Let L be the language generated by the grammar $G = (\{S\}, \{a, b\}, P, S)$, where P consists of the following productions

$$S \longrightarrow \varepsilon \mid Sa \mid bS \mid abS$$

Prove that *no* string generated by G has aab as a substring. Make all steps of the proof explicit.

[*Hint:* Use an induction on the length of the derivation according to G of a sentential form $w_1 S w_2$, establishing properties that hold for w_1 and w_2.]