Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2006/2007
Midterm Exam – 20/11/2006

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $(L_1^* \cap L_2^*)^* = (L_1 \cap L_2)^*$.

(b) If L_1 is regular and L_2 is non-regular, then $L_1 \cdot L_2$ can be regular.

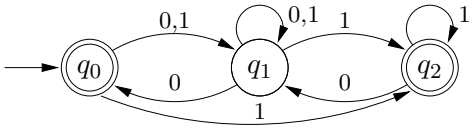
(c) A regular expression denotes an infinite language if and only if it contains the $*$ operator.

(d) For all languages L_1 and L_2, if L_1 is context free and $L_1 \subseteq L_2$, then L_2 cannot be regular.

Problem 1.2 [6 points] Given an NFA $N = (Q, \Sigma, \delta, q_0, F)$, we are interested in a DFA $D = (Q_D, \Sigma, \delta_D, q_{0D}, F_D)$ such that $\mathcal{L}(D) = \mathcal{L}(N)$.

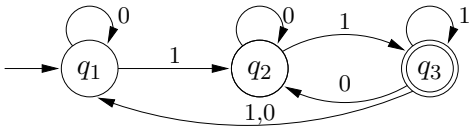
(a) Show in general how such a DFA D can be defined in terms of an NFA N.

(b) Illustrate the steps of the algorithm you have followed to construct D in the case of the DFA D over $\Sigma = \{0, 1\}$ specified by the following transition diagram:



Problem 1.3 [6 points] Describe an algorithm to construct from an NFA A a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(A)$.

Illustrate the steps of the algorithm you have chosen to construct E, on the example of the following NFA A over $\{0, 1\}$:



Problem 1.4 [6 points] Show that the following language L is not regular:

$$L = \{a^m b^n c^k \mid m, n, k \geq 0, m \neq n \text{ or } m \neq k \text{ or } n \neq k\}$$

[*Hint*: Consider first the language $\overline{L} \cap a^* b^* c^*$ and show that it is not regular using the pumping lemma. Then exploit the closure properties of regular languages.]

Problem 1.5 [6 points] Consider the language L over $\Sigma = \{0, 1, \#\}$ defined as follows:

$$L = \{x^R \# y \mid x, y \in \{0, 1\}^*,\ x \text{ is a substring of } y\}$$

where x^R denotes the reverse string of x.

(a) Show that L is context free by exhibiting a context free grammar G that generates L. Be precise in the specification of the grammar, by providing explicitly all its elements.

(b) Show the leftmost derivation according to G for the string 110#001110 and for the string 10#1010011. Draw the corresponding parse trees.

(c) Is the grammar you have provided ambiguous? Argue convincingly.