Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2005/2006

Final exam – 6/2/2006 – Part 2

Time: 90 minutes

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if both L_1 and L_2 are in NP, then also $L_1 \cup L_2$ is in NP.

(b) For all languages L_1 and L_2, if L_2 is in NP and $L_1 <_{poly} L_2$, then L_1 is in P.

(c) The class NP is closed under complement.

(d) There exists a language L such that both L and $\overline{L}$ are recursively enumerable.

Problem 2.2 [6 points] Consider the context free grammar $G = (\{S, A\}, \{0\}, P, S)$ where P consists of the following productions:

$$\begin{array}{rcl} S & \longrightarrow & ASA \mid A \mid \varepsilon \\ A & \longrightarrow & 00 \mid \varepsilon \end{array}$$

Convert G into Chomsky Normal Form. Illustrate the various steps of the algorithm.

Problem 2.3 [6 points] Let $L_p = \{w \in \{0,1\}^* \mid w \text{ is a palindrome}\}$ be the language of binary words that are palindromes. A word w is a palindrome if, when it is read from left to right, spells in the same way as when it is read from right to left. Construct a TM M_p that decides L_p, i.e., such that M_p always halts and $\mathcal{L}(M_p) = L_p$. Show the sequence of IDs of M_p on input string 1001 and on input string 10011.

Problem 2.4 [6 points] Show that the class of recursive languages is closed under concatenation. To do so, informally but precisely describe how to construct, from two halting TMs M_1 and M_2, a new halting TM that accepts $\mathcal{L}(M_1) \cdot \mathcal{L}(M_2)$. [*Hint*: Make use of standard TM constructions and extensions of the basic TM model, e.g., with multiple tapes or with non-determinism.]

Problem 2.5 [6 points] For a TM M with input alphabet $\Sigma = \{a, b\}$, let $\mathcal{E}(M)$ denote the encoding of M, and $\langle \mathcal{E}(M), w \rangle$ denote the encoding of M together with an input word w. Consider the language $L = \{\langle \mathcal{E}(M), w \rangle \mid M$, when started on an input word w, eventually overwrites an occurrence of symbol a with symbol b somewhere on the tape$\}$.

(a) Show that L is recursively enumerable. [*Hint*: Make use of a universal TM.]

(b) Show that L is not recursive. [*Hint*: Exploit a reduction from the halting problem.]