Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2005/2006

Final exam – 6/2/2006 – Part 1

Time: 90 minutes

Problem 1.1 [4.5 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $((L_1 \cup \{\varepsilon\}) \cdot (L_2 \cup \{\varepsilon\}))^* = (L_1 \cup L_2)^*$.

(b) For all languages L_1 and L_2, if L_1 is regular and $L_2 \subseteq L_1$, then L_2 is regular.

(c) There exist nonempty languages L_1 and L_2, with $L_1 \neq \{\varepsilon\}$, $L_2 \neq \{\varepsilon\}$, and $L_1 \neq L_2$, such that $L_1 \cdot L_2 = L_2 \cdot L_1$.

Problem 1.2 [1.5 points] Find a regular expression for the set of binary strings that have 00 but not 11 as substrings.

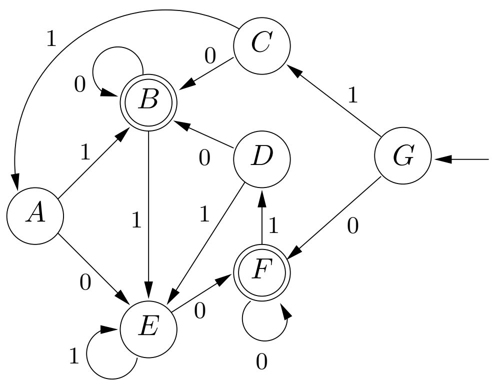
Problem 1.3 [4 points] Consider the language $L = \{x0^n y1^n z \mid n \geq 0, x \in L_1, y \in L_2, z \in L_3\}$, where L_1, L_2, L_3 are nonempty languages over the alphabet $\{0, 1\}$.

(a) Find nonempty regular languages L_1, L_2, L_3 such that L is regular.

(a) Find nonempty regular languages L_1, L_2, L_3 such that L is not regular.

Problem 1.4 [6 points] Consider the regular expression $E = (b + a)^* + (a \cdot b)^*$. Construct an ε-NFA A_ε such that $\mathcal{L}(A_\varepsilon) = \mathcal{L}(E)$. Simplify intermediate results whenever possible. Then, by eliminating ε-transitions from A_ε, construct an NFA A such that $\mathcal{L}(A) = \mathcal{L}(A_\varepsilon)$. Illustrate the steps of the algorithm you have followed to construct A_ε and A.

Problem 1.5 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct A_m.

Problem 1.6 [4 points] Show that the language $\{a^m b^n \mid m, n \geq 0, m \neq n\}$ is context free by exhibiting a context free grammar that generates it.

Problem 1.7 [4 points] A context-free grammar $G = (V_N, V_T, P, S)$ is said to be *linear* if every production in P is of the form $A \longrightarrow xB$ or $A \longrightarrow Bx$ or $A \longrightarrow x$, where $A, B \in V_N$ and $x \in V_T^*$. Show that the language generated by a linear grammar is not necessarily regular.