## Free University of Bozen-Bolzano – Faculty of Computer Science Master of Science in Computer Science Theory of Computing – A.A. 2004/2005 Final exam – 7/6/2005 – Part 2

## $Time: \ 90 \ minutes$

**Problem 2.1** [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

- (a) For all languages L, if L is in NP, then its complement  $\overline{L}$  is in P.
- (b) For all languages  $L_1$  and  $L_2$ , if  $L_1$  is in P and  $L_1 <_{poly} L_2$ , then  $L_2$  is in NP.
- (c) The class NP is closed under intersection.
- (d) There exists a language L such that L is recursively enumerable and  $\overline{L}$  is recursive.

**Problem 2.2** [6 points] Consider the context-free grammar  $G = (\{S, A, B\}, \{a, b, c\}, P, S)$ , where P consists of the following productions

$$\begin{array}{rcccc} S & \longrightarrow & aA \\ A & \longrightarrow & BA \mid a \\ B & \longrightarrow & bS \mid cS \end{array}$$

Construct a PDA M that accepts  $\mathcal{L}(G)$  by empty stack. Draw the parse tree of G for the string *abaaa*, and show the corresponding execution trace for M.

**Problem 2.3** [6 points] Consider the context free grammar  $G = (\{S, A, B, C\}, \{a, b\}, P, S)$  where P consists of the following productions:

Convert G into Chomsky Normal Form. Illustrate the various steps of the algorithm.

**Problem 2.4** [6 points] Describe how to obtain, for any given TM M, a new TM M' such that  $\mathcal{L}(M') = \mathcal{L}(M)$ , but when M' halts its tape is exactly as in the initial configuration (i.e., it contains the input string w, and the head of M' is positioned on the leftmost symbol of w). You can make use of standard constructions presented in the course.

**Problem 2.5** [6 points] For a TM M, let  $\mathcal{E}(M)$  denote the encoding of M. Consider the language  $L = \{\mathcal{E}(M) \mid M, \text{ when started on a blank tape eventually writes a 1 somewhere on the tape}.$ 

- (a) Show that L is recursively enumerable. [*Hint*: Make use of a universal TM.]
- (b) Show that L is not recursive. [*Hint*: Exploit a reduction from the halting problem.]