Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2004/2005
Final exam – 7/6/2005 – Part 1

Time: 90 minutes

Problem 1.1 [4.5 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $L_1^* \cap L_2^* = (L_1 \cap L_2)^*$.

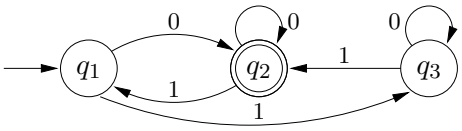
(b) If L_1 is regular and L_2 is non-regular, then $L_1 \cup L_2$ must be non-regular.

(c) There exists a language L such that L is not regular but L^* is regular.

Problem 1.2 [1.5 points] Show that $L^* = L \cdot L^*$ if and only if $\varepsilon \in L$.

Problem 1.3 [6 points] Consider the regular expression $E = ((1 \cdot 0)^* \cdot 0)^* + (1 \cdot 1)$. Construct an ε-NFA A_ε such that $\mathcal{L}(A_\varepsilon) = \mathcal{L}(E)$. Simplify intermediate results whenever possible. Then, by eliminating ε-transitions from A_ε, construct an NFA A such that $\mathcal{L}(A) = \mathcal{L}(A_\varepsilon)$. Illustrate the steps of the algorithm you have followed to construct A_ε and A.

Problem 1.4 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct E.

Problem 1.5 [5 points] The *quotient* L_1/L_2 of two languages L_1 and L_2 is defined as

$$L_1/L_2 = \{x \mid \text{ there is } y \in L_2 \text{ such that } xy \in L_1\}$$

For example, if $L_1 = \{w \in \{0,1\}^* \mid w \text{ has an even number of 0's}\}$, $L_2 = \{0\}$, and $L_3 = \{0, 00\}$, then $L_1/L_2 = \{w \in \{0,1\}^* \mid w \text{ has an odd number of 0's}\}$, and $L_1/L_3 = \{0,1\}^*$.

Show that, for an *arbitrary* language L_2, if L_1 is regular, then L_1/L_2 is also regular.

[*Hint*: Start from a DFA A for L_1, and show how to modify the set of final states of A to obtain a DFA for L_1/L_2.]

Problem 1.6 [3 points] Show that the language $\{uawb \mid u, w \in \{a, b\}^*, \text{ with } |u| = |w|\}$ is context free by exhibiting a context free grammar that generates it.

Problem 1.7 [4 points] Consider the grammar $G = (\{S, T\}, \{0, 1\}, P, S)$, where P consists of the following productions

$$\begin{aligned} S &\longrightarrow 0S \mid 1T \mid 0 \\ T &\longrightarrow 1T \mid 1 \end{aligned}$$

Show that no string in the language $\mathcal{L}(G)$ contains the substring 10.