Free University of Bozen-Bolzano - Faculty of Computer Science Master of Science in Computer Science Theory of Computing – A.A. 2004/2005 Final exam -4/2/2005 – Part 2

Time: 90 minutes

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

- (a) If a language L is in P, then its complement must belong to NP.
- (b) The class NP is not closed under union.
- (c) If B is recursive and $A <_{poly} B$, then A is recursive.
- (d) There exists a language L such that L is recursive and \overline{L} is R.E. but not recursive.

Problem 2.2 [4 points] Consider a context free grammar and the problem of transforming it in Chomsky Normal Form. What is the right order in which the following steps have to be performed?

- eliminate unit productions
- factor long productions

- eliminate ε -productions
- eliminate non-generating symbols
- eliminate unreachable symbols

Motivate your answer by describing which transformation may or may not affect which other one.

Problem 2.3 [6 points] Consider the context free grammar $G = (\{S, A, B, C, D\}, \{a, b\}, P, S)$ where P consists of the following productions:

Convert G into Chomsky Normal Form. Illustrate the various steps of the algorithm.

Problem 2.4 [4 points] Consider the problem of deciding whether a given context free language (CFL) over an alphabet Σ is equal to Σ^* . Suppose that you have already shown that this problem is undecidable. (You do not need to prove this, just assume you have already proved it!) Making use of this fact, prove that also the following problem is undecidable: Decide whether for two CFLs L_1 and L_2 , we have that $L_1 \subseteq L_2$.

Problem 2.5 [4 points] Construct, for any given TM M, a new TM M' such that $\mathcal{L}(M') = \mathcal{L}(M)$, but when M' halts its tape is always empty (i.e., it contains only the blank symbol). Describe how to modify the transitions of M and/or which transitions to add to obtain M'.

Problem 2.6 [6 points] Consider the language $L = \{ \langle \mathcal{E}(M), w, i \rangle \mid M \text{ enters state } q_i \text{ on input } w \},\$ where $\mathcal{E}(M)$ denotes the encoding of the TM M. Show that L is not recursive.

[*Hint*: Exploit a reduction from the halting problem.]