Coherence, Similarity, and Concept Generalisation

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What is this about?

- **Joint coherence** of concepts (wrt background ontology)
- Thagard-style **coherence maximising partition(s)**
- **Common Generalisation** of maximising partition(s)

What is this about?

White_Cats

Black_Cats

$$2^{74207281} - 1$$

Prime_Numbers

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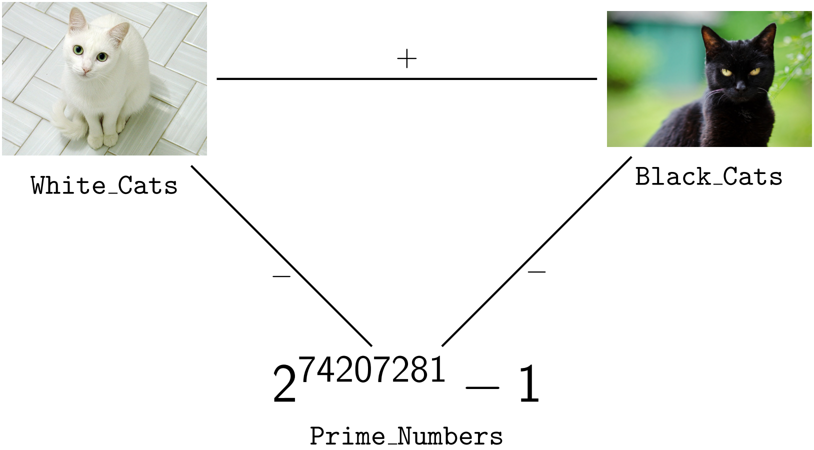

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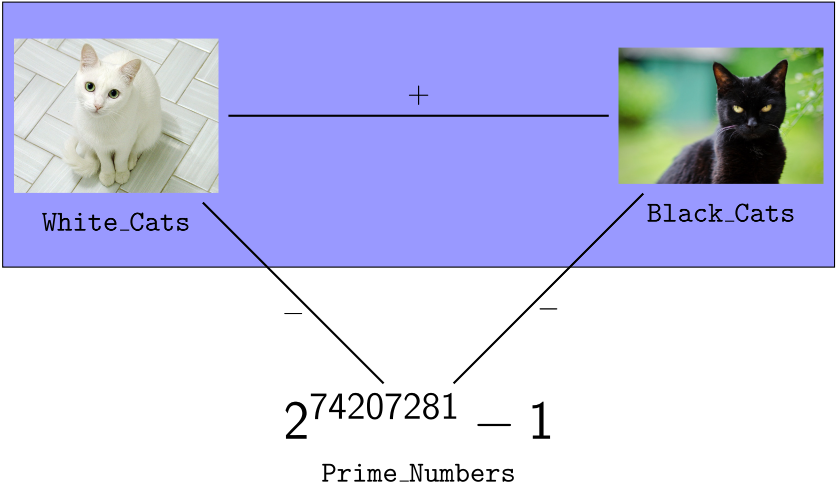

What is this about?

Greyscale_Cats

$$2^{74207281} - 1$$

Prime_Numbers

Main Procedure

Input:

- $\mathcal{ALC}$ TBox (background ontology) $\mathcal{T}$
- Concepts $C_1 \ldots C_n$

Output:

- Coherence Maximising Partitions $A \subseteq \{C_1 \ldots C_n\}$
- Common Generalisations (as justifications)

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1. Define **generalisation refinement operator** from $\mathcal{T}$;
2. Compute **similarity** between pairs (C_i, C_j);
3. Evaluate **coherence or incoherence** between pairs;
4. Find **coherence maximising partitions**;
5. Compute respective generalisations.

Generalisation Refinement Operator

Given a TBox $\mathcal{T}$, and a concept C, the generalisation operator $\gamma(C)$ is defined along the lines of (Confalonieri et al, 2016).

Main Idea: we generalise a concept C **either** by finding a concept $D \in \mathsf{sub}(\mathcal{T})$ which is immediately more general than C **or** by generalising one of the subformulas inside C. $\gamma(C)$ is the (finite) set of all such generalisations.

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- $\gamma^*(C)$ is the set of all concepts which can be obtained from C through repeated generalisations;
- For $D \in \gamma^*(C)$, $\lambda(C \xrightarrow{\gamma} D) =$ smallest number of steps from C to D.

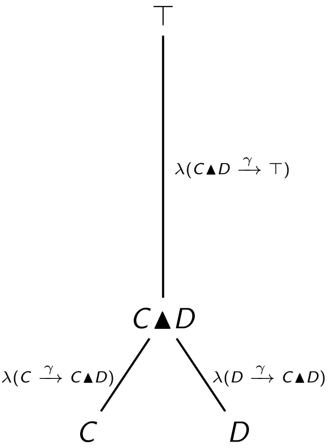
Common Generalisation(s)

$$C \blacktriangle D = G \in \gamma^*(C) \cap \gamma^*(D)$$

s.t. $\forall G' \in \gamma^*(C) \cap \gamma^*(D)$,

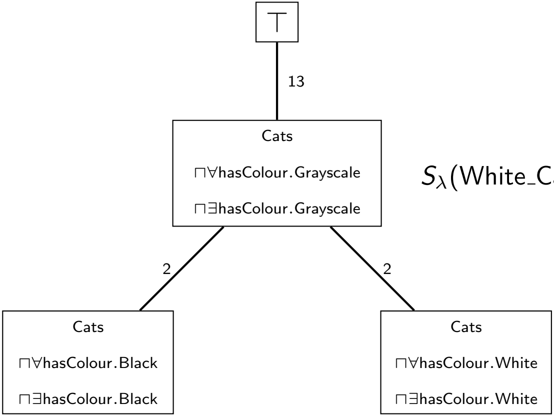
- $\lambda(C \xrightarrow{\gamma} G) + \lambda(D \xrightarrow{\gamma} G) < \lambda(C \xrightarrow{\gamma} G') + \lambda(D \xrightarrow{\gamma} G')$, or
- $\lambda(C \xrightarrow{\gamma} G) + \lambda(D \xrightarrow{\gamma} G) = \lambda(C \xrightarrow{\gamma} G') + \lambda(D \xrightarrow{\gamma} G')$
  and
  $\lambda(G \xrightarrow{\gamma} \top) \geq \lambda(G' \xrightarrow{\gamma} \top)$

Not Unique!



Concept Similarity

$$S_\lambda(C,D) = \begin{cases} \dfrac{\lambda(C \blacktriangle D \xrightarrow{\gamma} \top)}{\lambda(C \blacktriangle D \xrightarrow{\gamma} \top) + \lambda(C \xrightarrow{\gamma} C \blacktriangle D) + \lambda(D \xrightarrow{\gamma} C \blacktriangle D)} & \text{if } C \text{ or } D \neq \top \\ 1 & \text{otherwise.} \end{cases}$$



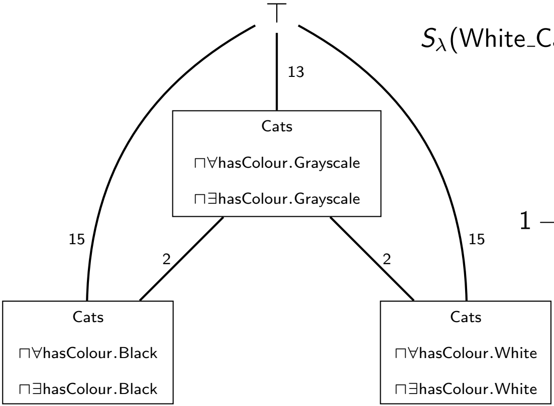
$$S_\lambda(\text{White\_Cats}, \text{Black\_Cats}) = \frac{13}{17}$$

Coherence and Incoherence

- C coheres with D $(+)$ if $S_\lambda(C, D) > 1 - \delta$
- C incoheres with D $(-)$ if $S_\lambda(C, D) \leq 1 - \delta$

where

$$\delta = \frac{\lambda(C \blacktriangle D \xrightarrow{\gamma} \top)}{\max\{\lambda(C \xrightarrow{\gamma} \top), \lambda(D \xrightarrow{\gamma} \top)\}}$$

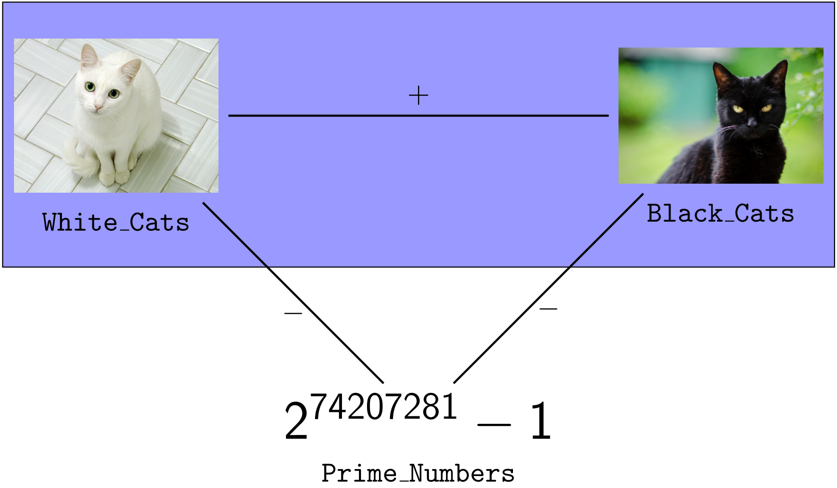


$$S_\lambda(\text{White\_Cats}, \text{Black\_Cats}) = \frac{13}{17}$$

$$\delta = \frac{13}{15}$$

$$1 - \delta < \frac{13}{17} : +$$

Coherence Maximising Partitions



Conclusions

What we have:

- Theoretical framework for **joint coherence** of concepts
- Common generalisation as justification

What we need:

- Implementation
- Real-world (not toy) tests
- Graded coherence/incoherence
- Exploration of different similarity measures (e.g. Resnik 1995)

References

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