11. Tableaux with TBoxes

Exercise 11.1 Consider the following $\mathcal{ALC}$ TBox $\mathcal{T}$:

$$
\begin{aligned}
A &\equiv \forall R.\neg(\neg B \sqcap C) \\
B &\equiv \exists R.C
\end{aligned}
$$

1. Analyze whether $\mathcal{T}$ is cyclic.

2. Determine, using tableaux, whether the concepts

   (a) $\exists R.(A \sqcap B)$ and
   (b) $A \sqcap \exists R.A \sqcap \neg(\exists R.\neg C \sqcup \exists R.\exists R.C)$

   are satisfiable w.r.t. $\mathcal{T}$.

3. Suppose that the assertion $C \equiv A \sqcup D$ is added to $\mathcal{T}$. Discuss whether the same tableaux technique used before can be applied, motivating your answer. If not, explain how the tableaux technique has to be extended.