5. Basics of Description Logics

Exercise 5.1 Translate the following DL expressions and axioms into first-order logic:

1. $\mathsf{Father} \sqcap \forall \mathsf{child}.(\mathsf{Doctor} \sqcup \exists \mathsf{managedBy}^-.(\mathsf{Company} \sqcap \neg \mathsf{MoreThan3Employees}))$

   Solution: $\mathsf{Father}(x) \land \forall y.\ (\mathsf{child}(x,y) \to (\mathsf{Doctor}(y) \lor \exists z.(\mathsf{managedBy}(z,y) \land \mathsf{Company}(z) \land \neg \mathsf{MoreThan3Employees}(z))))$

2. $\mathsf{Person} \sqcap \forall \mathsf{child}.\mathsf{HappyPerson} \ \sqsubseteq\ \exists \mathsf{child}.\forall \mathsf{child}.\mathsf{HappyPerson}$

   Solution: $\forall x.((\mathsf{Person}(x) \land \forall y.(\mathsf{child}(x,y) \to \mathsf{HappyPerson}(y))) \ \to (\exists y.(\mathsf{child}(x,y) \land \forall z.(\mathsf{child}(y,z) \to \mathsf{HappyPerson}(z))))$

3. $\mathsf{Person} \sqcap \exists \mathsf{child}.\mathsf{HappyPerson} \ \sqsubseteq\ \mathsf{Happy} \sqcap (\mathsf{Father} \sqcup \mathsf{Mother})$

   Solution: $\forall x.((\mathsf{Person}(x) \land \exists y.(\mathsf{child}(x,y) \land \mathsf{HappyPerson}(y))) \ \to (\mathsf{Happy}(x) \land (\mathsf{Father}(x) \lor \mathsf{Mother}(x))))$

Exercise 5.2 Translate the following sentences and first-order logic formulas into DL syntax, if possible:

1. Only humans have children that are humans.

   Solution: $\exists \mathsf{hasChild}.\mathsf{Human} \sqsubseteq \mathsf{Human}$

2. A node cannot have two distinct P-successors, such that one is a B and the second one is not a B.

   Solution: $\mathsf{Node} \sqsubseteq \neg((\exists P.B) \sqcap (\exists P.\neg B))$

3. $\forall x_1, x_2, y_1, y_2.\ P(x_1,y_1) \land P(x_1,y_2) \land P(x_2,y_2) \to x_1 = x_2 \lor y_1 = y_2$

   Solution: $\top \sqsubseteq (\leq 1 P) \sqcup (\forall P.(\leq 1 P^-))$

4. $\forall x, y, z.\ P(x,y) \land P(y,z) \land P(z,x) \to A(x)$

   Solution: Not expressible in DL. One cannot express triangles in DL.
   If we are allowed to use the identity role id, we can express it by $\exists((P \circ P \circ P) \cap id) \sqsubseteq A$

5. $\forall x, y, z.\ P(x,y) \land Q(y,z) \to R(x,z)$

   Solution: $P \circ Q \sqsubseteq R$

6. $\neg(\forall x.\ A(x) \to B(x)) \lor (\forall x.\ A(x) \to C(x))$

   Solution: Not expressible

7. $\exists x.\forall y.\ R(x,y) \lor S(x,y)$

   Solution: $\top \sqsubseteq \exists R^-.\{o\} \sqcup \exists S^-.\{o\}$

Exercise 5.3 Compute the certain answers to the query q over the KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$.

1. $q(x) = B(x), \quad \mathcal{A} = \{A(a), B(b), C(c)\}, \quad \mathcal{T} = \{A \sqsubseteq B,\ C \sqsubseteq \exists R,\ \exists R^- \sqsubseteq B\}$.

2. $q() = \exists x.B(x), \quad \mathcal{A} = \{A(a)\}$,

   (a) $\mathcal{T} = \{A \sqsubseteq \exists R, \quad \exists R^- \sqsubseteq B\}$.
   Solution: True

(b) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \quad \exists R^- \sqsubseteq B\}$.
Solution: False

(c) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcap (\exists S \sqcup \exists Q), \quad \exists R^- \sqsubseteq B, \quad \exists Q^- \sqsubseteq B\}$.
Solution: True

(d) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \quad \exists R^- \sqsubseteq B, \quad \exists S^- \sqsubseteq \exists R \sqcup \exists Q, \quad \exists Q^- \sqsubseteq \exists R\}$.
Solution: True

3. $q(x) = \exists y.R(x, y), \quad \mathcal{A} = \{A(a), R(b, c)\}, \quad \mathcal{T}$ as in Item 2.

   Solution:

   (a) $\{a, b\}$
   (b) $\{b\}$
   (c) $\{a, b\}$
   (d) $\{b\}$

4. $q(x) = \exists y.R(x, y), \quad \mathcal{A} = \{A(a), R(a, c)\}, \quad \mathcal{T}$ as in Item 2.

   Solution:

   (a) $\{a\}$
   (b) $\{a\}$
   (c) $\{a\}$
   (d) $\{a\}$