1. Basics of First-Order Logic

Exercise 1.1 In the following first-order sentences, $Bird(x)$ means "x is a bird", $Flies(x)$ means "x flies", $Person(x)$ means "x is a person", and $Mother(x, y)$ means "x is the mother of y". Translate the sentences into English:

1. $\forall x \, (Bird(x) \rightarrow Flies(x))$

2. $\forall x \, \exists y \, (Person(x) \rightarrow Mother(y, x))$

3. $\exists x \, \forall y \, (Person(x) \wedge Mother(x, y))$

Exercise 1.2 Convert the following English sentences into sentences of first-order logic. Use meaningful predicate names or state the abbreviation scheme that you are using.

1. All cats are mammals.

2. No cat is a reptile.

3. All computer scientists like some operating system.

4. The only good extraterrestrial is a drunk extraterrestrial.

5. The Barber of Seville shaves all men who do not shave themselves.

6. There are at least two mountains in England.

7. No mountain is higher than itself.

8. There is exactly one coin in the box.

9. There are exactly two coins in the box.

10. The largest coin in the box is a quarter.

11. All students get good grades if they study.

Exercise 1.3 Assume N is intended to mean "is a number"; I is intended to mean "is interesting"; $<$ is intended to mean "is less than"; and 0 is a constant symbol intended to denote zero. Translate into first-order logic sentences the English sentences listed below. If the English sentence is ambiguous, you will need more than one translation.

1. Zero is less than any number.

2. If any number is interesting, then zero is interesting.

3. No number is less than zero.

4. Any uninteresting number with the property that all smaller numbers are interesting certainly is interesting.

5. There is no number such that all numbers are less than it.

6. There is no number such that no number is less than it.

Exercise 1.4 For each of the following English sentences, write a corresponding sentence in FOL.

1. P is a person; T is a time, $F(x, y)$ means that you can fool x at time y.

   (a) You can fool some of the people all of the time.

   (b) You can fool all of the people some of the time.

   (c) You can't fool all of the people all of the time.

2. J is a job; a designates Adam; $D(x, y)$ means that x can do y right.

   (a) Adam can't do every job right.

   (b) Adam can't do any job right.

3. Nobody likes everybody. ($L(x, y)$ means x likes y.)

Exercise 1.5 Consider the following English sentences. Can you formalize them in first-order logic? If yes, how?

1. "There are three critics who admire only one another."

2. "There are some critics who admire only one another."

3. "It is not the case that there are some natural numbers smaller than 5 among which none is least."

4. "It is not the case that there are some numbers among which none is least."

Exercise 1.6 For each group of sentences, either write an interpretation under which the last sentence is false and all the rest are true or explain why such an interpretation does not exist.

1. $\forall x\ (P(x) \rightarrow Q(x))$
   $\forall x\ (R(x) \rightarrow Q(x))$
   $\exists x\ (R(x) \wedge P(x))$

2. $\forall x\ \exists y\ P(x, y)$
   $\exists y\ \forall x\ P(x, y)$

3. $\forall x\ (P(x) \rightarrow Q(\mathsf{a}))$
   $(\forall x\ P(x)) \rightarrow Q(\mathsf{a})$

Exercise 1.7 For each group of sentences, write an interpretation in which all sentences are true.

1. $\forall x\ (P(x) \vee Q(x)) \rightarrow \exists x\ R(x)$
   $\forall x\ (R(x) \rightarrow Q(x))$
   $\exists x\ (P(x) \wedge \neg Q(x))$

2. $\forall x\ \neg P(x, x)$
   $\forall x, y, z\ (P(x, y) \wedge P(y, z) \rightarrow P(x, z))$
   $\forall x\ \exists y\ P(x, y)$

3. $\forall x\ \exists y\ P(x, y)$
   $\forall x\ (Q(x) \rightarrow \exists y\ P(y, x))$
   $\exists x\ Q(x)$
   $\forall x\ \neg P(x, x)$