28/11/2013
Home exercise 1

Exercise 4

Consider the following problems:

1) Vertex-cover (VC)

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \leq k$ such that C covers all edges of G (i.e., for each edge $\{v_i, v_j\} \in E$ with $v_i \neq v_j$, $\{v_i, v_j\} \cap C \neq \emptyset$).

2) Independent-set (IS)

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \geq k$ such that for all $v_i, v_j \in C$ with $v_i \neq v_j$, $\{v_i, v_j\} \notin E$

3) Clique

Given an undirected graph $G=(V,E)$ and an integer $k \geq 2$, is there a subset C of V with $|C| \geq k$ such that for all $v_i, v_j \in C$ with $v_i \neq v_j$, $\{v_i, v_j\} \in E$.

Show that VC, IS, and Clique can be reduced to each other in polynomial time.

N.B. In the definitions of VC, IS, and Clique we have ignored self-loops (since we required $v_i \neq v_j$).