14/1/2011
E 11.1

Chomsky Normal Form

Exercise 1

Given the following grammar $G=(V_N, V_T, P, S)$ where V_N, V_T are sets of nonterminals and terminals, S is the start symbol and P is a set of production rules as follows:

$S \rightarrow 0S \mid BD \mid E0$

$B \rightarrow C \mid \varepsilon \mid 1$

$D \rightarrow 0 \mid A0 \mid S \mid \varepsilon$

$E \rightarrow BE \mid SE$

$C \rightarrow 0$

$F \rightarrow C$

a) Simplify the grammar.

b) tranform the simplified grammar into Chomsky Normal form.

Exercise 2: Given the following grammar $G=(V_N, V_T, P, S)$ where $V_N=\{S, A, B\}$, $V_T=\{0, 1\}$, S is the start symbol, and P is as follows.

$S \rightarrow A \mid B$

$A \rightarrow 0A0 \mid B \mid 00$

$B \rightarrow 1B0 \mid 10$

transform the grammar into chomsky normal form.

Exercise 3 Given the grammar $G=(V_N, V_T, P, S)$ where $V_N=\{S, A, B\}$, $V_T=\{0, 1\}$, S is the start symbol, P is as follows.

$S \rightarrow A \mid B$

$A \rightarrow 0S0 \mid B \mid 00$

$B \rightarrow 1B0 \mid 10$

Transform the grammar into chomsky normal form.

Solutions

1)a) Elimination of ε-production:

Nullable Symbols: It. 0: {B, D}
It. 1: {B, D, S}
It. 2: {B, D, S} ←

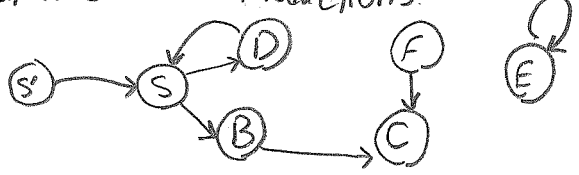
we get:
$S \to 0S \mid BD \mid E0 \mid 0 \mid B \mid D \mid \varepsilon$
$B \to C \mid \varepsilon \mid 1$
$D \to 0 \mid A0 \mid S \mid \varepsilon$
$E \to BE \mid SE \mid E$
$C \to 0$
$F \to C$

Since S is Nullable the resulting grammar is:
$G_1 = (V_{N_1}, V_{T_1}, P_1, S')$ where $V_{N_1} = V_N \cup \{S'\}$, $V_{T_1} = V_T$, P_1 is as follows:

$S' \to S \mid \varepsilon$
$S \to 0S \mid BD \mid E0 \mid 0 \mid B \mid D$
$B \to C \mid 1$
$D \to 0 \mid A0 \mid S$
$E \to BE \mid SE \mid E$
$C \to 0$
$F \to C$

Elimination of Unit Production.

Graph of unit Productions:



reachability:
$S' \Rightarrow^* S, S' \Rightarrow^* B, S' \Rightarrow^* D, S' \Rightarrow^* C$
$S \Rightarrow^* B, S \Rightarrow^* D, S \Rightarrow^* C$
$B \Rightarrow^* C$
$D \Rightarrow^* S, D \Rightarrow^* B, D \Rightarrow^* C$
$F \Rightarrow^* C$

we get: $G_2 = (V_{N_2}, V_{T_2}, P_2, S')$ where

$V_{N_2} = V_{N_1}$, $V_{T_2} = V_{T_1}$, P_2 is as follows:

$S' \longrightarrow \varepsilon \mid OS \mid O \mid BD \mid 1 \mid AO \mid EO$
$S \longrightarrow OS \mid O \mid BD \mid 1 \mid AO \mid EO$
$B \longrightarrow 0 \mid 1$
$D \longrightarrow 0 \mid AO \mid OS \mid BD \mid 1 \mid EO$
$E \longrightarrow BE \mid SE$
$C \longrightarrow 0$
$F \longrightarrow 0$

Elimination of useless symbols (non-generating)

Generating Symbols: it 0: $\{0, 1\}$
it 1: $\{0, 1, B, C, D, F, S, S'\}$
it 2: $\{0, 1, B, C, D, F, S, S'\}$

non-generating symbols $\{E, A\}$

we get $G_3 = (V_{N_3}, V_{T_3}, P_3, S')$ where $V_{N_3} = \{B, C, D, F, S, S'\}$

$V_T = \{0, 1\}$, P_3 is as follows:

$S' \longrightarrow \varepsilon \mid OS \mid O \mid BD \mid 1$
$S \longrightarrow OS \mid O \mid BD \mid 1$
$B \longrightarrow 0 \mid 1$
$D \longrightarrow 0 \mid OS \mid BD \mid 1$
$C \longrightarrow 0$
$F \longrightarrow 0$

Elimination of useless symbols - (unreachable)

Reachable Symbols: It. 0: $\{S'\}$

It. 1: $\{S', S, 0, B, D, 1\}$

It. 2: $\{S', S, 0, B, D, 1\}$

Unreachable Symbols: $\{C, f\}$

we get $G_4 = (V_{N_4}, V_{T_4}, P_4, S')$ where $V_{N_4} = \{S', S, B, D\}$ $V_{T_4} = \{0, 1\}$, and P_4 is defined as follows:

$$S' \longrightarrow \varepsilon \mid 0S \mid BD \mid 1 \mid 0$$
$$S \longrightarrow 0S \mid BD \mid 1 \mid 0$$
$$B \longrightarrow 0 \mid 1$$
$$D \longrightarrow 0 \mid 0S \mid BD \mid 1$$

1)b) Since we do not have long Productions, we only need to remove "mixed bodies" (such as $0S$) by introducing some new Productions.

we get $G_5 = (V_{N_5}, V_{T_5}, P_5, S')$ where $V_{N_5} = V_{N_4} \cup \{N_0\}$

and $V_{T_5} = \{0, 1\}$ and P_5 is as follows:

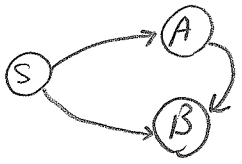
$$S' \longrightarrow \varepsilon \mid N_0 S \mid 0 \mid BD \mid 1$$
$$S \longrightarrow N_0 S \mid 0 \mid BD \mid 1$$
$$B \longrightarrow 0 \mid 1$$
$$D \longrightarrow 0 \mid N_0 S \mid BD \mid 1$$
$$N_0 \longrightarrow 0$$

2) Elimination of ε-Production:

Since there aren't any nullable symbol we skip this phase.

Elimination of Unit Productions:

Graph of unit productions.



reachability.

$S \Rightarrow^* A$, $S \Rightarrow^* B$

$A \Rightarrow^* B$

we get $G_1 = (V_{N_1}, V_{T_1}, P_1, S)$ where $V_{N_1} = V_N$, $V_{T_1} = V_T$, P_1 is as follows:

$$S \longrightarrow 0A0 \mid 00 \mid 1B0 \mid 10$$
$$A \longrightarrow 0A0 \mid 1B0 \mid 10 \mid 00$$
$$B \longrightarrow 1B0 \mid 10$$

Elimination of useless symbols: (non-generating)

Generating Symbols: It. 0: $\{1, 0\}$

It. 1: $\{1, 0, A, B, S\}$

It has all the symbols

Since non-generating symbols is an empty set, we also skip this phase.

Elimination of useless symbols: (un-reachable)

Reachable Symbols: It 0: $\{S\}$

It 1: $\{S, 0, 1, A, B\}$

It has all the symbols

Since unreachable symbols is an empty-set, we also skip this phase.

2) Con't Remove "mixed bodies"

we get $G_2 = (V_{T_2}, V_{N_2}, P_2, S)$ where

$V_{T_2} = V_{T_1} \cup \{N_0, N_1\}$, $V_{N_2} = V_N = \{0,1\}$, P_2 is as follows.

$S \longrightarrow N_0 A N_0 \mid N_0 N_0 \mid N_1 B N_0 \mid N_1 N_0$

$A \longrightarrow N_0 A N_0 \mid N_1 B N_0 \mid N_1 N_0 \mid N_0 N_0$

$B \longrightarrow N_1 B N_0 \mid N_1 N_0$

$N_1 \longrightarrow 1$

$N_0 \longrightarrow 0$

"Factor" long Productions:

we get $G_3 = (V_{T_3}, V_{N_3}, P_3, S)$ where

$V_{T_3} = V_{T_2} \cup \{B_1, B_2\}$, $V_{N_3} = \{0,1\}$, P_3 is as follows:

$S \longrightarrow N_0 B_1 \mid N_0 N_0 \mid N_1 B_2 \mid N_1 N_0$

$A \longrightarrow N_0 B_1 \mid N_1 B_2 \mid N_1 N_0 \mid N_0 N_0$

$B \longrightarrow N_1 B_2 \mid N_1 N_0$

$N_1 \longrightarrow 1$

$N_0 \longrightarrow 0$

$B_1 \longrightarrow A N_0$

$B_2 \longrightarrow B N_0$

3) we only provide the solution:

Note that we have eliminated the useless symbol A.

The resulting grammar is $G_1 = (V_{N_1}, V_{T_1}, P_1, S)$

where $V_{N_1} = \{S, B, N_0, N_1, B_1, B_2\}$, $V_{T_1} = \{0, 1\}$

and P_1 is as follows:

$$S \longrightarrow N_0 B_1 \mid N_0 N_0 \mid N_1 B_2 \mid N_1 N_0$$

$$B \longrightarrow N_1 B_2 \mid N_1 N_0$$

$$N_0 \longrightarrow 0$$

$$N_1 \longrightarrow 1$$

$$B_1 \longrightarrow S N_0$$

$$B_2 \longrightarrow B N_0$$