Exercise

10/12/2010
E8.6

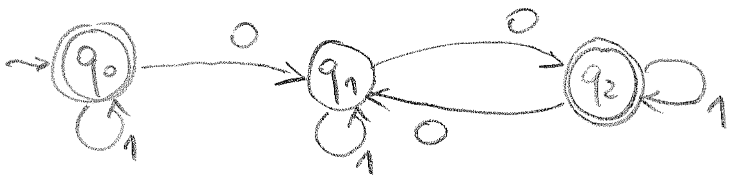
Using the characterization of regular languages in terms of DFAs, show the following:

If L_1 and L_2 are regular, then so is $L_1 \cap L_2$.

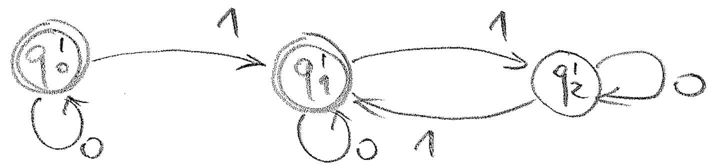
Do <u>not</u> rely on De Morgan's law $L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$.

Apply the construction of a DFA for $L_1 \cap L_2$ to the following DFAs A_1 for L_1 and A_2 for L_2:

A_1 :



A_2 :



Note: we can assume that L_1 and L_2 are RLs over the same alphabet Σ.

Solution:

E8.7

Let $A_1 = (Q_1, \Sigma, \delta_1, q_{01}, F_1)$ be a DFA s.t. $\mathcal{L}(A_1) = L_1$.

Let $A_2 = (Q_2, \Sigma, \delta_2, q_{02}, F_2)$ ... $\mathcal{L}(A_2) = L_2$.

Consider a string w accepted by both A_1 and A_2. Let $w = a_1 a_2 \cdots a_n$. Then we have

in A_1 $\quad q_{01} \xrightarrow{a_1} p_1 \xrightarrow{a_2} p_2 \xrightarrow{a_3} \cdots \xrightarrow{a_n} p_n \in F_1$

in A_2 $\quad q_{02} \xrightarrow{a_1} p_1' \xrightarrow{a_2} p_2' \xrightarrow{a_3} \cdots \xrightarrow{a_n} p_n' \in F_2$

Hence we can construct a DFA $A_\cap = (Q_\cap, \Sigma, \delta_\cap, q_{0\cap}, F_\cap)$ that simulates the transitions of both A_1 and A_2:

- Each state of $A_\cap$ is a pair of states (q_1, q_2), where $q_1 \in Q_1$ and $q_2 \in Q_2$.

  Hence $Q_\cap = Q_1 \times Q_2$.

- The initial state $q_{0\cap}$ is the pair of initial states of Q_1 and Q_2. Hence $q_{0\cap} = (q_{01}, q_{02})$

- The set of final states is such that both A_1 and A_2 accept if $A_\cap$ accepts. Hence $F_\cap = F_1 \times F_2$

- The transition function $\delta_\cap$ simulates the transitions of both A_1 and A_2: If $A_\cap$ is in state (q_1, q_2), then on input a it goes to a state (q_1', q_2'), where $q_1' = \delta_1(q_1, a)$ and $q_2' = \delta_2(q_2, a)$.

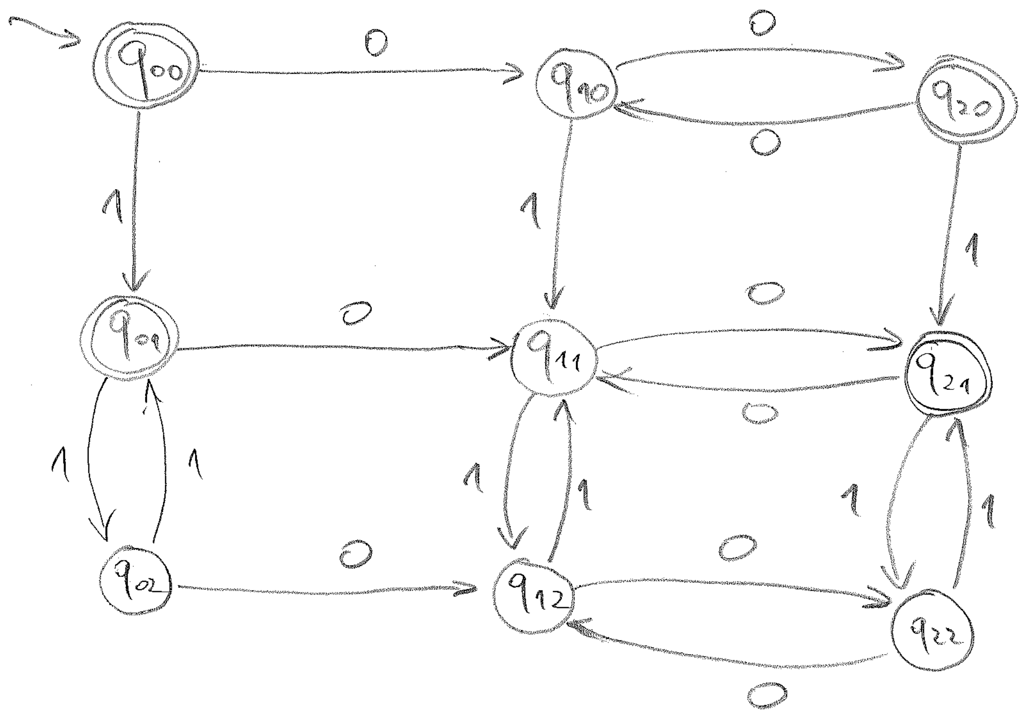
  Hence: for all $a \in \Sigma$, $q_1 \in Q_1$, $q_2 \in Q_2$:

$$\delta_\cap((q_1, q_2), a) = (\delta_1(q_1, a), \delta_2(q_2, a)).$$

One can show that $A_\cap$ constructed in this way accepts $\mathcal{L}(A_1) \cap \mathcal{L}(A_2)$.

$A_\cap$ is called the <u>product</u> automaton.

By applying this construction to the automata A_1 and A_2, we obtain



We have used q_{ij} to denote (q_i, q'_j)