

EXERCISE 1

Define context-free grammars for the following languages:

a) $\{0^i 1^j \mid i \geq 0, i \leq j \leq 2i\};$

b) $\{0^m 1^n 1^n 0^m \mid m+n > 0\};$

c) $\{0^n 1^m 0^{m+n} \mid m+n > 0\}.$

EXERCISE 2

Define a context-free grammar that generates strings of balanced parentheses. Examples of strings of balanced parentheses are $(())$, $()()$, $(())()$ and ϵ , while $)()$ and $((()$ are not.

EXERCISE 3

Is the following context-free grammar ambiguous?

$$S \rightarrow a \mid S; S \mid i S \mid i S e S$$

One may think of the grammar as a rudimentary programming language where i stands for "if-then" and e for "else". Motivate your answer.

EXERCISE 4

Define a context-sensitive grammar for the language $\{a^i b^j c^k \mid 0 < i < j < k\}$. Provide derivations of the strings $abbbcccc$ and $aabbbccccc$.

$$1)a) S \rightarrow \epsilon \mid 0S1 \mid 0S11$$

$$1)b) S \rightarrow 00 \mid 0S0 \mid A$$

$$A \rightarrow 11 \mid 1A1$$

$$1)c) S \rightarrow A \mid B$$

$$A \rightarrow 00 \mid 0A0 \mid B$$

$$B \rightarrow 10 \mid 1B0$$

$$2) \textcircled{1} B \rightarrow BB$$

$$\textcircled{2} B \rightarrow (B)$$

$$\textcircled{3} B \rightarrow \epsilon$$

$$\begin{aligned} B &\stackrel{2}{\Rightarrow} (\underline{B}) \stackrel{1}{\Rightarrow} (\underline{B}B) \stackrel{2}{\Rightarrow} ((\underline{B})B) \stackrel{1}{\Rightarrow} ((BB)\underline{B}) \stackrel{2}{\Rightarrow} ((\underline{B}B)(B)) \\ &\stackrel{2}{\Rightarrow} (((B)\underline{B})(B)) \stackrel{3}{\Rightarrow} (((B))(\underline{B})) \stackrel{3}{\Rightarrow} (((\underline{B}))()) \stackrel{3}{\Rightarrow} (((\underline{B}))()) \end{aligned}$$

3) The context-free grammar is ambiguous. Indeed, the sentential form $iiSeS$ has two derivations from S :

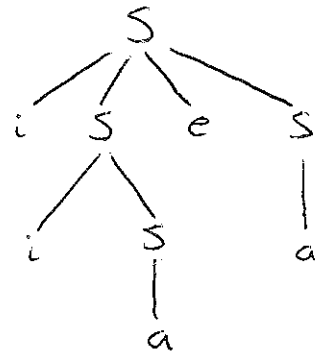
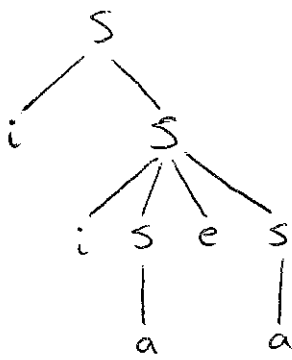
$$a) S \Rightarrow i\underline{S} \Rightarrow iiSeS;$$

$$b) S \Rightarrow i\underline{S}eS \Rightarrow iiSeS.$$

From this one can easily see that the string $iiacea$ has two different parse trees.

3) can't

E13.3



- 4)
- ① $S \rightarrow aSBC$
 - ② $S \rightarrow abBCC$
 - ③ $CB \rightarrow BC$
 - ④ $bB \rightarrow bbBC$
 - ⑤ $bB \rightarrow bb$
 - ⑥ $bC \rightarrow bCc$
 - ⑦ $bC \rightarrow bcc$
 - ⑧ $cC \rightarrow cc$

Derivation of $abbbbcccc$:

$$\begin{aligned}
 S &\xRightarrow{2} ab\underline{B}CC \xRightarrow{4} abb\underline{B}CCC \xRightarrow{5} abbb\underline{C}CC \\
 &\xRightarrow{6} abbb\underline{b}CcCC \xRightarrow{7} abbbbcccc\underline{C}C \\
 &\xRightarrow{8} abbbbcccc\underline{c}C \xRightarrow{8} abbbbcccccc
 \end{aligned}$$

Derivation of $aabbbbcccc$:

$$\begin{aligned}
 S &\xRightarrow{1} a\underline{S}BC \xRightarrow{2} aab\underline{B}CC\underline{B}C \xRightarrow{3} aab\underline{B}C\underline{B}CC \\
 &\xRightarrow{3} aab\underline{B}BCCC \xRightarrow{5} aabb\underline{B}CCC \xRightarrow{5} aabbb\underline{C}CC \\
 &\xRightarrow{7} aabbbbcc\underline{C}C \xRightarrow{8} aabbbbcccc\underline{C} \xRightarrow{8} aabbbbcccccc
 \end{aligned}$$