FINITE AUTOMATA VS. REGULAR EXPRESSIONS

EXERCISE 1

Convert the following regular expressions to ε-NFA's.

a) $(\varepsilon + 1)(01)^*(\varepsilon + 0)$ b) $a^* b^* c^*$

EXERCISE 2

Convert the following DFA to a regular expression, using the state elimination technique.

	0	1
$\rightarrow$ p	s	p
q	p	s
r	r	q
s	q	r

EXERCISE 3

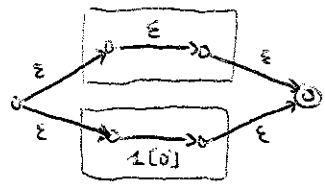
Repeat exercise 2, eliminating the states in a different order. Then, verify that the following binary strings (accepted by the above DFA) are in the language of the regular expressions obtained here and in exercise 2.

a) 00101100101 b) 01000110 1110

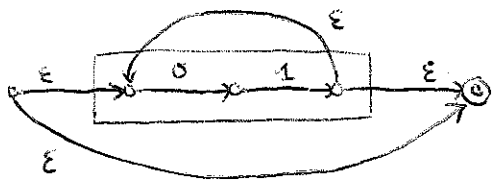
SOLUTIONS (27/11/2008)

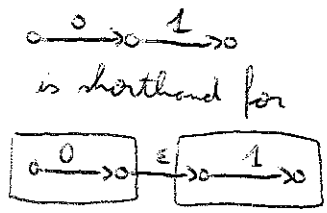
1) a) ε-NFA's for $\varepsilon+1$, $\varepsilon+0$, and $(01)^*$ are as follows:

$\varepsilon+1$
$[\varepsilon+0]$

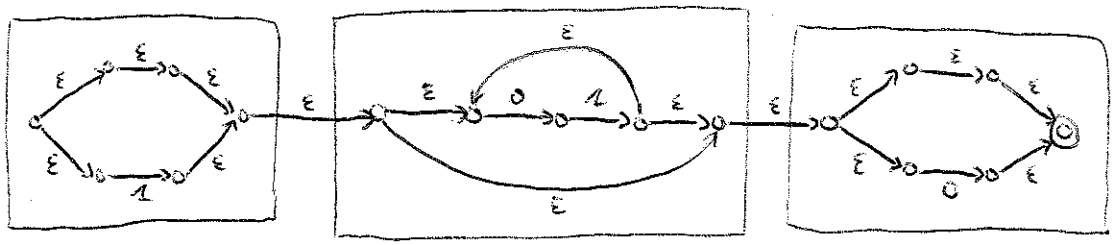


$(01)^*$

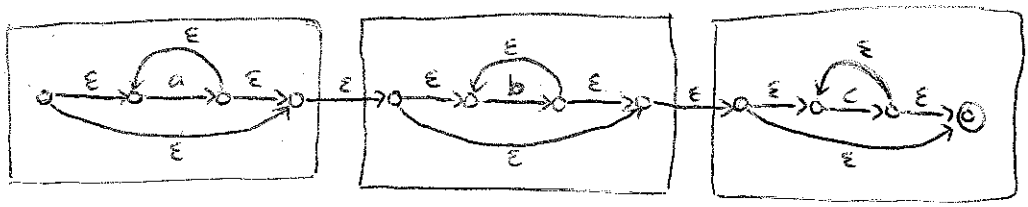




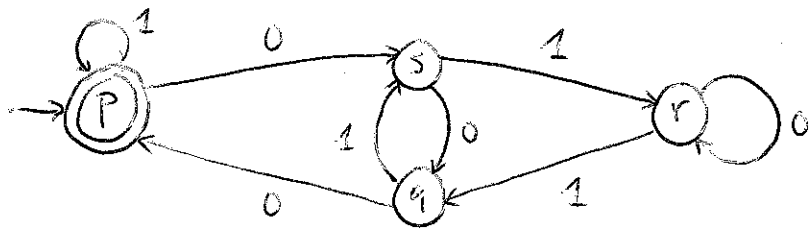
Composition of the above ε-NFA's yields:



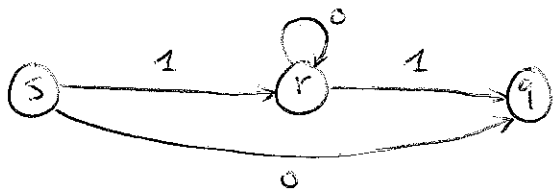
1) b) We get the following ε-NFA:



2) The DFA looks as follows:



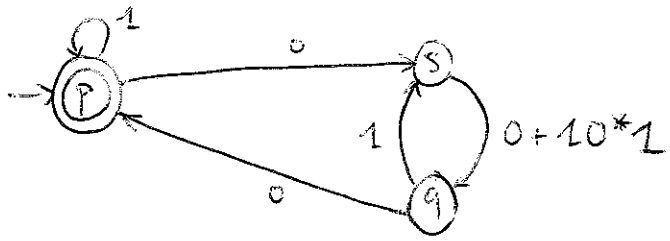
First, we eliminate the state r



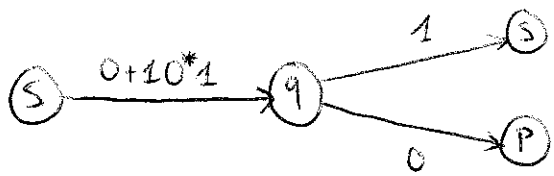
$(0+10^*1$ for $s \to q)$

2) con't

and obtain



Second, we eliminate the state q



$((0+10^*1)\cdot 1$ for $s \to s)$

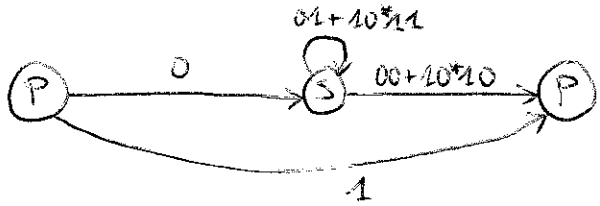
$((0+10^*1)\cdot 0$ for $s \to p)$

and obtain

$(0+10^*1)1$
$= 01 + 10^*11$

$(0+10^*1)0$
$= 00 + 10^*10$

Third, we eliminate the state s



and obtain

$1 + 0(01+10^*11)^*(00+10^*10)$

The regular expression is therefore

$$(1+0(01+10^*11)^*(00+10^*10))^* = E$$

3) Note that, if we eliminate first the state r, but then s before q, we get the following regular expression:

$$E' = (1 + (00 + 010^*1)(10 + 110^*1)^*0)^*$$

3)a) $00101100101 \in \mathcal{L}(E)$:

$(0\underbrace{(01)(01)}_{\text{from } (01 + 10^*11)^*}\underbrace{(10010)}_{\text{from } (00 + 10^*10)})1$

$00101100101 \in \mathcal{L}(E')$

$((\underbrace{00}_{\text{from } (00 + 010^*1)})\underbrace{(10)(11001)}_{\text{from } (10 + 110^*1)^*}0)1$

3)b) $0\underbrace{(100011)(01)}_{\text{from } (01 + 10^*11)^*}\underbrace{(110)}_{\text{from } (00 + 10^*10)} \in \mathcal{L}(E)$

$\underbrace{(010001)}_{\text{from } (00 + 010^*1)}\underbrace{(10)(111)}_{\text{from } (10 + 110^*1)^*}0 \in \mathcal{L}(E')$