Knowledge Bases and Databases

Part 2: Ontology-Based Access to Information

Diego Calvanese

Faculty of Computer Science
Master of Science in Computer Science

A.Y. 2008/2009

Freie Universität Bozen
Libera Università di Bolzano
Free University of Bozen · Bolzano

Overview of Part 2: Ontology-based access to information

Chapter I

Introduction to ontology-based access to information

Outline

Outline

Different meanings of "**Semantics**"

1. Part of **linguistics** that studies the meaning of words and phrases.
2. **Meaning** of a set of symbols in some **representation scheme**.
   Provides a means to specify and communicate the intended meaning of a set of "syntactic" objects.
3. **Formal semantics of a language** (e.g., an artificial language).
   (Meta-mathematical) mechanism to associate to each sentence in a language an element of a symbolic domain that is "outside the language".

In information systems, meanings 2 and 3 are the relevant ones:

- In order to talk about semantics we need a representation scheme, i.e., an **ontology**.
- ... but 2 makes no sense without 3.

Ontologies

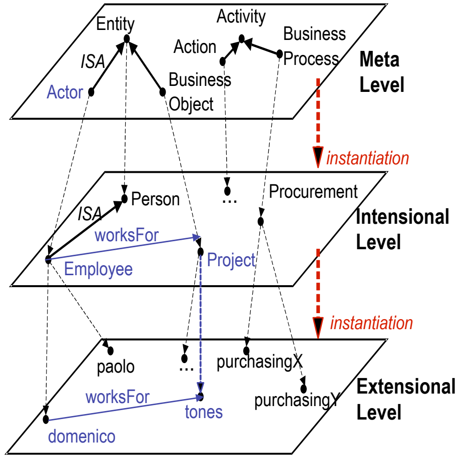
Def.: Ontology

is a representation scheme that describes a **formal conceptualization** of a domain of interest.

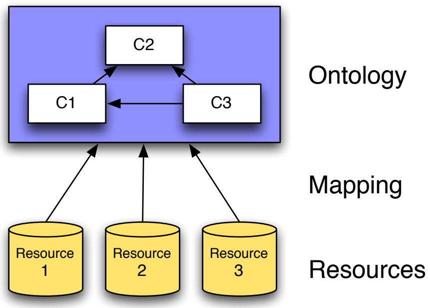
The specification of an ontology comprises several levels:

- **Meta-level**: specifies a set of modeling categories.
- **Intensional level**: specifies a set of conceptual elements (instances of categories) and of rules to describe the conceptual structures of the domain.
- **Extensional level**: specifies a set of instances of the conceptual elements described at the intensional level.

The three levels of an ontology



Ontologies at the core of information systems



The usage of all system resources (data and services) is done through the domain conceptualization.

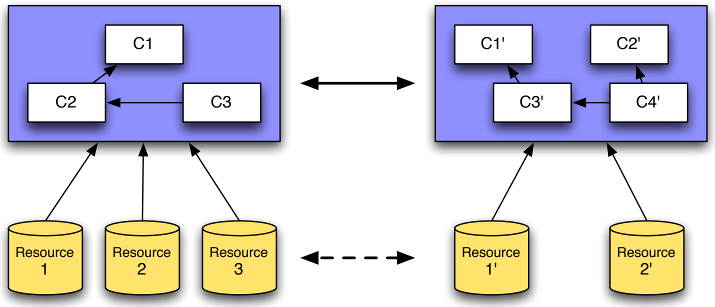
Ontology mediated access to data

Desiderata: achieve **logical transparency** in access to data:

- **Hide** to the user where and how data are stored.
- Present to the user a **conceptual view** of the data.
- Use a **semantically rich formalism** for the conceptual view.

We will see that this setting is similar to the one of Data Integration. The difference is that here the ontology provides a rich conceptual description as the information managed by the system.

Ontologies at the core of cooperation



The cooperation between systems is done at the level of the conceptualization.

Three novel challenges

1. **Languages**
2. Methodologies
3. **Tools**

... for specifying, building, and managing ontologies to be used in information systems.

Challenge 1: Ontology languages

- Several proposals for ontology languages have been made.
- **Tradeoff** between **expressive power** of the language and **computational complexity** of dealing with (i.e., performing inference over) ontologies specified in that language.
- Usability needs to be addressed.

In this course:

- We discuss variants of ontology languages suited for managing **ontologies in information systems**.
- We study the above mentioned **tradeoff** . . .
- . . . paying particular attention to the aspects related to data management.

Challenge 2: Methodologies

- Developing and dealing with ontologies is a complex and challenging task.
- Developing **good ontologies** is even more challenging.
- It requires to master the technologies based on semantics, which in turn requires good knowledge about the languages, their semantics, and the implications it has w.r.t. reasoning over the ontology.

In this course:

- We study in depth the **semantics of ontologies**, with an emphasis on their relationship to data in information sources.
- We thus lay the **foundations for the development of methodologies**, though we do not discuss specific ontology-development methodologies here.

Challenge 3: Tools

- According to the principle that "there is no meaning without a language with a formal semantics", the formal semantics becomes the solid basis for dealing with ontologies.
- Hence every kind of access to an ontology (to extract information, to modify it, etc.), requires to **fully** take into account its semantics.
- We need to resort to tools that provide capabilities to perform **automated reasoning** over the ontology, and the kind of reasoning should be **sound** and **complete** w.r.t. the formal semantics.

In this course:

- We discuss the requirements for such ontology management tools.
- We will work with a tool that has been specifically designed for optimized access to information sources through ontologies.

A challenge across the three challenges: Scalability

When we want to use ontologies to access information sources, we have to address the three challenges of languages, methodologies, and tools by taking into account **scalability** w.r.t.:

- the size of (the intensional level of) the ontology
- the number of ontologies
- the **size of the information sources** that are accessed through the ontology/ontologies.

In this course we pay particular attention to the third aspect, since we work under the realistic assumption that the extensional level (i.e., the data) largely dominates in size the intensional level of an ontology.

Outline

Elements of an ontology language

- **Syntax**
  - Alphabet
  - Languages constructs
  - Sentences to assert knowledge
- **Semantics**
  - Formal meaning
- **Pragmatics**
  - Intended meaning
  - Usage

Static vs. dynamic aspects

The aspects of the domain of interest that can be modeled by an ontology language can be classified into:

- **Static aspects**
  - Are related to the structuring of the domain of interest.
  - Supported by virtually all languages.
- Dynamic aspects
  - Are related to how the elements of the domain of interest evolve over time.
  - Supported only by some languages, and only partially (cf. services).

Before delving into the dynamic aspects, we need a good understanding of the static ones.

In this course we concentrate essentially on the static aspects.

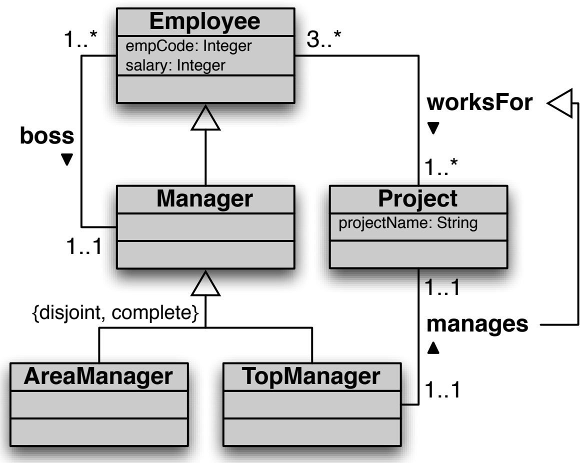
Intensional level of an ontology language

An ontology language for expressing the intensional level usually includes:

- Concepts
- Properties of concepts
- Relationships between concepts, and their properties
- Axioms
- Queries

Ontologies are typically **rendered as diagrams** (e.g., Semantic Networks, Entity-Relationship schemas, UML Class Diagrams).

Example: ontology rendered as UML Class Diagram



Concepts

Def.: Concept

Is an element of an ontology that denotes a collection of instances (e.g., the set of "employees").

We distinguish between:

- Intensional definition:
  specification of name, properties, relations, . . .
- Extensional definition:
  specification of the instances

Concepts are also called **classes**, **entity types**, **frames**.

Properties

Def.: Property

Is an element of an ontology that qualifies another element (e.g., a concept or a relationship).

Property definition (intensional and extensional):

- Name
- Type: may be either
  - atomic (integer, real, string, enumerated, . . .), or
    e.g., eye-color → { blu, brown, green, grey }
  - structured (date, set, list, . . .)
    e.g., date → day/month/year
- The definition may also specify a default value.

Properties are also called **attributes**, **features**, **slots**.

Relationships

Def.: Relationship

Is an element of an ontology that expresses an association among concepts.

We distinguish between:

- Intensional definition:
  specification of involved concepts
  e.g., worksFor is defined on Employee and Project
- Extensional definition:
  specification of the instances of the relationship, called facts
  e.g., worksFor(domenico, tones)

Relationships are also called **associations**, **relationship types**, **roles**.

Axioms

Def.: Axiom

Is a logical formula that expresses at the intensional level a condition that must be satisified by the elements at the extensional level.

Different kinds of axioms/conditions:

- subclass relationships, e.g., Manager $\sqsubseteq$ Employee
- equivalences, e.g., Manager $\equiv$ AreaManager $\sqcup$ TopManager
- disjointness, e.g., AreaManager $\sqcap$ TopManager $\equiv \perp$
- (cardinality) restrictions,
  e.g., each Employee worksFor at least 3 Project
- ...

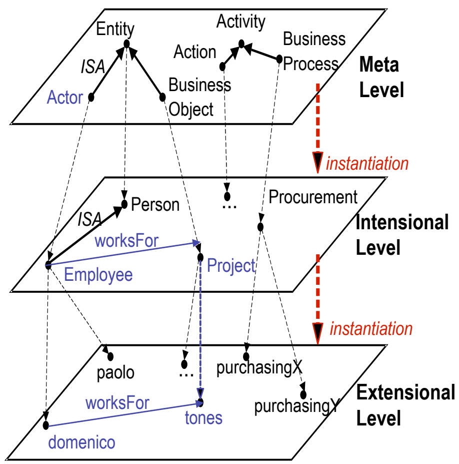
Axioms are also called **assertions**.
A special kind of axioms are **definitions**.

Extensional level of an ontology language

At the extensional level we have individuals and facts:

- An **instance** represents an individual (or object) in the extension of a concept.
  e.g., domenico is an instance of Employee
- A **fact** represents a relationship holding between instances.
  e.g., worksFor(domenico, tones)

The three levels of an ontology



Comparison with other formalisms

- Ontology languages vs. knowledge representation languages:
  Ontologies **are** knowledge representation schemas.
- Ontology vs. logic:
  Logic is **the** tool for assigning semantics to ontology languages.
- Ontology languages vs. conceptual data models:
  Conceptual schemas **are** special ontologies, suited for conceptualizing a single logical model (database).
- Ontology languages vs. programming languages:
  Class definitions **are** special ontologies, suited for conceptualizing a single structure for computation.

Classification of ontology languages

- Graph-based
  - Semantic networks
  - Conceptual graphs
  - UML class diagrams, Entity-Relationship schemas
- Frame based
  - Frame Systems
  - OKBC, XOL
- Logic based
  - **Description Logics** (e.g., $\mathcal{SHOIQ}$, $\mathcal{DLR}$, *DL-Lite*, OWL, ...)
  - Rules (e.g., RuleML, LP/Prolog, F-Logic)
  - First Order Logic (e.g., KIF)
  - Non-classical logics (e.g., non-monotonic, probabilistic)

Queries

Queries may be posed over an ontology.

Def.: Query

Is an expression at the intensional level denoting a (possibly structured) collection of individuals satisfying a given condition.

Def.: Meta-Query

Is an expression at the meta level denoting a collection of ontology elements satisfying a given condition.

Note: One may also conceive queries that span across levels (**object-meta queries**), cf. [RDF], [CK06]

Ontology languages vs. query languages

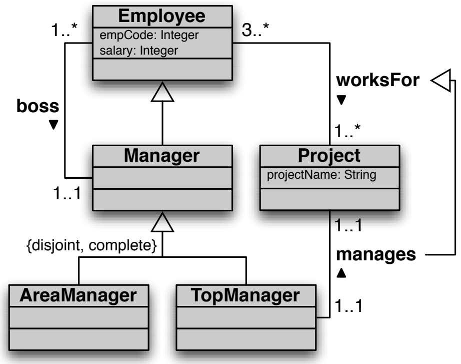
Ontology languages:

- Tailored for capturing intensional relationships.
- Are quite **poor as query languages**:
  - Cannot refer to same object via multiple navigation paths in the ontology,
  - i.e., allow only for a limited form of JOIN, namely chaining.

Instead, **when querying** a data source (either directly, or via the ontology), to retrieve the data of interest, general forms of **joins** are required.

It follows that the constructs for queries may be quite different from the constructs used in the ontology to form concepts and relationships.

Example of query



$q(ce, cm, se, sm) \leftarrow \exists e, p, m.$
$\quad \mathsf{worksFor}(e, p) \wedge \mathsf{manages}(m, p) \wedge \mathsf{boss}(m, e) \wedge \mathsf{empCode}(e, ce) \wedge$
$\quad \mathsf{empCode}(m, cm) \wedge \mathsf{salary}(e, se) \wedge \mathsf{salary}(m, sm) \wedge se \geq sm$

Query answering under different assumptions

Depending on the setting, query answering may have different meanings:

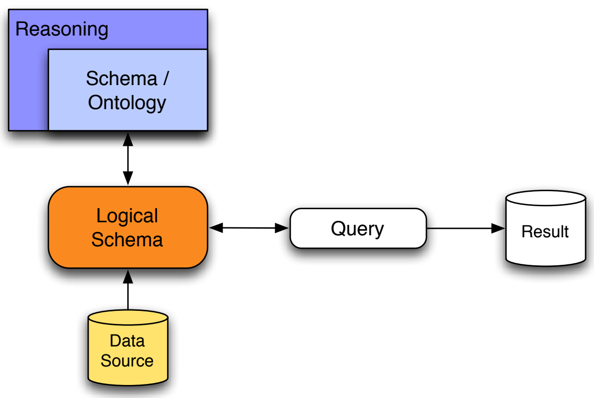
- Traditional databases $\rightsquigarrow$ **complete information**
- Ontologies (or knowledge bases) $\rightsquigarrow$ **incomplete information**

Query answering in traditional databases

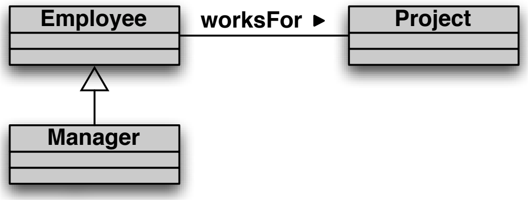
- Data are completely specified (CWA), and typically large.
- Schema/intensional information used in the design phase.
- At **runtime**, the data is assumed to satisfy the schema, and therefore the **schema is not used**.
- Queries allow for complex navigation paths in the data (cf. SQL).

$\rightsquigarrow$ Query answering amounts to **query evaluation**, which is computationally easy.

Query answering in traditional databases (cont'd)



Query answering in traditional databases – Example



For each concept/relationship we have a (complete) table in the DB.

DB: Employee = { `john`, `mary`, `nick` }
Manager = { `john`, `nick` }
Project = { `prA`, `prB` }
worksFor = { (`john`,`prA`), (`mary`,`prB`) }

Query: $q(x) \leftarrow \exists p.\,\text{Manager}(x), \text{Project}(p), \text{worksFor}(x,p)$

Answer: { `john` }

Query answering over ontologies

- An ontology (or conceptual schema, or knowledge base) imposes constraints on the data.
- Actual data may be incomplete or inconsistent w.r.t. such constraints.
- The system has to take into account the constraints during query answering, and overcome incompleteness or inconsistency.

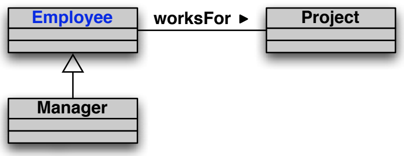
$\leadsto$ Query answering amounts to **logical inference**, which is computationally more costly.

Note:

- The size of the data is not considered critical (comparable to the size of the intensional information).
- Queries are typically simple, i.e., atomic (a class name), and query answering amounts to instance checking.

Query answering over ontologies (cont'd)

Query answering over ontologies – Example



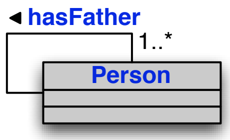
The tables in the database may be **incompletely specified**, or even missing for some classes/properties.

DB: Manager $\supseteq$ { `john`, `nick` }
Project $\supseteq$ { `prA`, `prB` }
worksFor $\supseteq$ { (`john`,`prA`), (`mary`,`prB`) }

Query: $q(x) \leftarrow \text{Employee}(x)$

Answer: { `john`, `nick`, `mary` }

Query answering over ontologies – Example 2



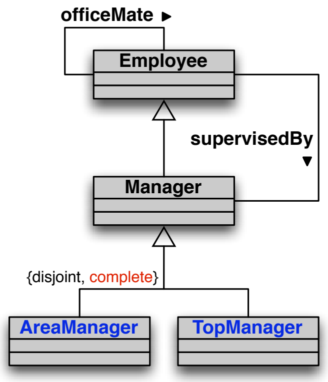
Each person has a father, who is a person.

DB: Person $\supseteq$ { `john`, `nick`, `toni` }
hasFather $\supseteq$ { (`john`,`nick`), (`nick`,`toni`) }

Queries: $q_1(x, y) \leftarrow \text{hasFather}(x, y)$
$q_2(x) \leftarrow \exists y.\, \text{hasFather}(x, y)$
$q_3(x) \leftarrow \exists y_1, y_2, y_3.\, \text{hasFather}(x, y_1), \text{hasFather}(y_1, y_2), \text{hasFather}(y_2, y_3)$
$q_4(x, y_3) \leftarrow \exists y_1, y_2.\, \text{hasFather}(x, y_1), \text{hasFather}(y_1, y_2), \text{hasFather}(y_2, y_3)$

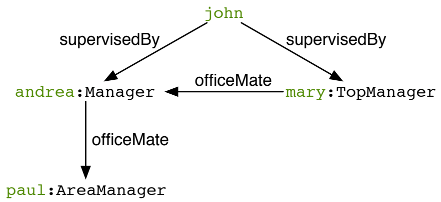
Answers: to q_1: { (`john`,`nick`), (`nick`,`toni`) }
to q_2: { `john`, `nick`, `toni` }
to q_3: { `john`, `nick`, `toni` }
to q_4: { }

QA over ontologies – Andrea's Example(*)



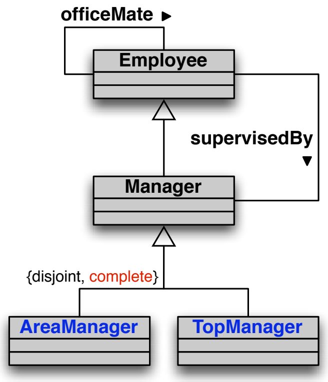
Manager is **partitioned into** AreaManager and TopManager.

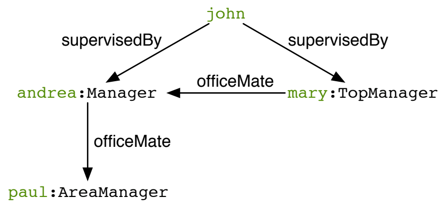
Employee ⊇ { andrea, nick, mary, john }
Manager ⊇ { andrea, nick, mary }
AreaManager ⊇ { nick }
TopManager ⊇ { mary }
supervisedBy ⊇ { (john,andrea), (john,mary) }
officeMate ⊇ { (mary,andrea), (andrea,nick) }



(*) Due to Andrea Schaerf [Sch93].

QA over ontologies – Andrea's Example (cont'd)





$$q(x) \leftarrow \exists y, z.\, \mathsf{supervisedBy}(x, y),\ \mathsf{TopManager}(y),\ \mathsf{officeMate}(y, z),\ \mathsf{AreaManager}(z)$$

Answer: { john }

To determine this answer, we need to resort to **reasoning by cases**.

Query answering in Ontology-Based Data Access

In OBDA, we have to face the difficulties of both assumptions:

- The actual **data** is stored in external information sources (i.e., databases), and thus its size is typically **very large**.
- The ontology introduces **incompleteness** of information, and we have to do logical inference, rather than query evaluation.
- We want to take into account at **runtime** the **constraints** expressed in the ontology.
- We want to answer **complex database-like queries**.
- We may have to deal with multiple information sources, and thus face also the problems that are typical of data integration.

Researchers are starting only now to tackle this difficult and challenging problem. In this course we will study state-of-the-art technology in this area.

Chapter II

Description Logics and the *DL-Lite* family

Outline

Outline

What are Description Logics?

Description Logics (DLs) [BCM+03] are **logics** specifically designed to represent and reason on structured knowledge.

The domain of interest is composed of objects and is structured into:

- concepts, which correspond to classes, and denote sets of objects
- roles, which correspond to (binary) relationships, and denote binary relations on objects

The knowledge is asserted through so-called assertions, i.e., logical axioms.

Origins of Description Logics

DLs stem from early days Knowledge Representation formalisms (late '70s, early '80s):

- Semantic Networks: graph-based formalism, used to represent the meaning of sentences
- Frame Systems: frames used to represent prototypical situations, antecedents of object-oriented formalisms

Problems: no clear semantics, reasoning not well understood

Description Logics (a.k.a. Concept Languages, Terminological Languages) developed starting in the mid '80s, with the aim of providing semantics and inference techniques to knowledge representation systems.

Current applications of Description Logics

DLs have evolved from being used "just" in KR.

Novel applications of DLs:

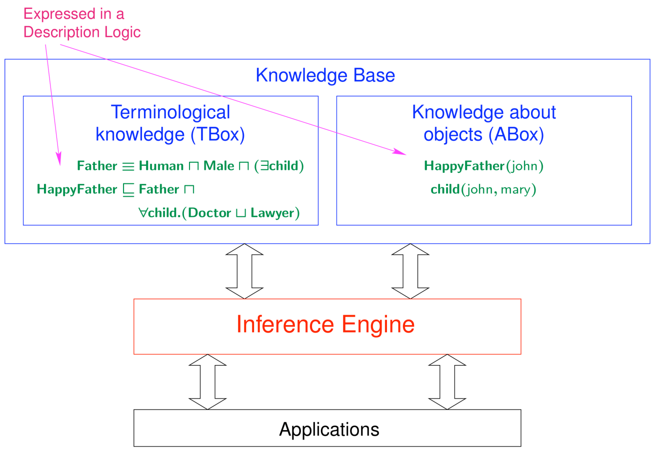
- Databases:
  - schema design, schema evolution
  - query optimization
  - integration of heterogeneous data sources, data warehousing
- Conceptual modeling
- Foundation for the Semantic Web (variants of OWL correspond to specific DLs)
- ...

Ingredients of a Description Logic

A **DL** is characterized by:

1. A **description language**: how to form concepts and roles
   $\mathsf{Human} \sqcap \mathsf{Male} \sqcap \exists \mathsf{hasChild} \sqcap \forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$
2. A mechanism to **specify knowledge** about concepts and roles (i.e., a TBox)
   $$\mathcal{T} = \{ \mathsf{Father} \equiv \mathsf{Human} \sqcap \mathsf{Male} \sqcap \exists \mathsf{hasChild}, \quad \mathsf{HappyFather} \sqsubseteq \mathsf{Father} \sqcap \forall \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer}) \}$$
3. A mechanism to specify **properties of objects** (i.e., an ABox)
   $\mathcal{A} = \{ \mathsf{HappyFather}(\mathtt{john}), \quad \mathsf{hasChild}(\mathtt{john}, \mathtt{mary}) \}$
4. A set of **inference services**: how to reason on a given KB
   $\mathcal{T} \models \mathsf{HappyFather} \sqsubseteq \exists \mathsf{hasChild}.(\mathsf{Doctor} \sqcup \mathsf{Lawyer})$
   $\mathcal{T} \cup \mathcal{A} \models (\mathsf{Doctor} \sqcup \mathsf{Lawyer})(\mathtt{mary})$

Architecture of a Description Logic system



Description language

A description language provides the means for defining:

- concepts, corresponding to classes: interpreted as sets of objects;
- roles, corresponding to relationships: interpreted as binary relations on objects.

To define concepts and roles:

- We start from a (finite) alphabet of **atomic concepts** and **atomic roles**, i.e., simply names for concept and roles.
- Then, by applying specific **constructors**, we can build **complex concepts** and **roles**, starting from the atomic ones.

A **description language** is characterized by the set of constructs that are available for that.

Semantics of a description language

The **formal semantics** of DLs is given in terms of interpretations.

Def.: An **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ consists of:

- a nonempty set $\Delta^{\mathcal{I}}$, the domain of $\mathcal{I}$
- an interpretation function $\cdot^{\mathcal{I}}$, which maps
  - each individual a to an element $a^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic concept A to a subset $A^{\mathcal{I}}$ of $\Delta^{\mathcal{I}}$
  - each atomic role P to a subset $P^{\mathcal{I}}$ of $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

Note: A DL interpretation is analogous to a FOL interpretation, except that, by tradition, it is specified in terms of a function $\cdot^{\mathcal{I}}$ rather than a set of (unary and binary) relations.

The interpretation function is extended to complex concepts and roles according to their syntactic structure.

Concept constructors

Construct	Syntax	Example	Semantics
atomic concept	A	Doctor	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	P	hasChild	$P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
atomic negation	$\neg A$	¬Doctor	$\Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
conjunction	$C \sqcap D$	Hum ⊓ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
(unqual.) exist. res.	$\exists R$	∃hasChild	$\{ a \mid \exists b. (a,b) \in R^{\mathcal{I}} \}$
value restriction	$\forall R.C$	∀hasChild.Male	$\{a \mid \forall b. (a,b) \in R^{\mathcal{I}} \to b \in C^{\mathcal{I}}\}$
bottom	$\bot$		$\emptyset$

(C, D denote arbitrary concepts and R an arbitrary role)

The above constructs form the basic language $\mathcal{AL}$ of the family of $\mathcal{AL}$ languages.

Additional concept and role constructors

Construct	$\mathcal{AL}\cdot$	Syntax	Semantics
disjunction	$\mathcal{U}$	$C \sqcup D$	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
top		$\top$	$\Delta^{\mathcal{I}}$
qual. exist. res.	$\mathcal{E}$	$\exists R.C$	$\{ a \mid \exists b. (a,b) \in R^{\mathcal{I}} \land b \in C^{\mathcal{I}} \}$
(full) negation	$\mathcal{C}$	$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
number restrictions	$\mathcal{N}$	$(\geq k\, R)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}}\} \geq k \}$
		$(\leq k\, R)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}}\} \leq k \}$
qual. number restrictions	$\mathcal{Q}$	$(\geq k\, R.C)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}} \land b \in C^{\mathcal{I}}\} \geq k \}$
		$(\leq k\, R.C)$	$\{ a \mid \#\{b \mid (a,b) \in R^{\mathcal{I}} \land b \in C^{\mathcal{I}}\} \leq k \}$
inverse role	$\mathcal{I}$	R^-	$\{ (a,b) \mid (b,a) \in R^{\mathcal{I}} \}$
role closure	reg	$\mathcal{R}^*$	$(R^{\mathcal{I}})^*$

Many different DL constructs and their combinations have been investigated.

Further examples of DL constructs

- Disjunction: ∀hasChild.(Doctor ⊔ Lawyer)
- Qualified existential restriction: ∃hasChild.Doctor
- Full negation: ¬(Doctor ⊔ Lawyer)
- Number restrictions: (≥ 2 hasChild) ⊓ (≤ 1 sibling)
- Qualified number restrictions: (≥ 2 hasChild. Doctor)
- Inverse role: ∀hasChild$^-$.Doctor
- Reflexive-transitive role closure: ∃hasChild*.Doctor

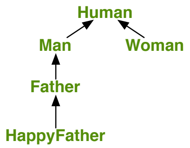
Reasoning on concept expressions

An interpretation $\mathcal{I}$ is a **model** of a concept C if $C^{\mathcal{I}} \neq \emptyset$.

Basic reasoning tasks:

1. **Concept satisfiability**: does C admit a model?
2. **Concept subsumption** $C \sqsubseteq D$: does $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ hold for all interpretations $\mathcal{I}$?

Subsumption is used to build the concept hierarchy:



Note: (1) and (2) are mutually reducible if DL is propositionally closed.

Complexity of reasoning on concept expressions

Complexity of concept satisfiability: [DLNN97]

$\mathcal{AL}$, $\mathcal{ALN}$	PTIME
$\mathcal{ALU}$, $\mathcal{ALUN}$	NP-complete
$\mathcal{ALE}$	coNP-complete
$\mathcal{ALC}$, $\mathcal{ALCN}$, $\mathcal{ALCI}$, $\mathcal{ALCQI}$	PSPACE-complete

Observations:

- Two sources of complexity:
  - union ($\mathcal{U}$) of type NP,
  - existential quantification ($\mathcal{E}$) of type coNP.

  When they are combined, the complexity jumps to PSPACE.
- Number restrictions ($\mathcal{N}$) do not add to the complexity.

Structural properties vs. asserted properties

We have seen how to build complex **concept and roles expressions**, which allow one to denote classes with a complex structure.

However, in order to represent real world domains, one needs the ability to **assert properties** of classes and relationships between them (e.g., as done in UML class diagrams).

The assertion of properties is done in DLs by means of an **ontology** (or knowledge base).

Description Logics ontology (or knowledge base)

Is a pair $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, where $\mathcal{T}$ is a **TBox** and $\mathcal{A}$ is an **ABox**:

Def.: Description Logics TBox

Consists of a set of **assertions** on concepts and roles:

- Inclusion assertions on concepts: $C_1 \sqsubseteq C_2$
- Inclusion assertions on roles: $R_1 \sqsubseteq R_2$
- Property assertions on (atomic) roles:
  (**transitive** P) (**symmetric** P) (**domain** P C)
  (**functional** P) (**reflexive** P) (**range** P C) $\cdots$

Def.: Description Logics ABox

Consists of a set of **membership assertions** on individuals:

- for concepts: $A(c)$
- for roles: $P(c_1, c_2)$ (we use c_i to denote individuals)

Description Logics knowledge base – Example

Note: We use $C_1 \equiv C_2$ as an abbreviation for $C_1 \sqsubseteq C_2,\ C_2 \sqsubseteq C_1$.

TBox assertions:

- Inclusion assertions on concepts:

$$\begin{array}{rcl} \text{Father} & \equiv & \text{Human} \sqcap \text{Male} \sqcap \exists\text{hasChild} \\ \text{HappyFather} & \sqsubseteq & \text{Father} \sqcap \forall\text{hasChild}.(\text{Doctor} \sqcup \text{Lawyer} \sqcup \text{HappyPerson}) \\ \text{HappyAnc} & \sqsubseteq & \forall\text{descendant}.\text{HappyFather} \\ \text{Teacher} & \sqsubseteq & \neg\text{Doctor} \sqcap \neg\text{Lawyer} \end{array}$$

- Inclusion assertions on roles:
  hasChild $\sqsubseteq$ descendant   hasFather $\sqsubseteq$ hasChild$^-$
- Property assertions on roles:
  (**transitive** descendant), (**reflexive** descendant), (**functional** hasFather)

ABox membership assertions:

- Teacher(`mary`), hasFather(`mary`, `john`), HappyAnc(`john`)

Semantics of a Description Logics knowledge base

The semantics is given by specifying when an interpretation $\mathcal{I}$ **satisfies** an assertion:

- $C_1 \sqsubseteq C_2$ is satisfied by $\mathcal{I}$ if $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$.
- $R_1 \sqsubseteq R_2$ is satisfied by $\mathcal{I}$ if $R_1^{\mathcal{I}} \subseteq R_2^{\mathcal{I}}$.
- A property assertion (**prop** P) is satisfied by $\mathcal{I}$ if $P^{\mathcal{I}}$ is a relation that has the property **prop**.
  (Note: domain and range assertions can be expressed by means of concept inclusion assertions.)
- $A(c)$ is satisfied by $\mathcal{I}$ if $c^{\mathcal{I}} \in A^{\mathcal{I}}$.
- $P(c_1, c_2)$ is satisfied by $\mathcal{I}$ if $(c_1^{\mathcal{I}}, c_2^{\mathcal{I}}) \in P^{\mathcal{I}}$.

We adopt **the unique name assumption**, i.e., $c_1^{\mathcal{I}} \neq c_2^{\mathcal{I}}$, for $c_1 \neq c_2$.

Models of a Description Logics ontology

Def.: **Model** of a DL knowledge base

An interpretation $\mathcal{I}$ is a **model** of $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ if it satisfies all assertions in $\mathcal{T}$ and all assertions in $\mathcal{A}$.

$\mathcal{O}$ is said to be **satisfiable** if it admits a model.

The fundamental reasoning service from which all other ones can be easily derived is ...

Def.: **Logical implication**

$\mathcal{O}$ **logically implies** an assertion α, written $\mathcal{O} \models \alpha$, if α is satisfied by all models of $\mathcal{O}$.

TBox reasoning

- **Concept Satisfiability:** C is satisfiable wrt $\mathcal{T}$, if there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is not empty, i.e., $\mathcal{T} \not\models C \equiv \bot$.
- **Subsumption:** C_1 is subsumed by C_2 wrt $\mathcal{T}$, if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \sqsubseteq C_2$.
- **Equivalence:** C_1 and C_2 are equivalent wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \equiv C_2$.
- **Disjointness:** C_1 and C_2 are disjoint wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} = \emptyset$, i.e., $\mathcal{T} \models C_1 \sqcap C_2 \equiv \bot$.
- **Functionality implication:** A functionality assertion (**funct** R) is logically implied by $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$, we have that $(o, o_1) \in R^{\mathcal{I}}$ and $(o, o_2) \in R^{\mathcal{I}}$ implies $o_1 = o_2$, i.e., $\mathcal{T} \models$ (**funct** R).

Analogous definitions hold for role satisfiability, subsumption, equivalence, and disjointness.

Reasoning over an ontology

- **Ontology Satisfiability:** Verify whether an ontology $\mathcal{O}$ is satisfiable, i.e., whether $\mathcal{O}$ admits at least one model.
- **Concept Instance Checking:** Verify whether an individual c is an instance of a concept C in $\mathcal{O}$, i.e., whether $\mathcal{O} \models C(c)$.
- **Role Instance Checking:** Verify whether a pair (c_1, c_2) of individuals is an instance of a role R in $\mathcal{O}$, i.e., whether $\mathcal{O} \models R(c_1, c_2)$.
- **Query Answering:** see later ...

Reasoning in Description Logics – Example

TBox:

- Inclusion assertions on concepts:

$$\begin{array}{rcl} \text{Father} & \equiv & \text{Human} \sqcap \text{Male} \sqcap \exists\text{hasChild} \\ \text{HappyFather} & \sqsubseteq & \text{Father} \sqcap \forall\text{hasChild}.(\text{Doctor} \sqcup \text{Lawyer} \sqcup \text{HappyPerson}) \\ \text{HappyAnc} & \sqsubseteq & \forall\text{descendant}.\text{HappyFather} \\ \text{Teacher} & \sqsubseteq & \neg\text{Doctor} \sqcap \neg\text{Lawyer} \end{array}$$

- Inclusion assertions on roles:

$$\text{hasChild} \sqsubseteq \text{descendant} \qquad \text{hasFather} \sqsubseteq \text{hasChild}^-$$

- Property assertions on roles:
  (**transitive** descendant), (**reflexive** descendant), (**functional** hasFather)

The above TBox logically implies: $\text{HappyAncestor} \sqsubseteq \text{Father}$.

- Membership assertions:
  $\text{Teacher}(\mathtt{mary})$, $\text{hasFather}(\mathtt{mary}, \mathtt{john})$, $\text{HappyAnc}(\mathtt{john})$

The above TBox and ABox logically imply: $\text{HappyPerson}(\mathtt{mary})$

Complexity of reasoning over DL ontologies

Reasoning over DL ontologies is much more complex than reasoning over concept expressions:

Bad news:

- without restrictions on the form of TBox assertions, reasoning over DL ontologies is already **ExpTime-hard**, even for very simple DLs (see, e.g., [Don03]).

Good news:

- We can add a lot of expressivity (i.e., essentially all DL constructs seen so far), while still staying within the ExpTime upper bound.
- There are DL reasoners that perform reasonably well in practice for such DLs (e.g, Racer, Pellet, Fact++, ...) [MH03].

Outline

Relationship between DLs and ontology formalisms

- DLs are nowadays advocated to provide the foundations for ontology languages.
- Different versions of the W3C standard **Web Ontology Language (OWL)** have been defined as syntactic variants of certain DLs.
- DLs are also ideally suited to capture the fundamental features of conceptual modeling formalims used in information systems design:
  - **Entity-Relationship diagrams**, used in database conceptual modeling
  - **UML Class Diagrams**, used in the design phase of software applications

We briefly overview these correspondences, highlighting essential DL constructs, also in light of the tradeoff between expressive power and computational complexity of reasoning.

DLs vs. OWL

The Web Ontology Language (OWL) comes in different variants:

- **OWL1 Lite** is a variant of the DL $\mathcal{SHIF}(\mathcal{D})$, where:
  - $\mathcal{S}$ stands for $\mathcal{ALC}$ extended with transitive roles,
  - $\mathcal{H}$ stands for role hierarchies (i.e., role inclusion assertions),
  - $\mathcal{I}$ stands for inverse roles,
  - $\mathcal{F}$ stands for functionality of roles,
  - $(\mathcal{D})$ stand for data types, which are necessary in any practical knowledge representation language.
- **OWL1 DL** is a variant of $\mathcal{SHOIN}(\mathcal{D})$, where:
  - $\mathcal{O}$ stands for nominals, which means the possibility of using individuals in the TBox (i.e., the intensional part of the ontology),
  - $\mathcal{N}$ stands for (unqualified) number restrictions,

DLs vs. OWL2

A new version of OWL, **OWL2**, is currently being standardized:

- **OWL2 DL** is a variant of $\mathcal{SROIQ}(\mathcal{D})$, which adds to OWL1 DL several constructs, while still preserving satisfiability of reasoning.
  - $\mathcal{Q}$ stands for qualified number restrictions.
  - $\mathcal{R}$ stands for regular role hierarchies, where role chaining might be used in the left-hand side of role inclusion assertions, with suitable acyclicity conditions.
- OWL2 defines also three **profiles**: OWL2 QL, OWL2 EL, OWL2 EL.
  - Each profile corresponds to a syntactic fragment (i.e., a sub-language) of OWL2 DL that is targeted towards a specific use.
  - The restrictions in each profile guarantee better computational properties than those of OWL2 DL.
  - The **OWL2 QL** profile is derived from the DLs of the *DL-Lite* family (see later).

DL constructs vs. OWL constructs

OWL contructor	**DL constructor**	**Example**
`ObjectIntersectionOf`	$C_1 \sqcap \cdots \sqcap C_n$	Human $\sqcap$ Male
`ObjectUnionOf`	$C_1 \sqcup \cdots \sqcup C_n$	Doctor $\sqcup$ Lawyer
`ObjectComplementOf`	$\neg C$	$\neg$Male
`ObjectOneOf`	$\{a_1\} \sqcup \cdots \sqcup \{a_n\}$	{john} $\sqcup$ {mary}
`ObjectAllValuesFrom`	$\forall P.C$	$\forall$hasChild.Doctor
`ObjectSomeValuesFrom`	$\exists P.C$	$\exists$hasChild.Lawyer
`ObjectMaxCardinality`	$(\leq n\, P)$	$(\leq 1$ hasChild$)$
`ObjectMinCardinality`	$(\geq n\, P)$	$(\geq 2$ hasChild$)$

...

Note: all constructs come also in the `Data...` instead of `Object...` variant.

DL axioms vs. OWL axioms

OWL axiom	DL syntax	Example
SubClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
EquivalentClasses	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
DisjointClasseses	$C_1 \sqsubseteq \neg C_2$	Man $\sqsubseteq$ $\neg$Female
SameIndividual	$\{a_1\} \equiv \{a_2\}$	{presBush} $\equiv$ {G.W.Bush}
DifferentIndividuals	$\{a_1\} \sqsubseteq \neg\{a_2\}$	{john} $\sqsubseteq$ $\neg${peter}
SubObjectPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
EquivalentObjectProperties	$P_1 \equiv P_2$	hasCost $\equiv$ hasPrice
InverseObjectProperties	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent$^-$
TransitiveObjectProperty	$P^+ \sqsubseteq P$	ancestor$^+$ $\sqsubseteq$ ancestor
FunctionalObjectProperty	$\top \sqsubseteq (\leq 1\, P)$	$\top \sqsubseteq (\leq 1$ hasFather$)$

...

DLs vs. UML Class Diagrams

There is a tight correspondence between variants of DLs and UML Class Diagrams [BCDG05].

- We can devise two transformations:
  - one that associates to each UML Class Diagram $\mathcal{D}$ a DL TBox $\mathcal{T}_\mathcal{D}$.
  - one that associates to each DL TBox $\mathcal{T}$ a UML Class Diagram $\mathcal{D}_\mathcal{T}$.
- The transformations are not model-preserving, but are based on a correspondence between instantiations of the Class Diagram and models of the associated ontology.
- The transformations are **satisfiability-preserving**, i.e., a class C is consistent in $\mathcal{D}$ iff the corresponding concept is satisfiable in $\mathcal{T}$.

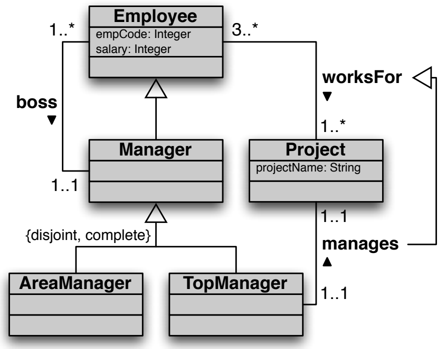
Encoding UML Class Diagrams in DLs

The ideas behind the encoding of a UML Class Diagram $\mathcal{D}$ in terms of a DL TBox $\mathcal{T}_\mathcal{D}$ are quite natural:

- Each class is represented by an atomic concept.
- Each attribute is represented by a role.
- Each binary association is represented by a role.
- Each non-binary association is reified, i.e., represented as a concept connected to its components by roles.
- Each part of the diagram is encoded by suitable assertions.

We illustrate the encoding by means of an example.

Encoding UML Class Diagrams in DLs – Example



Manager	$\sqsubseteq$	Employee
AreaManager	$\sqsubseteq$	Manager
TopManager	$\sqsubseteq$	Manager
Manager	$\sqsubseteq$	AreaManager $\sqcup$ TopManager
AreaManager	$\sqsubseteq$	$\neg$TopManager
Employee	$\sqsubseteq$	$\exists$salary
$\exists$salary$^-$	$\sqsubseteq$	Integer
$\exists$worksFor	$\sqsubseteq$	Employee
$\exists$worksFor$^-$	$\sqsubseteq$	Project
Employee	$\sqsubseteq$	$\exists$worksFor
Project	$\sqsubseteq$	$(\geq 3$ worksFor$^-)$

(**funct** manages)
(**funct** manages$^-$)

manages	$\sqsubseteq$	worksFor

...

Note: Domain and range of associations are expressed by means of concept inclusions.

Encoding DL TBoxes in UML Class Diagrams

The encoding of an $\mathcal{ALC}$ TBox $\mathcal{T}$ in terms of a UML Class Diagram $\mathcal{T}_{\mathcal{D}}$ is based on the following observations:

- We can restrict the attention to $\mathcal{ALC}$ TBoxes, that are constituted by concept inclusion assertions of a simplified form (single atomic concept on the left, and a single concept constructor on the right).
- For each such inclusion assertion, the encoding introduces a portion of UML Class Diagram, that may refer to some common classes.

Reasoning in the encoded $\mathcal{ALC}$-fragment is already **ExpTime-hard**.
From this, we obtain:

Theorem

Reasoning over UML Class Diagrams is ExpTime-hard.

Reasoning on UML Class Diagrams using DLs

- The two encodings show that DL TBoxes and UML Class Diagrams essentially have the **same expressive power**.
- Hence, reasoning over UML Class Diagrams has the same complexity as reasoning over ontologies in expressive DLs, i.e., ExpTime-complete.
- The high complexity is caused by:
  1. the possibility to use disjunction (covering constraints)
  2. the interaction between role inclusions and functionality constraints (maximum 1 cardinality)

Without (1) and restricting (2), reasoning becomes simpler [ACK+07]:

- NLogSpace-complete in combined complexity
- in LogSpace in data complexity (see later)

Efficient reasoning on UML Class Diagrams

We are interested in using UML Class Diagrams to specify ontologies in the context of Ontology-Based Data Access.

Questions

- Which is the right combination of constructs to allow in UML Class Diagrams to be used for OBDA?
- Are there techniques for query answering in this case that can be derived from Description Logics?
- Can query answering be done efficiently in the size of the data?
- If yes, can we leverage relational database technology for query answering?

Outline

Queries over Description Logics ontologies

Traditionally, simple concept (or role) expressions have been considered as queries over DL ontologies.

We need more complex forms of queries, as those used in databases.

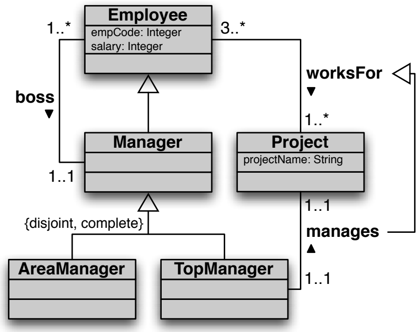
Def.: A **conjunctive query** $q(\vec{x})$ over an ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$

is a conjunctive query $q(\vec{x}) \leftarrow \vec{y}.\, conj(\vec{x}, \vec{y})$ where each atom in the body $conj(\vec{x}, \vec{y})$:

- has **as predicate symbol an atomic concept or role** of $\mathcal{T}$,
- may use variables in $\vec{x}$ and $\vec{y}$,
- may use constants that are individuals of $\mathcal{A}$.

Note: a CQ corresponds to a select-project-join SQL query.

Queries over Description Logics ontologies – Example



Conjunctive query over the above ontology:

$$q(x, y) \leftarrow \exists p.\, \mathsf{Employee}(x), \mathsf{Employee}(y), \mathsf{Project}(p), \mathsf{boss}(x, y), \mathsf{worksFor}(x, p), \mathsf{worksFor}(y, p)$$

Certain answers to a query

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be an ontology, $\mathcal{I}$ an interpretation for $\mathcal{O}$, and $q(\vec{x}) \leftarrow \exists \vec{y}.\, conj(\vec{x}, \vec{y})$ a CQ.

Def.: The **answer** to $q(\vec{x})$ over $\mathcal{I}$, denoted $q^{\mathcal{I}}$

is the set of **tuples** $\vec{c}$ **of constants of** $\mathcal{A}$ such that the formula $\exists \vec{y}.\, conj(\vec{c}, \vec{y})$ evaluates to true in $\mathcal{I}$.

We are interested in finding those answers that hold in all models of an ontology.

Def.: The **certain answers** to $q(\vec{x})$ over $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted $cert(q, \mathcal{O})$

are the **tuples** $\vec{c}$ **of constants of** $\mathcal{A}$ such that $\vec{c} \in q^{\mathcal{I}}$, for every model $\mathcal{I}$ of $\mathcal{O}$.

Query answering over ontologies

Def.: **Query answering** over an ontology $\mathcal{O}$

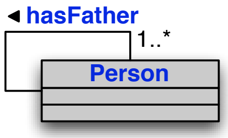
Is the problem of **computing the certain answers** to a query over $\mathcal{O}$.

Computing certain answers is a form of **logical implication**:

$$\vec{c} \in cert(q, \mathcal{O}) \quad \text{iff} \quad \mathcal{O} \models q(\vec{c})$$

Note: A special case of query answering is **instance checking**: it amounts to answering the boolean query $q() \leftarrow A(c)$ (resp., $q() \leftarrow P(c_1, c_2)$) over $\mathcal{O}$ (in this case $\vec{c}$ is the empty tuple).

Query answering over ontologies – Example



TBox $\mathcal{T}$:
$$\exists \text{hasFather} \sqsubseteq \text{Person}$$
$$\exists \text{hasFather}^- \sqsubseteq \text{Person}$$
$$\text{Person} \sqsubseteq \exists \text{hasFather}$$

ABox $\mathcal{A}$: Person(john), Person(nick), Person(toni)
hasFather(john,nick), hasFather(nick,toni)

Queries:

$q_1(x, y) \leftarrow \text{hasFather}(x, y)$

$q_2(x) \leftarrow \exists y. \text{hasFather}(x, y)$

$q_3(x) \leftarrow \exists y_1, y_2, y_3. \text{hasFather}(x, y_1) \wedge \text{hasFather}(y_1, y_2) \wedge \text{hasFather}(y_2, y_3)$

$q_4(x, y_3) \leftarrow \exists y_1, y_2. \text{hasFather}(x, y_1) \wedge \text{hasFather}(y_1, y_2) \wedge \text{hasFather}(y_2, y_3)$

Certain answers:
$cert(q_1, \langle\mathcal{T}, \mathcal{A}\rangle) = \{$ (john,nick), (nick,toni) $\}$
$cert(q_2, \langle\mathcal{T}, \mathcal{A}\rangle) = \{$ john, nick, toni $\}$
$cert(q_3, \langle\mathcal{T}, \mathcal{A}\rangle) = \{$ john, nick, toni $\}$
$cert(q_4, \langle\mathcal{T}, \mathcal{A}\rangle) = \{ \}$

Unions of conjunctive queries

We consider also unions of CQs over an ontology.

A **union of conjunctive queries** (UCQ) has the form:

$$q(\vec{x}) = \exists \vec{y_1}. \mathit{conj}(\vec{x}, \vec{y_1}) \vee \cdots \vee \exists \vec{y_k}. \mathit{conj}(\vec{x}, \vec{y_k})$$

where each $\exists \vec{y_i}. \mathit{conj}(\vec{x}, \vec{y_i})$ is the body of a CQ.

The (certain) answers to a UCQ are defined analogously to those for CQs.

Example

$$q(x) \leftarrow (\text{Manager}(x) \wedge \text{worksFor}(x, \texttt{tones})) \vee (\exists y. \text{boss}(x, y) \wedge \text{worksFor}(y, \texttt{tones}))$$

We typically use the Datalog notation:
$$q(x) \leftarrow \text{Manager}(x), \text{worksFor}(x, \texttt{tones})$$
$$q(x) \leftarrow \exists y. \text{boss}(x, y) \wedge \text{worksFor}(y, \texttt{tones})$$

Data and combined complexity

When measuring the complexity of answering a query $q(\vec{x})$ over an ontology $\mathcal{O} = \langle\mathcal{T}, \mathcal{A}\rangle$, various parameters are of importance.

Depending on which parameters we consider, we get different complexity measures:

- **Data complexity**: TBox and query are considered fixed, and only the size of the ABox (i.e., the data) matters.
- **Query complexity**: TBox and ABox are considered fixed, and only the size of the query matters.
- **Schema complexity**: ABox and query are considered fixed, and only the size of the TBox (i.e., the schema) matters.
- **Combined complexity**: no parameter is considered fixed.

In the OBDA setting, **the size of the data largely dominates** the size of the conceptual layer (and of the query).
$\leadsto$ **Data complexity** is the relevant complexity measure.

Complexity of query answering in DLs

Answering (U)CQs over DL ontologies has been studied extensively:

- Combined complexity:
  - NP-complete for plain databases (i.e., with an empty TBox)
  - ExpTime-complete for $\mathcal{ALC}$ [CDGL98, Lut07]
  - 2ExpTime-complete for very expressive DLs (with inverse roles) [CDGL98, Lut07]
- Data complexity:
  - in LogSpace for plain databases
  - coNP-hard with disjunction in the TBox [DLNS94, CDGL+06b]
  - coNP-complete for very expressive DLs [LR98, OCE06, GHLS07]

Questions

- Can we find interesting families of DLs for which the query answering problem can be solved efficiently?
- If yes, can we leverage relational database technology for query answering?

Outline

The *DL-Lite* family

- Is a family of DLs optimized according to the tradeoff between expressive power and data complexity of query answering.
- We present first two incomparable languages of this family, *DL-Lite*$_\mathcal{F}$, *DL-Lite*$_\mathcal{R}$ (we use *DL-Lite* to refer to both).
- We will see that *DL-Lite* has nice computational properties:
  - PTIME in the size of the TBox (schema complexity)
  - LOGSPACE in the size of the ABox (data complexity)
  - enjoys FOL-rewritability
- We will see that *DL-Lite*$_\mathcal{F}$ and *DL-Lite*$_\mathcal{R}$ are in some sense the maximal DLs with these nice computational properties, which are lost if the two logics are combined, and with minimal additions of constructs.
- We will see, however, that a restricted combination of *DL-Lite*$_\mathcal{F}$ and *DL-Lite*$_\mathcal{R}$ is possible, without losing the computational properties.

Hence, *DL-Lite* provides a positive answer to our basic questions, and sets the foundations for Ontology-Based Data Access.

DL-Lite$_\mathcal{F}$ ontologies

TBox assertions:

- Concept inclusion assertions: $Cl \sqsubseteq Cr$, with:

$$\begin{array}{rcl} Cl & \longrightarrow & A \mid \exists Q \\ Cr & \longrightarrow & A \mid \exists Q \mid \neg A \mid \neg\exists Q \\ Q & \longrightarrow & P \mid P^- \end{array}$$

- Functionality assertions: $(\textbf{funct}\ Q)$

ABox assertions: $A(c)$, $P(c_1, c_2)$, with c_1, c_2 constants

Observations:

- Captures all the basic constructs of UML Class Diagrams and ER
- Notable exception: covering constraints in generalizations.

DL-Lite$_\mathcal{R}$ ontologies

TBox assertions:

- Concept inclusion assertions: $Cl \sqsubseteq Cr$, with:

$$\begin{array}{rcl} Cl & \longrightarrow & A \mid \exists Q \\ Cr & \longrightarrow & A \mid \exists Q \mid \neg A \mid \neg\exists Q \end{array}$$

- Role inclusion assertions: $Q \sqsubseteq R$, with:

$$\begin{array}{rcl} Q & \longrightarrow & P \mid P^- \\ R & \longrightarrow & Q \mid \neg Q \end{array}$$

ABox assertions: $A(c)$, $P(c_1, c_2)$, with c_1, c_2 constants

Observations:

- Drops functional restrictions in favor of ISA between roles.
- Extends (the DL fragment of) the ontology language RDFS.

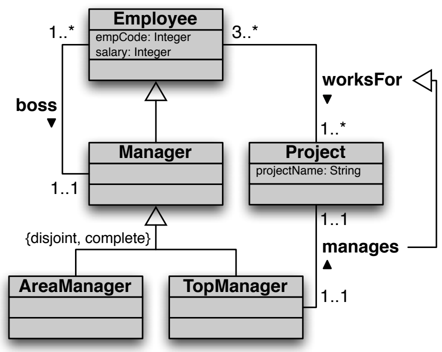
Semantics of *DL-Lite*

Construct	Syntax	Example	Semantics
atomic conc.	A	Doctor	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
exist. restr.	$\exists Q$	$\exists$child$^-$	$\{d \mid \exists e.\,(d,e) \in Q^{\mathcal{I}}\}$
at. conc. neg.	$\neg A$	$\neg$Doctor	$\Delta^{\mathcal{I}} \setminus A^{\mathcal{I}}$
conc. neg.	$\neg \exists Q$	$\neg\exists$child	$\Delta^{\mathcal{I}} \setminus (\exists Q)^{\mathcal{I}}$
atomic role	P	child	$P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
inverse role	P^-	child$^-$	$\{(o,o') \mid (o',o) \in P^{\mathcal{I}}\}$
role negation	$\neg Q$	$\neg$manages	$(\Delta_O^{\mathcal{I}} \times \Delta_O^{\mathcal{I}}) \setminus Q^{\mathcal{I}}$
conc. incl.	$Cl \sqsubseteq Cr$	Father $\sqsubseteq \exists$child	$Cl^{\mathcal{I}} \subseteq Cr^{\mathcal{I}}$
role incl.	$Q \sqsubseteq R$	hasFather $\sqsubseteq$ child$^-$	$Q^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
funct. asser.	(**funct** Q)	(**funct** succ)	$\forall d,e,e'.(d,e) \in Q^{\mathcal{I}} \wedge (d,e') \in Q^{\mathcal{I}} \rightarrow e = e'$
mem. asser.	$A(c)$	Father(bob)	$c^{\mathcal{I}} \in A^{\mathcal{I}}$
mem. asser.	$P(c_1, c_2)$	child(bob, ann)	$(c_1^{\mathcal{I}}, c_2^{\mathcal{I}}) \in P^{\mathcal{I}}$

Capturing basic ontology constructs in *DL-Lite*

ISA between classes	$A_1 \sqsubseteq A_2$
Disjointness between classes	$A_1 \sqsubseteq \neg A_2$
Domain and range of relations	$\exists P \sqsubseteq A_1$ $\quad \exists P^- \sqsubseteq A_2$
Mandatory participation	$A_1 \sqsubseteq \exists P$ $\quad A_2 \sqsubseteq \exists P^-$
Functionality of relations (in *DL-Lite*$_{\mathcal{F}}$)	(**funct** P) $\quad$ (**funct** P^-)
ISA between relations (in *DL-Lite*$_{\mathcal{R}}$)	$Q_1 \sqsubseteq Q_2$
Disjointness between relations (in *DL-Lite*$_{\mathcal{R}}$)	$Q_1 \sqsubseteq \neg Q_2$

DL-Lite – Example



$$
\begin{array}{rcl}
\text{Manager} & \sqsubseteq & \text{Employee} \\
\text{AreaManager} & \sqsubseteq & \text{Manager} \\
\text{TopManager} & \sqsubseteq & \text{Manager} \\
\text{AreaManager} & \sqsubseteq & \neg\text{TopManager} \\
\text{Employee} & \sqsubseteq & \exists\text{salary} \\
\exists\text{salary}^- & \sqsubseteq & \text{Integer} \\
\exists\text{worksFor} & \sqsubseteq & \text{Employee} \\
\exists\text{worksFor}^- & \sqsubseteq & \text{Project} \\
\text{Employee} & \sqsubseteq & \exists\text{worksFor} \\
\text{Project} & \sqsubseteq & \exists\text{worksFor}^- \\
 & \vdots &
\end{array}
$$

Additionally, in **DL-Lite**$_{\mathcal{F}}$: (**funct** manages), (**funct** manages$^-$), ...
in **DL-Lite**$_{\mathcal{R}}$: manages $\sqsubseteq$ worksFor

Note: in *DL-Lite* we cannot capture: – **completeness of the hierarchy**,
– number restrictions

Properties of *DL-Lite*

- The TBox may contain **cyclic dependencies** (which typically increase the computational complexity of reasoning).

  Example: $A \sqsubseteq \exists P, \quad \exists P^- \sqsubseteq A$

- We have not included in the syntax $\sqcap$ on the right hand-side of inclusion assertions, but it can trivially be added, since

  $Cl \sqsubseteq Cr_1 \sqcap Cr_2$ is equivalent to $\begin{array}{l} Cl \sqsubseteq Cr_1 \\ Cl \sqsubseteq Cr_2 \end{array}$

- A domain assertion on role P has the form: $\exists P \sqsubseteq A_1$
  A range assertion on role P has the form: $\exists P^- \sqsubseteq A_2$

Properties of *DL-Lite*$_\mathcal{F}$

DL-Lite$_\mathcal{F}$ does **not** enjoy the **finite model property**.

Example

TBox $\mathcal{T}$: $\mathsf{Nat} \sqsubseteq \exists\mathsf{succ}$ $\quad \exists\mathsf{succ}^- \sqsubseteq \mathsf{Nat}$

$\mathsf{Zero} \sqsubseteq \mathsf{Nat} \sqcap \neg\exists\mathsf{succ}^-$ $\quad (\textbf{funct}\ \mathsf{succ}^-)$

ABox $\mathcal{A}$: $\mathsf{Zero}(0)$

$\mathcal{O} = \langle\mathcal{T}, \mathcal{A}\rangle$ admits only infinite models.
Hence, it is satisfiable, but **not finitely satisfiable**.

Hence, reasoning w.r.t. arbitrary models is different from reasoning w.r.t. finite models only.

Properties of *DL-Lite*$_\mathcal{R}$

- The TBox may contain **cyclic dependencies**.
- *DL-Lite*$_\mathcal{R}$ **does enjoy the finite model property**. Hence, reasoning w.r.t. finite models is the same as reasoning w.r.t. arbitrary models.
- With role inclusion assertions, we can simulate **qualified existential quantification** in the rhs of an inclusion assertion $A_1 \sqsubseteq \exists Q.A_2$.

  To do so, we introduce a new role Q_{A_2} and:
  - the role inclusion assertion $Q_{A_2} \sqsubseteq Q$
  - the concept inclusion assertions: $A_1 \sqsubseteq \exists Q_{A_2}$, $\exists Q_{A_2}^- \sqsubseteq A_2$

  In this way, we can consider $\exists Q.A$ in the right-hand side of an inclusion assertion as an abbreviation.

What is missing in *DL-Lite* wrt popular data models?

Let us consider UML class diagrams that have the following features:

- functionality of associations (i.e., roles)
- inclusion (i.e., ISA) between associations
- attributes of concepts and associations, possibly functional
- covering constraints in hierarchies

What can we capture of these while maintaining FOL-rewritability?

1. We can **forget about covering constraints**, since they make query answering coNP-hard in data complexity (see Part 3).
2. Attributes of concepts are "syntactic sugar" (they could be modeled by means of roles), but their functionality is an issue.
3. We could also add attributes of roles (we won't discuss this here).
4. *Functionality and role inclusions* are present separately (in *DL-Lite*$_\mathcal{F}$ and *DL-Lite*$_\mathcal{R}$), but *were not allowed to be used together*.

DL-Lite$_\mathcal{A}$: a DL combining *DL-Lite*$_\mathcal{F}$ and *DL-Lite*$_\mathcal{R}$

DL-Lite$_\mathcal{A}$ is a Description Logic designed to capture as much features as possible of conceptual data models, while preserving nice computational properties for query answering.

- Allows for **both functionality assertions and role inclusion assertions**, but restricts in a suitable way their interaction.
- Takes into account the distinction between **objects** and **values**:
  - Objects are elements of an abstract interpretation domain.
  - Values are elements of concrete data types, such as integers, strings, ecc.
- Values are connected to objects through **attributes**, rather than roles (we consider here only concept attributes and not role attributes [CDGL+06a]).
- Enjoys **FOL-rewritability**, and hence is LOGSPACE in data complexity.

Syntax of the $DL\text{-}Lite_{\mathcal{A}}$ description language

- Concept expressions:

$$\begin{array}{lcl} B & \longrightarrow & A \mid \exists Q \mid \delta(U) \\ C & \longrightarrow & \top_C \mid B \mid \neg B \mid \exists Q.C \end{array}$$

- Role expressions:

$$\begin{array}{lcl} Q & \longrightarrow & P \mid P^- \\ R & \longrightarrow & Q \mid \neg Q \end{array}$$

- Value-domain expressions: (each T_i is one of the RDF datatypes)

$$\begin{array}{lcl} E & \longrightarrow & \rho(U) \\ F & \longrightarrow & \top_D \mid T_1 \mid \cdots \mid T_n \end{array}$$

- Attribute expressions:

$$V \longrightarrow U \mid \neg U$$

Semantics of $DL\text{-}Lite_{\mathcal{A}}$ – Objects vs. values

We make use of an alphabet Γ of constants, partitioned into:

- an alphabet Γ_O of object constants.
- an alphabet Γ_V of value constants, in turn partitioned into alphabets Γ_{V_i}, one for each RDF datatype T_i.

The interpretation domain $\Delta^{\mathcal{I}}$ is partitioned into:

- a domain of objects $\Delta_O^{\mathcal{I}}$
- a domain of values $\Delta_V^{\mathcal{I}}$

The semantics of $DL\text{-}Lite_{\mathcal{A}}$ descriptions is determined as usual, considering the following:

- The interpretation $C^{\mathcal{I}}$ of a concept C is a subset of $\Delta_O^{\mathcal{I}}$.
- The interpretation $R^{\mathcal{I}}$ of a role R is a subset of $\Delta_O^{\mathcal{I}} \times \Delta_O^{\mathcal{I}}$.
- The interpretation $val(v)$ of each value constant v in Γ_V and RDF datatype T_i is given a priori (e.g., all strings for `xsd:string`).
- The interpretation $V^{\mathcal{I}}$ of an attribute V is a subset of $\Delta_O^{\mathcal{I}} \times \Delta_V^{\mathcal{I}}$.

Semantics of the $DL\text{-}Lite_{\mathcal{A}}$ constructs

Construct	Syntax	Example	Semantics
top concept	$\top_C$		$\top_C^{\mathcal{I}} = \Delta_O^{\mathcal{I}}$
atomic concept	A	Doctor	$A^{\mathcal{I}} \subseteq \Delta_O^{\mathcal{I}}$
existential restriction	$\exists Q$	$\exists$child$^-$	$\{o \mid \exists o'. (o,o') \in Q^{\mathcal{I}}\}$
qualified exist. restriction	$\exists Q.C$	$\exists$child.Male	$\{o \mid \exists o'. (o,o') \in Q^{\mathcal{I}} \wedge o' \in C^{\mathcal{I}}\}$
concept negation	$\neg B$	$\neg\exists$child	$\Delta^{\mathcal{I}} \setminus B^{\mathcal{I}}$
attribute domain	$\delta(U)$	δ(salary)	$\{o \mid \exists v. (o,v) \in U^{\mathcal{I}}\}$
atomic role	P	child	$P^{\mathcal{I}} \subseteq \Delta_O^{\mathcal{I}} \times \Delta_O^{\mathcal{I}}$
inverse role	P^-	child$^-$	$\{(o,o') \mid (o',o) \in P^{\mathcal{I}}\}$
role negation	$\neg Q$	$\neg$manages	$(\Delta_O^{\mathcal{I}} \times \Delta_O^{\mathcal{I}}) \setminus Q^{\mathcal{I}}$
top domain	$\top_D$		$\top_D^{\mathcal{I}} = \Delta_V^{\mathcal{I}}$
datatype	T_i	`xsd:int`	$val(T_i) \subseteq \Delta_V^{\mathcal{I}}$
attribute range	$\rho(U)$	ρ(salary)	$\{v \mid \exists o. (o,v) \in U^{\mathcal{I}}\}$
atomic attribute	U	salary	$U^{\mathcal{I}} \subseteq \Delta_O^{\mathcal{I}} \times \Delta_V^{\mathcal{I}}$
attribute negation	$\neg U$	$\neg$salary	$(\Delta_O^{\mathcal{I}} \times \Delta_V^{\mathcal{I}}) \setminus U^{\mathcal{I}}$
object constant	c	`john`	$c^{\mathcal{I}} \in \Delta_O^{\mathcal{I}}$
value constant	v	`'john'`	$val(v) \in \Delta_V^{\mathcal{I}}$

$DL\text{-}Lite_{\mathcal{A}}$ assertions

TBox assertions can have the following forms:

$B \sqsubseteq C$	concept inclusion assertion
$Q \sqsubseteq R$	role inclusion assertion
$E \sqsubseteq F$	value-domain inclusion assertion
$U \sqsubseteq V$	attribute inclusion assertion
$(\textbf{funct}\ Q)$	role functionality assertion
$(\textbf{funct}\ U)$	attribute functionality assertion

ABox assertions: $A(c)$, $P(c, c')$, $U(c, d)$,
where c, c' are object constants
d is a value constant

Semantics of the *DL-Lite*$_{\mathcal{A}}$ assertions

Assertion	Syntax	Example	Semantics
conc. incl.	$B \sqsubseteq C$	Father $\sqsubseteq \exists$child	$B^\mathcal{I} \subseteq C^\mathcal{I}$
role incl.	$Q \sqsubseteq R$	father $\sqsubseteq$ anc	$Q^\mathcal{I} \subseteq R^\mathcal{I}$
v.dom. incl.	$E \sqsubseteq F$	ρ(age) $\sqsubseteq$ `xsd:int`	$E^\mathcal{I} \subseteq F^\mathcal{I}$
attr. incl.	$U \sqsubseteq V$	offPhone $\sqsubseteq$ phone	$U^\mathcal{I} \subseteq V^\mathcal{I}$
role funct.	(**funct** Q)	(**funct** father)	$\forall o, o, o''.(o, o') \in Q^\mathcal{I} \wedge (o, o'') \in Q^\mathcal{I} \rightarrow o' = o''$
att. funct.	(**funct** U)	(**funct** ssn)	$\forall o, v, v'.(o, v) \in U^\mathcal{I} \wedge (o, v') \in U^\mathcal{I} \rightarrow v = v'$
mem. asser.	$A(c)$	Father(bob)	$c^\mathcal{I} \in A^\mathcal{I}$
mem. asser.	$P(c_1, c_2)$	child(bob, ann)	$(c_1^\mathcal{I}, c_2^\mathcal{I}) \in P^\mathcal{I}$
mem. asser.	$U(c, d)$	phone(bob, '2345')	$(c^\mathcal{I}, val(d)) \in U^\mathcal{I}$

Restriction on TBox assertions in *DL-Lite*$_{\mathcal{A}}$ ontologies

We will see that, to ensure FOL-rewritability, we have to impose a **restriction** on the use of functionality and role/attribute inclusions.

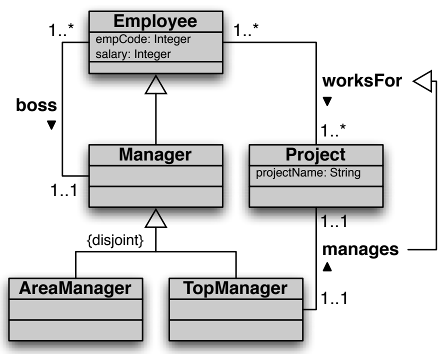
Restriction on *DL-Lite*$_{\mathcal{A}}$ TBoxes

No functional role or attribute can be specialized by using it in the right-hand side of a role or attribute inclusion assertion.

Formally:

- If $\exists P.C$ or $\exists P^-.C$ appears in $\mathcal{T}$, then (**funct** P) and (**funct** P^-) are **not in** $\mathcal{T}$.
- If $Q \sqsubseteq P$ or $Q \sqsubseteq P^-$ is in $\mathcal{T}$, then (**funct** P) and (**funct** P^-) are **not in** $\mathcal{T}$.
- If $U_1 \sqsubseteq U_2$ is in $\mathcal{T}$, then (**funct** U_2) is **not in** $\mathcal{T}$.

DL-Lite$_{\mathcal{A}}$ – Example



Manager	$\sqsubseteq$	Employee
AreaManager	$\sqsubseteq$	Manager
TopManager	$\sqsubseteq$	Manager
AreaManager	$\sqsubseteq$	$\neg$TopManager
Employee	$\sqsubseteq$	δ(salary)
δ(salary)	$\sqsubseteq$	Employee
ρ(salary)	$\sqsubseteq$	`xsd:int`
	(**funct** salary)	
$\exists$worksFor	$\sqsubseteq$	Employee
$\exists$worksFor$^-$	$\sqsubseteq$	Project
Employee	$\sqsubseteq$	$\exists$worksFor
Project	$\sqsubseteq$	$\exists$worksFor$^-$
	(**funct** manages)	
	(**funct** manages$^-$)	
manages	$\sqsubseteq$	worksFor
	$\vdots$	

Note: in *DL-Lite*$_{\mathcal{A}}$ we still cannot capture:
– completeness of the hierarchy
– number restrictions

Complexity results for *DL-Lite*

1. We have seen that *DL-Lite*$_{\mathcal{A}}$ can capture the essential features of prominent conceptual modeling formalisms.
2. In the following, we will analyze reasoning in *DL-Lite*, and establish the following characterization of its computational properties:
   - Ontology satisfiability is **polynomial** in the size of **TBox** and **ABox**.
   - Query answering is:
     - **PTime** in the size of the **TBox**.
     - **LogSpace** in the size of the **ABox**, and **FOL-rewritable**, which means that we can leverage for it relational database technology.
3. We will also see that *DL-Lite* is essentially the maximal DL enjoying these nice computational properties.

From (1), (2), and (3) we get the following claim:

DL-Lite is the representation formalism that is best suited to underly Ontology-Based Data Management systems.

Chapter III

Linking ontologies to data

Outline

Outline

Managing ABoxes

In the traditional DL setting, it is assumed that the data is maintained in the ABox of the ontology:

- The ABox is perfectly compatible with the TBox:
  - the vocabulary of concepts, roles, and attributes is the one used in the TBox.
  - The ABox "stores" abstract objects, and these objects and their properties are those returned by queries over the ontology.
- There may be different ways to manage the ABox from a physical point of view:
  - Description Logics reasoners maintain the ABox is main-memory data structures.
  - When an ABox becomes large, managing it in secondary storage may be required, but this is again handled directly by the reasoner.

Data in external sources

There are several situations where the assumptions of having the data in an ABox managed directly by the ontology system (e.g., a Description Logics reasoner) is not feasible or realistic:

- When the ABox is very large, so that it requires relational database technology.
- When we have no direct control over the data since it belongs to some external organization, which controls the access to it.
- When multiple data sources need to be accessed, such as in Information Integration.

We would like to deal with such a situation by keeping the data in the external (relational) storage, and performing **query answering** by leveraging the capabilities of the **relational engine**.

The impedance mismatch problem

We have to deal with the **impedance mismatch problem**:

- Sources store data, which is constituted by values taken from concrete domains, such as strings, integers, codes, ...
- Instead, instances of concepts and relations in an ontology are (abstract) objects.

Solution:

- We need to specify how to construct from the data values in the relational sources the (abstract) objects that populate the ABox of the ontology.
- This specification is embedded in the mappings between the data sources and the ontology.

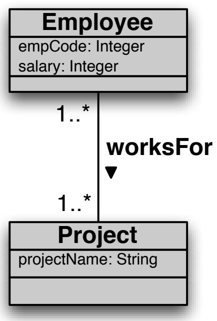
Note: the **ABox** is only **virtual**, and the objects are not materialized.

Solution to the impedance mismatch problem

We need to define a **mapping language** that allows for specifying how to transform data into abstract objects:

- Each mapping assertion maps:
  - a query that retrieves values from a data source to ...
  - a set of atoms specified over the ontology.
- Basic idea: use **Skolem functions** in the atoms over the ontology to "generate" the objects from the data values.
- Semantics of mappings:
  - Objects are denoted by terms (of exactly one level of nesting).
  - Different terms denote different objects (i.e., we make the unique name assumption on terms).

Impedance mismatch – Example



Actual data is stored in a DB:
– An Employee is identified by her *SSN*.
– A Project is identified by its *name*.

D_1[*SSN*: String, *PrName*: String]
Employees and Projects they work for

D_2[*Code*: String, *Salary*: Int]
Employee's Code with salary

D_3[*Code*: String, *SSN*: String]
Employee's Code with SSN

...

Intuitively:

- An employee should be created from her *SSN*: **pers**(*SSN*)
- A project should be created from its *Name*: **proj**(*PrName*)

Creating object identifiers

We need to associate to the data in the tables objects in the ontology.

- We introduce an alphabet Λ of **function symbols**, each with an associated arity.
- To denote values, we use value constants from an alphabet Γ_V.
- To denote objects, we use **object terms** instead of object constants. An object term has the form $\mathbf{f}(d_1, \ldots, d_n)$, with $\mathbf{f} \in \Lambda$, and each d_i a value constant in Γ_V.

Example

- If a person is identified by its *SSN*, we can introduce a function symbol **pers**/1. If VRD56B25 is a *SSN*, then **pers**(VRD56B25) denotes a person.
- If a person is identified by its *name* and *dateOfBirth*, we can introduce a function symbol **pers**/2. Then **pers**(Vardi, 25/2/56) denotes a person.

Mapping assertions

Mapping assertions are used to extract the data from the DB to populate the ontology.

We make use of **variable terms**, which are like object terms, but with variables instead of values as arguments of the functions.

Def.: Mapping assertion between a database and a TBox

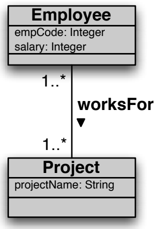
A **mapping assertion** between a database $\mathcal{D}$ and a TBox $\mathcal{T}$ has the form

$$\Phi \rightsquigarrow \Psi$$

where

- Φ is an arbitrary SQL query of arity $n > 0$ over $\mathcal{D}$.
- Ψ is a conjunctive query over $\mathcal{T}$ of arity $n' > 0$ **without non-distinguished variables**, possibly involving variable terms.

Mapping assertions – Example



D_1[*SSN*: String, *PrName*: String]
Employees and Projects they work for

D_2[*Code*: String, *Salary*: Int]
Employee's Code with salary

D_3[*Code*: String, *SSN*: String]
Employee's Code with SSN

...

m_1: `SELECT SSN, PrName FROM D1` $\rightsquigarrow$ Employee(**pers**(*SSN*)), Project(**proj**(*PrName*)), projectName(**proj**(*PrName*), *PrName*), worksFor(**pers**(*SSN*), **proj**(*PrName*))

m_2: `SELECT SSN, Salary FROM D2, D3 WHERE D2.Code = D3.Code` $\rightsquigarrow$ Employee(**pers**(*SSN*)), salary(**pers**(*SSN*), *Salary*)

Outline

Ontology-Based Data Access System

The mapping assertions are a crucial part of an Ontology-Based Data Access System.

Def.: Ontology-Based Data Access System

is a triple $\mathcal{O} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$, where

- $\mathcal{T}$ is a TBox.
- $\mathcal{D}$ is a relational database.
- $\mathcal{M}$ is a set of mapping assertions between $\mathcal{T}$ and $\mathcal{D}$.

We need to specify the syntax and semantics of mapping assertions.

Mapping assertions

A mapping assertion in $\mathcal{M}$ has the form

$$\Phi(\vec{x}) \leadsto \Psi(\vec{t}, \vec{y})$$

where

- Φ is an arbitrary SQL query of arity $n > 0$ over $\mathcal{D}$;
- Ψ is a conjunctive query over $\mathcal{T}$ of arity $n' > 0$ **without non-distinguished variables**;
- $\vec{x}$, $\vec{y}$ are variables, with $\vec{y} \subseteq \vec{x}$;
- $\vec{t}$ are variable terms of the form $f(\vec{z})$, with $f \in \Lambda$ and $\vec{z} \subseteq \vec{x}$.

Note: we could consider also mapping assertions between the datatypes of the database and those of the ontology.

Semantics of mappings

To define the semantics of an OBDA system $\mathcal{O} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$, we first need to define the semantics of mappings.

Def.: Satisfaction of a mapping assertion with respect to a database

An interpretation $\mathcal{I}$ **satisfies** a mapping assertion $\Phi(\vec{x}) \leadsto \Psi(\vec{t}, \vec{y})$ in $\mathcal{M}$ **with respect to a database** $\mathcal{D}$, if for each tuple of values $\vec{v} \in Eval(\Phi, \mathcal{D})$, and for each ground atom in $\Psi[\vec{x}/\vec{v}]$, we have that:

- if the ground atom is $A(s)$, then $s^{\mathcal{I}} \in A^{\mathcal{I}}$.
- if the ground atom is $P(s_1, s_2)$, then $(s_1^{\mathcal{I}}, s_2^{\mathcal{I}}) \in P^{\mathcal{I}}$.

Intuitively, $\mathcal{I}$ **satisfies** $\Phi \leadsto \Psi$ w.r.t. $\mathcal{D}$ if all facts obtained by evaluating Φ over $\mathcal{D}$ and then propagating the answers to Ψ, hold in $\mathcal{I}$.

Note: $Eval(\Phi, \mathcal{D})$ denotes the result of evaluating Φ over the database $\mathcal{D}$.
$\Psi[\vec{x}/\vec{v}]$ denotes Ψ where each x_i has been substituted with v_i.

Semantics of an OBDA system

Def.: Model of an OBDA system

An interpretation $\mathcal{I}$ is a **model** of $\mathcal{O} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$ if:

- $\mathcal{I}$ is a model of $\mathcal{T}$;
- $\mathcal{I}$ satisfies $\mathcal{M}$ w.r.t. $\mathcal{D}$, i.e., $\mathcal{I}$ satisfies every assertion in $\mathcal{M}$ w.r.t. $\mathcal{D}$.

An OBDA system $\mathcal{O}$ is **satisfiable** if it admits at least one model.

Outline

Answering queries over an OBDA system

In an OBDA system $\mathcal{O} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$

- Queries are posed over the TBox $\mathcal{T}$.
- The data needed to answer queries is stored in the database $\mathcal{D}$.
- The mapping $\mathcal{M}$ is used to bridge the gap between $\mathcal{T}$ and $\mathcal{D}$.

Two approaches to exploit the mapping:

- bottom-up approach: simpler, but less efficient
- top-down approach: more sophisticated, but also more efficient

Note: Both approaches require to first **split** the TBox queries in the mapping assertions into their constituent atoms.

Splitting of mappings

A mapping assertion $\Phi \rightsquigarrow \Psi$, where the TBox query Ψ is constituted by the atoms $X_1, \ldots, X_k$, can be split into several mapping assertions:

$$\Phi \rightsquigarrow X_1 \qquad \cdots \qquad \Phi \rightsquigarrow X_k$$

This is possible, since Ψ does not contain non-distinguished variables.

Example

m_1: `SELECT SSN, PrName FROM D`$_1$ $\rightsquigarrow$ Employee(**pers**(*SSN*)), Project(**proj**(*PrName*)), projectName(**proj**(*PrName*), *PrName*), worksFor(**pers**(*SSN*), **proj**(*PrName*))

is split into

m_1^1: `SELECT SSN, PrName FROM D`$_1$ $\rightsquigarrow$ Employee(**pers**(*SSN*))
m_1^2: `SELECT SSN, PrName FROM D`$_1$ $\rightsquigarrow$ Project(**proj**(*PrName*))
m_1^3: `SELECT SSN, PrName FROM D`$_1$ $\rightsquigarrow$ projectName(**proj**(*PrName*), *PrName*)
m_1^4: `SELECT SSN, PrName FROM D`$_1$ $\rightsquigarrow$ worksFor(**pers**(*SSN*), **proj**(*PrName*))

Bottom-up approach to query answering

Consists in a straightforward application of the mappings:

1. Propagate the data from $\mathcal{D}$ through $\mathcal{M}$, materializing an ABox $\mathcal{A}_{\mathcal{M},\mathcal{D}}$ (the constants in such an ABox are values and object terms).
2. Apply to $\mathcal{A}_{\mathcal{M},\mathcal{D}}$ and to the TBox $\mathcal{T}$, the satisfiability and query answering algorithms developed for *DL-Lite*$_{\mathcal{A}}$.

This approach has several drawbacks (hence is only theoretical):

- The technique is no more LogSpace in the data, since the ABox $\mathcal{A}_{\mathcal{M},\mathcal{D}}$ to materialize is in general polynomial in the size of the data.
- $\mathcal{A}_{\mathcal{M},\mathcal{D}}$ may be very large, and thus it may be infeasible to actually materialize it.
- Freshness of $\mathcal{A}_{\mathcal{M},\mathcal{D}}$ with respect to the underlying data source(s) may be an issue, and one would need to propagate source updates (cf. Data Warehousing).

Top-down approach to query answering

Consists of three steps:

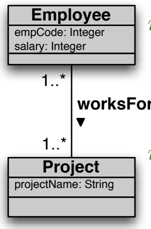
1. **Reformulation:** Compute the perfect reformulation $q_{pr} = \mathit{PerfectRef}(q, \mathcal{T}_P)$ of the original query q, using the inclusion assertions of the TBox $\mathcal{T}$ (see later).
2. **Unfolding:** Compute from q_{pr} a new query q_{unf} by unfolding q_{pr} using (the split version of) the mappings $\mathcal{M}$.
   - Essentially, each atom in q_{pr} that unifies with an atom in Ψ is substituted with the corresponding query Φ over the database.
   - The unfolded query is such that $\mathit{Eval}(q_{unf}, \mathcal{D}) = \mathit{Eval}(q_{pr}, \mathcal{A}_{\mathcal{M},\mathcal{D}})$.
3. **Evaluation:** Delegate the evaluation of q_{unf} to the relational DBMS managing $\mathcal{D}$.

Unfolding

To unfold a query q_{pr} with respect to a set of mapping assertions:

1. For each non-split mapping assertion $\Phi_i(\vec{x}) \leadsto \Psi_i(\vec{t}, \vec{y})$:
   1. Introduce a **view symbol** Aux_i of arity equal to that of Φ_i.
   2. Add a **view definition** $\mathsf{Aux}_i(\vec{x}) \leftarrow \Phi_i(\vec{x})$.
2. For each split version $\Phi_i(\vec{x}) \leadsto X_j(\vec{t}, \vec{y})$ of a mapping assertion, introduce a **clause** $X_j(\vec{t}, \vec{y}) \leftarrow \mathsf{Aux}_i(\vec{x})$.
3. Obtain from q_{pr} in all possible ways queries q_{aux} defined over the view symbols Aux_i as follows:
   1. Find a most general unifier ϑ that unifies each atom $X(\vec{z})$ in the body of q_{pr} with the head of a clause $X(\vec{t}, \vec{y}) \leftarrow \mathsf{Aux}_i(\vec{x})$.
   2. Substitute each atom $X(\vec{z})$ with $\vartheta(\mathsf{Aux}_i(\vec{x}))$, i.e., with the body the unified clause to which the unifier ϑ is applied.
4. The unfolded query q_{unf} is the **union** of all queries q_{aux}, together with the view definitions for the predicates Aux_i appearing in q_{aux}.

Unfolding – Example



m_1: `SELECT SSN, PrName FROM D1` $\leadsto$ $\mathsf{Employee}(\mathbf{pers}(\mathit{SSN}))$, $\mathsf{Project}(\mathbf{proj}(\mathit{PrName}))$, $\mathsf{projectName}(\mathbf{proj}(\mathit{PrName}), \mathit{PrName})$, $\mathsf{worksFor}(\mathbf{pers}(\mathit{SSN}), \mathbf{proj}(\mathit{PrName}))$

m_2: `SELECT SSN, Salary FROM D2, D3 WHERE D2.Code = D3.Code` $\leadsto$ $\mathsf{Employee}(\mathbf{pers}(\mathit{SSN}))$, $\mathsf{salary}(\mathbf{pers}(\mathit{SSN}), \mathit{Salary})$

We define a view Aux_i for the source query of each mapping m_i.

For each (split) mapping assertion, we introduce a clause:

$$
\begin{array}{rcl}
\mathsf{Employee}(\mathbf{pers}(\mathit{SSN})) & \leftarrow & \mathsf{Aux}_1(\mathit{SSN}, \mathit{PrName}) \\
\mathsf{projectName}(\mathbf{proj}(\mathit{PrName}), \mathit{PrName}) & \leftarrow & \mathsf{Aux}_1(\mathit{SSN}, \mathit{PrName}) \\
\mathsf{Project}(\mathbf{proj}(\mathit{PrName})) & \leftarrow & \mathsf{Aux}_1(\mathit{SSN}, \mathit{PrName}) \\
\mathsf{worksFor}(\mathbf{pers}(\mathit{SSN}), \mathbf{proj}(\mathit{PrName})) & \leftarrow & \mathsf{Aux}_1(\mathit{SSN}, \mathit{PrName}) \\
\mathsf{Employee}(\mathbf{pers}(\mathit{SSN})) & \leftarrow & \mathsf{Aux}_2(\mathit{SSN}, \mathit{Salary}) \\
\mathsf{salary}(\mathbf{pers}(\mathit{SSN}), \mathit{Salary}) & \leftarrow & \mathsf{Aux}_2(\mathit{SSN}, \mathit{Salary})
\end{array}
$$

Unfolding – Example (cont'd)

Query over ontology: employees who work for `tones` and their salary:
$q(e, s) \leftarrow \mathsf{Employee}(e), \mathsf{salary}(e, s), \mathsf{worksFor}(e, p), \mathsf{projectName}(p, \mathtt{tones})$

A unifier between the atoms in q and the clause heads is:

$$
\begin{array}{ll}
\vartheta(e) = \mathbf{pers}(\mathit{SSN}) & \vartheta(s) = \mathit{Salary} \\
\vartheta(\mathit{PrName}) = \mathtt{tones} & \vartheta(p) = \mathbf{proj}(\mathtt{tones})
\end{array}
$$

After applying ϑ to q, we obtain:
$q(\mathbf{pers}(\mathit{SSN}), \mathit{Salary}) \leftarrow \mathsf{Employee}(\mathbf{pers}(\mathit{SSN})),\ \mathsf{salary}(\mathbf{pers}(\mathit{SSN}), \mathit{Salary}),$
$\mathsf{worksFor}(\mathbf{pers}(\mathit{SSN}), \mathbf{proj}(\mathtt{tones})),$
$\mathsf{projectName}(\mathbf{proj}(\mathtt{tones}), \mathtt{tones})$

Substituting the atoms with the bodies of the unified clauses, we obtain:
$q(\mathbf{pers}(\mathit{SSN}), \mathit{Salary}) \leftarrow \mathsf{Aux}_1(\mathit{SSN}, \mathtt{tones}),\ \mathsf{Aux}_2(\mathit{SSN}, \mathit{Salary}),$
$\mathsf{Aux}_1(\mathit{SSN}, \mathtt{tones}),\ \mathsf{Aux}_1(\mathit{SSN}, \mathtt{tones})$

Exponential blowup in the unfolding

When there are multiple mapping assertions for each atom, the unfolded query may be exponential in the original one.

Consider a query: $q(y) \leftarrow A_1(y), A_2(y), \ldots, A_n(y)$

and the mappings: $m_i^1\colon \Phi_i^1(x) \rightsquigarrow A_i(\mathbf{f}(x))$ (for $i \in \{1, \ldots, n\}$)
$m_i^2\colon \Phi_i^2(x) \rightsquigarrow A_i(\mathbf{f}(x))$

We add the view definitions: $\text{Aux}_i^j(x) \leftarrow \Phi_i^j(x)$
and introduce the clauses: $A_i(\mathbf{f}(x)) \leftarrow \text{Aux}_i^j(x)$ (for $i \in \{1, \ldots, n\}$, $j \in \{1, 2\}$).

There is a single unifier, namely $\vartheta(y) = \mathbf{f}(x)$, but each atom $A_i(y)$ in the query unifies with the head of two clauses.

Hence, we obtain one unfolded query

$$q(\mathbf{f}(x)) \leftarrow \text{Aux}_1^{j_1}(x), \text{Aux}_2^{j_2}(x), \ldots, \text{Aux}_n^{j_n}(x)$$

for each possible combination of $j_i \in \{1, 2\}$, for $i \in \{1, \ldots, n\}$.
Hence, we obtain 2^n **unfolded queries**.

Computational complexity of query answering

From the top-down approach to query answering, and the complexity results for *DL-Lite*, we obtain the following result.

Theorem

Query answering in a *DL-Lite* OBDM system $\mathcal{O} = \langle \mathcal{T}, \mathcal{M}, \mathcal{D} \rangle$ is

1. **NP-complete** in the size of the query.
2. **PTime** in the size of the **TBox** $\mathcal{T}$ and the **mappings** $\mathcal{M}$.
3. **LogSpace** in the size of the **database** $\mathcal{D}$.

Note: The LogSpace result is a consequence of the fact that query answering in such a setting can be reduced to evaluating an SQL query over the relational database.

Implementation of top-down approach to query answering

To implement the top-down approach, we need to generate an SQL query.

We can follow different strategies:

1. Substitute each view predicate in the unfolded queries with the corresponding SQL query over the source:
   - \+ joins are performed on the DB attributes;
   - \+ does not generate doubly nested queries;
   - − the number of unfolded queries may be exponential.
2. Construct for each atom in the original query a new view. This view takes the union of all SQL queries corresponding to the view predicates, and constructs also the Skolem terms:
   - \+ avoids exponential blow-up of the resulting query, since the union (of the queries coming from multiple mappings) is done before the joins;
   - − joins are performed on Skolem terms;
   - − generates doubly nested queries.

Which method is better, depends on various parameters.
Experiments have shown that (1) behaves better in most cases.

Chapter IV

Reasoning in the *DL-Lite* family

Outline

Outline

Remark on used notation

In the following,

- We use "TBox" to denote either a $DL\text{-}Lite_{\mathcal{R}}$ or a $DL\text{-}Lite_{\mathcal{F}}$ TBox.
- Q, possibly with subscript, denotes a **basic role**, i.e.,

$$Q \longrightarrow P \mid P^-$$

- C, possibly with subscript, denotes a **general concept**, i.e.,

$$C \longrightarrow A \mid \neg A \mid \exists Q \mid \neg\exists Q$$

  where A is an atomic concept and P is an atomic role.
- R, possibly with subscript, denotes a **general role**, i.e.,

$$R \longrightarrow Q \mid \neg Q$$

TBox Reasoning services

- **Concept Satisfiability:** C is satisfiable wrt $\mathcal{T}$, if there is a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is not empty, i.e., $\mathcal{T} \not\models C \equiv \bot$
- **Subsumption:** C_1 is subsumed by C_2 wrt $\mathcal{T}$, if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \sqsubseteq C_2$.
- **Equivalence:** C_1 and C_2 are equivalent wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$, i.e., $\mathcal{T} \models C_1 \equiv C_2$.
- **Disjointness:** C_1 and C_2 are disjoint wrt $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$ we have $C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}} = \emptyset$, i.e., $\mathcal{T} \models C_1 \sqcap C_2 \equiv \bot$
- **Functionality implication:** A functionality assertion $(\textbf{funct } Q)$ is logically implied by $\mathcal{T}$ if for every model $\mathcal{I}$ of $\mathcal{T}$, we have that $(o, o_1) \in Q^{\mathcal{I}}$ and $(o, o_2) \in Q^{\mathcal{I}}$ implies $o_1 = o_2$, i.e., $\mathcal{T} \models (\textbf{funct } Q)$.

Analogous definitions hold for role satisfiability, subsumption, equivalence, and disjointness.

From TBox reasoning to ontology (un)satisfiability

Basic reasoning service:

- **Ontology satisfiability**: Verify whether an ontology $\mathcal{O}$ is satisfiable, i.e., whether $\mathcal{O}$ admits at least one model.

In the following, we show how to reduce TBox reasoning to ontology unsatisfiability:

1. We show how to reduce TBox reasoning services to concept/role subsumption.
2. We provide reductions from concept/role subsumption to ontology unsatisfiability.

Concept/role satisfiability, equivalence, and disjointness

Theorem

1. C is unsatisfiable wrt $\mathcal{T}$ iff $\mathcal{T} \models C \sqsubseteq \neg C$.
2. $\mathcal{T} \models C_1 \equiv C_2$ iff $\mathcal{T} \models C_1 \sqsubseteq C_2$ and $\mathcal{T} \models C_2 \sqsubseteq C_1$.
3. C_1 and C_2 are disjoint iff $\mathcal{T} \models C_1 \sqsubseteq \neg C_2$.

Proof (sketch).

1. "$\Leftarrow$" if $\mathcal{T} \models C \sqsubseteq \neg C$, then $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$, for every model $\mathcal{I} = \langle \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}} \rangle$ of $\mathcal{T}$; but this holds iff $C^{\mathcal{I}} = \emptyset$.
   "$\Rightarrow$" if C is unsatisfiable, then $C^{\mathcal{I}} = \emptyset$, for every model $\mathcal{I}$ of $\mathcal{T}$. Therefore $C^{\mathcal{I}} \subseteq (\neg C)^{\mathcal{I}}$.
2. Trivial.
3. Trivial. □

Analogous reductions for role satisfiability, equivalence and disjointness.

From implication of functionalities to subsumption

Theorem

$\mathcal{T} \models$ (**funct** Q) iff either (**funct** Q) $\in \mathcal{T}$ (only for *DL-Lite*$_{\mathcal{F}}$ ontologies), or $\mathcal{T} \models Q \sqsubseteq \neg Q$.

Proof (sketch).

"$\Leftarrow$" The case in which (**funct** Q) $\in \mathcal{T}$ is trivial.
Instead, if $\mathcal{T} \models Q \sqsubseteq \neg Q$, then $Q^{\mathcal{I}} = \emptyset$ and hence $\mathcal{I} \models$ (**funct** Q), for every model $\mathcal{I}$ of $\mathcal{T}$.

"$\Rightarrow$" When neither (**funct** Q) $\in \mathcal{T}$ nor $\mathcal{T} \models Q \sqsubseteq \neg Q$, we can construct a model of $\mathcal{T}$ that is not a model of (**funct** Q). □

From concept subsumption to ontology unsatisfiability

Theorem

$\mathcal{T} \models C_1 \sqsubseteq C_2$ iff the ontology $\mathcal{O}_{C_1 \sqsubseteq C_2} = \langle \mathcal{T} \cup \{\hat{A} \sqsubseteq C_1, \hat{A} \sqsubseteq \neg C_2\},\ \{\hat{A}(c)\} \rangle$ is unsatisfiable, where $\hat{A}$ is an atomic concept not in $\mathcal{T}$, and c is a constant.

Intuitively, C_1 is subsumed by C_2 iff the smallest ontology containing $\mathcal{T}$ and implying both $C_1(c)$ and $\neg C_2(c)$ is unsatisfiable.

Proof (sketch).

"$\Leftarrow$" Let $\mathcal{O}_{C_1 \sqsubseteq C_2}$ be unsatisfiable, and suppose that $\mathcal{T} \not\models C_1 \sqsubseteq C_2$. Then there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C_1^{\mathcal{I}} \not\subseteq C_2^{\mathcal{I}}$. Hence $C_1^{\mathcal{I}} \setminus C_2^{\mathcal{I}} \neq \emptyset$. We can extend $\mathcal{I}$ to a model of $\mathcal{O}_{C_1 \sqsubseteq C_2}$ by taking $c^{\mathcal{I}} = o$, for some $o \in C_1^{\mathcal{I}} \setminus C_2^{\mathcal{I}}$, and $\hat{A}^{\mathcal{I}} = \{c^{\mathcal{I}}\}$. This contradicts $\mathcal{O}_{C_1 \sqsubseteq C_2}$ being unsatisfiable.
"$\Rightarrow$" Let $\mathcal{T} \models C_1 \sqsubseteq C_2$, and suppose that $\mathcal{O}_{C_1 \sqsubseteq C_2}$ is satisfiable. Then there exists a model $\mathcal{I}$ be of $\mathcal{O}_{C_1 \sqsubseteq C_2}$. Then $\mathcal{I} \models \mathcal{T}$, and $\mathcal{I} \models C_1(c)$ and $\mathcal{I} \models \neg C_2(c)$, i.e., $\mathcal{I} \not\models C_1 \sqsubseteq C_2$. This contradicts $\mathcal{T} \models C_1 \sqsubseteq C_2$. □

From role subsumption to ont. unsatisfiability for *DL-Lite*$_\mathcal{R}$

Theorem

Let $\mathcal{T}$ be a **DL-Lite$_\mathcal{R}$ TBox** and R_1, R_2 two general roles.
Then $\mathcal{T} \models R_1 \sqsubseteq R_2$ iff the ontology
$\mathcal{O}_{R_1 \sqsubseteq R_2} = \langle \mathcal{T} \cup \{\hat{P} \sqsubseteq R_1, \hat{P} \sqsubseteq \neg R_2\},\ \{\hat{P}(c_1, c_2)\}\rangle$ is unsatisfiable,
where $\hat{P}$ is an atomic role not in $\mathcal{T}$, and c_1, c_2 are two constants.

Intuitively, R_1 is subsumed by R_2 iff the smallest ontology containing $\mathcal{T}$ and implying both $R_1(c_1, c_2)$ and $\neg R_2(c_1, c_2)$ is unsatisfiable.

Proof (sketch).

Analogous to the one for concept subsumption. □

Notice that $\mathcal{O}_{R_1 \sqsubseteq R_2}$ *is inherently a DL-Lite*$_\mathcal{R}$ *ontology.*

From role subsumption to ont. unsatisfiability for *DL-Lite*$_\mathcal{F}$

Theorem

Let $\mathcal{T}$ be a **DL-Lite$_\mathcal{F}$ TBox**, and Q_1, Q_2 two basic roles such that $Q_1 \neq Q_2$.
Then,

1. $\mathcal{T} \models Q_1 \sqsubseteq Q_2$ iff Q_1 is unsatisfiable iff
   either $\exists Q_1$ or $\exists Q_1^-$ is unsatisfiable wrt $\mathcal{T}$,
   which can again be reduced to ontology unsatisfiability.
2. $\mathcal{T} \models \neg Q_1 \sqsubseteq Q_2$ iff $\mathcal{T}$ is unsatisfiable.
3. $\mathcal{T} \models Q_1 \sqsubseteq \neg Q_2$ iff
   either $\exists Q_1$ and $\exists Q_2$ are disjoint, or $\exists Q_1^-$ and $\exists Q_2^-$ are disjoint, iff
   either $\mathcal{T} \models \exists Q_1 \sqsubseteq \neg\exists Q_2$, or $\mathcal{T} \models \exists Q_1^- \sqsubseteq \neg\exists Q_2^-$,
   which can again be reduced to ontology unsatisfiability.

Notice that an inclusion of the form $\neg Q_1 \sqsubseteq \neg Q_2$ is equivalent to $Q_2 \sqsubseteq Q_1$, and therefore is considered in the first item.

Summary

- The results above tell us that we can support TBox reasoning services by relying on the ontology (un)satisfiability service.
- Ontology satisfiability is a form of reasoning over both the TBox and the ABox of the ontology.
- In the following, we first consider other TBox & ABox reasoning services, in particular **query answering**, and then turn back to ontology satisfiability.

Outline

TBox and ABox reasoning services

- **Ontology Satisfiability:** Verify wether an ontology $\mathcal{O}$ is satisfiable, i.e., whether $\mathcal{O}$ admits at least one model.
- **Concept Instance Checking:** Verify wether an individual c is an instance of a concept C in an ontology $\mathcal{O}$, i.e., whether $\mathcal{O} \models C(c)$.
- **Role Instance Checking:** Verify wether a pair (c_1, c_2) of individuals is an instance of a role Q in an ontology $\mathcal{O}$, i.e., whether $\mathcal{O} \models Q(c_1, c_2)$.
- **Query Answering** Given a query q over an ontology $\mathcal{O}$, find all tuples $\vec{c}$ of constants such that $\mathcal{O} \models q(\vec{c})$.

Query answering and instance checking

For atomic concepts and roles, **instance checking is a special case of query answering**, in which the query is **boolean** and constituted by a single atom in the body.

- $\mathcal{O} \models A(c)$ iff $q() \leftarrow A(c)$ evaluated over $\mathcal{O}$ is true.
- $\mathcal{O} \models P(c_1, c_2)$ iff $q() \leftarrow P(c_1, c_2)$ evaluated over $\mathcal{O}$ is true.

From instance checking to ontology unsatisfiability

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a **DL-Lite** ontology, C a *DL-Lite* concept, and P an atomic role. Then:

- $\mathcal{O} \models C(c)$ iff $\mathcal{O}_{C(c)} = \langle \mathcal{T} \cup \{\hat{A} \sqsubseteq \neg C\},\ \mathcal{A} \cup \{\hat{A}(c)\} \rangle$ is unsatisfiable, where $\hat{A}$ is an atomic concept not in $\mathcal{O}$.
- $\mathcal{O} \models \neg P(c_1, c_2)$ iff $\mathcal{O}_{\neg P(c_1,c_2)} = \langle \mathcal{T},\ \mathcal{A} \cup \{P(c_1, c_2)\} \rangle$ is unsatisfiable.

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a **DL-Lite$_{\mathcal{F}}$** ontology and P an atomic role.
Then $\mathcal{O} \models P(c_1, c_2)$ iff $\mathcal{O}$ is unsatisfiable or $P(c_1, c_2) \in \mathcal{A}$.

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a **DL-Lite$_{\mathcal{R}}$** ontology and P an atomic role.
Then $\mathcal{O} \models P(c_1, c_2)$ iff $\mathcal{O}_{P(c_1,c_2)} = \langle \mathcal{T} \cup \{\hat{P} \sqsubseteq \neg P\},\ \mathcal{A} \cup \{\hat{P}(c_1, c_2)\} \rangle$ is unsatisfiable, where $\hat{P}$ is an atomic role not in $\mathcal{O}$.

Certain answers

We recall that

Query answering over an ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ is a form of **logical implication**:

find all tuples $\vec{c}$ of constants of $\mathcal{A}$ s.t. $\mathcal{O} \models q(\vec{c})$

A.k.a. **certain answers** in databases, i.e., the tuples that are answers to q in **all** models of $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$:

$$cert(q, \mathcal{O}) \;=\; \{\ \vec{c} \mid \vec{c} \in q^{\mathcal{I}},\ \text{ for every model } \mathcal{I} \text{ of } \mathcal{O}\ \}$$

Note: We have assumed that the answer $q^{\mathcal{I}}$ to a query q over an interpretation $\mathcal{I}$ is constituted by a set of tuples of **constants** of $\mathcal{A}$, rather than objects in $\Delta^{\mathcal{I}}$.

Data complexity of query answering

When studying the complexity of query answering, we need to consider the associated decision problem:

Def.: Recognition problem for query answering

Given an ontology $\mathcal{O}$, a query q over $\mathcal{O}$, and a tuple $\vec{c}$ of constants, **check whether** $\vec{c} \in cert(q, \mathcal{O})$.

We consider a setting where the size of the data largely dominates the size of the conceptual layer, hence, we concentrate on efficiency in the size of the data.

We look at **data complexity** of query answering, i.e., complexity of the recognition problem computed **w.r.t. the size of the ABox only**.

Basic questions associated to query answering

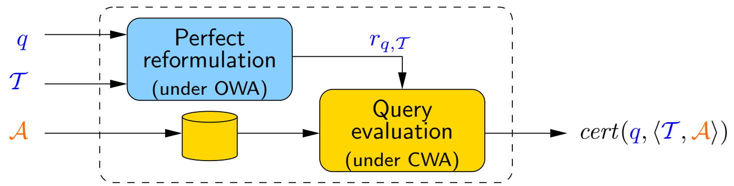
1. For which ontology languages can we answer queries over an ontology efficiently?
2. How complex becomes query answering over an ontology when we consider more expressive ontology languages?

Inference in query answering

To be able to deal with data efficiently, we need to separate the contribution of $\mathcal{A}$ from the contribution of q and $\mathcal{T}$.

$\rightsquigarrow$ Query answering by **query rewriting**.

Query rewriting



Query answering can **always** be thought as done in two phases:

1. **Perfect rewriting**: produce from q and the TBox $\mathcal{T}$ a new query $r_{q,\mathcal{T}}$ (called the perfect rewriting of q w.r.t. $\mathcal{T}$).
2. **Query evaluation**: evaluate $r_{q,\mathcal{T}}$ over the ABox $\mathcal{A}$ seen as a complete database (and without considering the TBox $\mathcal{T}$).
   $\rightsquigarrow$ Produces $cert(q, \langle\mathcal{T}, \mathcal{A}\rangle)$.

Note: The "always" holds if we pose no restriction on the language in which to express the rewriting $r_{q,\mathcal{T}}$.

$\mathcal{Q}$-rewritability (cont'd)

Let Q be a query language and $\mathcal{L}$ be an ontology language.

Def.: $\mathcal{Q}$-rewritability

For an ontology language $\mathcal{L}$, query answering is **$\mathcal{Q}$-rewritable** if for every TBox $\mathcal{T}$ of $\mathcal{L}$ and for every query q, the perfect reformulation $r_{q,\mathcal{T}}$ of q w.r.t. $\mathcal{T}$ can be expressed in the query language $\mathcal{Q}$.

Notice that the complexity of computing $r_{q,\mathcal{T}}$ or the size of $r_{q,\mathcal{T}}$ do **not** affect data complexity.

Hence, $\mathcal{Q}$-rewritability is tightly related to **data complexity**, i.e.:

- complexity of computing $cert(q, \langle\mathcal{T}, \mathcal{A}\rangle)$ measured in the size of the ABox $\mathcal{A}$ only,
- which corresponds to the **complexity of evaluating $r_{q,\mathcal{T}}$ over $\mathcal{A}$**.

$\mathcal{Q}$-rewritability: interesting cases

Consider an ontology language $\mathcal{L}$ that enjoys $\mathcal{Q}$-rewritability, for a query language $\mathcal{Q}$:

- When $\mathcal{Q}$ is FOL (i.e., the language enjoys **FOL-rewritability**)
  $\leadsto$ query evaluation can be done in SQL, i.e., via an RDBMS
  (*Note:* FOL is in LogSpace).
- When $\mathcal{Q}$ is an NLogSpace-hard language
  $\leadsto$ query evaluation requires (at least) linear recursion.
- When $\mathcal{Q}$ is a PTime-hard language
  $\leadsto$ query evaluation requires (at least) recursion (e.g., Datalog).
- When $\mathcal{Q}$ is a coNP-hard language
  $\leadsto$ query evaluation requires (at least) power of Disjunctive Datalog.

$\mathcal{Q}$-rewritability for *DL-Lite*

- We now study $\mathcal{Q}$-rewritability of query answering over *DL-Lite* ontologies.
- In particular we will show that both *DL-Lite*$_{\mathcal{R}}$ and *DL-Lite*$_{\mathcal{F}}$ enjoy FOL-rewritability of conjunctive query answering.

Query answering over unsatisfiable ontologies

- In the case in which an ontology is unsatisfiable, according to the "ex falso quod libet" principle, reasoning is trivialized.
- In particular, query answering is meaningless, since every tuple is in the answer to every query.
- We are not interested in encoding meaningless query answering into the perfect reformulation of the input query. Therefore, before query answering, we will always check ontology satisfiability to single out meaningful cases.
- Thus, in the following, we focus on query answering over **satisfiable ontologies.**
- We first consider **satisfiable DL-Lite$_{\mathcal{R}}$ ontologies.**

Remark

We call positive inclusions (PIs) assertions of the form

$$
\begin{array}{rcl}
Cl & \sqsubseteq & A \mid \exists Q \\
Q_1 & \sqsubseteq & Q_2
\end{array}
$$

We call negative inclusions (NIs) assertions of the form

$$
\begin{array}{rcl}
Cl & \sqsubseteq & \neg A \mid \neg\exists Q \\
Q_1 & \sqsubseteq & \neg Q_2
\end{array}
$$

Query answering in *DL-Lite*$_{\mathcal{R}}$

Given a CQ q and a satisfiable ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$, we compute $cert(q, \mathcal{O})$ as follows:

1. Using $\mathcal{T}$, **reformulate** q as a union $r_{q,\mathcal{T}}$ of CQs.
2. Evaluate $r_{q,\mathcal{T}}$ directly over $\mathcal{A}$ managed in **secondary storage via a RDBMS**.

Correctness of this procedure shows FOL-rewritability of query answering in *DL-Lite*$_{\mathcal{R}}$.

$\leadsto$ Query answering over *DL-Lite*$_{\mathcal{R}}$ ontologies can be done using RDMBS technology.

Query reformulation

Consider the query $\quad \mathsf{q}(x) \leftarrow \mathsf{Professor}(x)$

Intuition: Use the PIs as basic rewriting rules:

$$\mathsf{AssistantProf} \sqsubseteq \mathsf{Professor}$$

as a logic rule: $\quad \mathsf{Professor}(z) \leftarrow \mathsf{AssistantProf}(z)$

Basic rewriting step:

when an atom in the query unifies with the **head** of the rule,

substitute the atom with the **body** of the rule.

We say that the PI inclusion **applies to** the atom.

In the example, the PI $\mathsf{AssistantProf} \sqsubseteq \mathsf{Professor}$ applies to the atom $\mathsf{Professor}(x)$. Towards the computation of the perfect reformulation, we add to the input query above, the query

$$\mathsf{q}(x) \leftarrow \mathsf{AssistantProf}(x)$$

Query reformulation (cont'd)

Consider the query $\quad \mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$

and the PI $\quad \exists\mathsf{teaches}^- \sqsubseteq \mathsf{Course}$

as a logic rule: $\quad \mathsf{Course}(z_2) \leftarrow \mathsf{teaches}(z_1, z_2)$

The PI applies to the atom $\mathsf{Course}(y)$, and we add to the perfect reformulation the query

$$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{teaches}(z_1, y)$$

Consider now the query $\quad \mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y)$

and the PI $\quad \mathsf{Professor} \sqsubseteq \exists\mathsf{teaches}$

as a logic rule: $\quad \mathsf{teaches}(z, f(z)) \leftarrow \mathsf{Professor}(z)$

The PI applies to the atom $\mathsf{teaches}(x, y)$, and we add to the perfect reformulation the query

$$\mathsf{q}(x) \leftarrow \mathsf{Professor}(x)$$

Query reformulation – Constants

Conversely, for the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, \mathtt{kbdb})$

and the same PI as before $\mathsf{Professor} \sqsubseteq \exists\mathsf{teaches}$

as a logic rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{Professor}(z)$

$\mathsf{teaches}(x, \mathtt{kbdb})$ does not unify with $\mathsf{teaches}(z, f(z))$, since the **skolem term** $f(z)$ in the head of the rule **does not unify** with the constant $\mathtt{kbdb}$.

In this case, the PI **does not apply** to the atom $\mathsf{teaches}(x, \mathtt{kbdb})$.

The same holds for the following query, where y is **distinguished**, since unifying $f(z)$ with y would correspond to returning a skolem term as answer to the query:

$$\mathsf{q}(x, y) \leftarrow \mathsf{teaches}(x, y)$$

Query reformulation – Join variables

An analogous behavior to the one with constants and with distinguished variables holds when the atom contains **join variables** that would have to be unified with skolem terms.

Consider the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$

and the PI $\mathsf{Professor} \sqsubseteq \exists\mathsf{teaches}$

as a logic rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{Professor}(z)$

The PI above does **not** apply to the atom $\mathsf{teaches}(x, y)$.

Query reformulation – Reduce step

Consider now the query $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{teaches}(z, y)$

and the PI $\mathsf{Professor} \sqsubseteq \exists\mathsf{teaches}$

as a logic rule: $\mathsf{teaches}(z, f(z)) \leftarrow \mathsf{Professor}(z)$

This PI does not apply to $\mathsf{teaches}(x, y)$ or $\mathsf{teaches}(z, y)$, since y is in join, and we would introduce the skolem term in the rewritten query.

However, we can transform the above query by unifying the atoms $\mathsf{teaches}(x, y)$ and $\mathsf{teaches}(z, y)$. This rewriting step is called **reduce**, and produces the query

$$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y)$$

Now, we can apply the PI above, and add to the reformulation the query

$$\mathsf{q}(x) \leftarrow \mathsf{Professor}(x)$$

Query reformulation – Summary

Reformulate the CQ q into a set of queries: apply to q and the computed queries in all possible ways the PIs in the TBox $\mathcal{T}$:

$$\begin{array}{rccc}
A_1 \sqsubseteq A_2 & \ldots, A_2(x), \ldots & \rightsquigarrow & \ldots, A_1(x), \ldots \\
\exists P \sqsubseteq A & \ldots, A(x), \ldots & \rightsquigarrow & \ldots, P(x, _), \ldots \\
\exists P^- \sqsubseteq A & \ldots, A(x), \ldots & \rightsquigarrow & \ldots, P(_, x), \ldots \\
A \sqsubseteq \exists P & \ldots, P(x, _), \ldots & \rightsquigarrow & \ldots, A(x), \ldots \\
A \sqsubseteq \exists P^- & \ldots, P(_, x), \ldots & \rightsquigarrow & \ldots, A(x), \ldots \\
\exists P_1 \sqsubseteq \exists P_2 & \ldots, P_2(x, _), \ldots & \rightsquigarrow & \ldots, P_1(x, _), \ldots \\
P_1 \sqsubseteq P_2 & \ldots, P_2(x, y), \ldots & \rightsquigarrow & \ldots, P_1(x, y), \ldots \\
\cdots & & &
\end{array}$$

(_ denotes an **unbound** variable, i.e., a variable that appears only once)

This corresponds to exploiting ISAs, role typing, and mandatory participation to obtain new queries that could contribute to the answer.

Unifying atoms can make rules applicable that were not so before.

The UCQ resulting from this process is the **perfect reformulation** $r_{q,\mathcal{T}}$.

Query reformulation algorithm

Algorithm *PerfectRef*$(q, \mathcal{T}_P)$
Input: conjunctive query q, set of *DL-Lite*$_{\mathcal{R}}$ PIs $\mathcal{T}_P$
Output: union of conjunctive queries PR
$PR := \{q\}$;
repeat
  $PR' := PR$;
  for each $q \in PR'$ **do**
    for each g in q **do**
      for each PI I in $\mathcal{T}_P$ **do**
        if I is applicable to g
        then $PR := PR \cup \{\, q[g/(g, I)] \,\}$
    for each g_1, g_2 in q **do**
      if g_1 and g_2 unify
      then $PR := PR \cup \{\tau(\mathit{reduce}(q, g_1, g_2))\}$;
until $PR' = PR$;
return PR

Notice that NIs do not play any role in the reformulation of the query.

ABox storage

ABox $\mathcal{A}$ stored as a **relational database** in a standard RDBMS as follows:

- For each **atomic concept** A used in the ABox:
  - define a **unary relational table** $\mathtt{tab}_A$
  - populate $\mathtt{tab}_A$ with each $\langle c\rangle$ such that $A(c) \in \mathcal{A}$
- For each atomic role P used in the ABox,
  - define a binary relational table $\mathtt{tab}_P$
  - populate $\mathtt{tab}_P$ with each $\langle c_1, c_2\rangle$ such that $P(c_1, c_2) \in \mathcal{A}$

We denote with **DB**$(\mathcal{A})$ the database obtained as above.

Query evaluation

Let $r_{q,\mathcal{T}}$ be the UCQ returned by the algorithm *PerfectRef*$(q, \mathcal{T})$.

- We denote by **SQL**$(r_{q,\mathcal{T}})$ the encoding of $r_{q,\mathcal{T}}$ into an SQL query over *DB*$(\mathcal{A})$.
- We indicate with **Eval(SQL**$(r_{q,\mathcal{T}})$**, DB**$(\mathcal{A})$**)** the evaluation of *SQL*$(r_{q,\mathcal{T}})$ over *DB*$(\mathcal{A})$.

Query answering in *DL-Lite*$_{\mathcal{R}}$

Theorem

Let $\mathcal{T}$ be a *DL-Lite*$_{\mathcal{R}}$ TBox, $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$, q a CQ over $\mathcal{T}$, and let $r_{q,\mathcal{T}} = $ *PerfectRef*$(q, \mathcal{T}_P)$. Then, for each ABox $\mathcal{A}$ such that $\langle\mathcal{T}, \mathcal{A}\rangle$ is satisfiable, we have that $\mathit{cert}(q, \langle\mathcal{T}, \mathcal{A}\rangle) = $ **Eval(SQL**$(r_{q,\mathcal{T}})$**, DB**$(\mathcal{A})$**)**.

In other words, query answering over a satisfiable *DL-Lite*$_{\mathcal{R}}$ ontology is FOL-rewritable.

Notice that we did not mention NIs of $\mathcal{T}$ in the theorem above. Indeed, **when the ontology is satisfiable, we can ignore NIs and answer queries as if NIs were not specified in** $\mathcal{T}$.

Query answering in $DL\text{-}Lite_{\mathcal{R}}$ – Example

TBox: Professor $\sqsubseteq$ $\exists$teaches
$\exists$teaches$^-$ $\sqsubseteq$ Course

Query: $\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$

Perfect Reformulation:
$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{Course}(y)$
$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, y), \mathsf{teaches}(\_, y)$
$\mathsf{q}(x) \leftarrow \mathsf{teaches}(x, \_)$
$\mathsf{q}(x) \leftarrow \mathsf{Professor}(x)$

ABox: teaches(john, kbdb)
Professor(mary)

It is easy to see that $Eval(SQL(r_{q,\mathcal{T}}), DB(\mathcal{A}))$ in this case produces as answer $\{$john, mary$\}$.

Query answering in $DL\text{-}Lite_{\mathcal{R}}$ – An interesting example

TBox: Person $\sqsubseteq$ $\exists$hasFather
$\exists$hasFather$^-$ $\sqsubseteq$ Person

ABox: Person(mary)

Query: $\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(y_2, y_3)$

$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(y_2, \_)$
$\Downarrow$ **Apply** Person $\sqsubseteq$ $\exists$hasFather to the atom $\mathsf{hasFather}(y_2, \_)$
$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{Person}(y_2)$
$\Downarrow$ **Apply** $\exists$hasFather$^-$ $\sqsubseteq$ Person to the atom $\mathsf{Person}(y_2)$
$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2), \mathsf{hasFather}(\_, y_2)$
$\Downarrow$ **Unify** atoms $\mathsf{hasFather}(y_1, y_2)$ and $\mathsf{hasFather}(\_, y_2)$
$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, y_1), \mathsf{hasFather}(y_1, y_2)$
$\Downarrow$
...
$\mathsf{q}(x) \leftarrow \mathsf{Person}(x), \mathsf{hasFather}(x, \_)$
$\Downarrow$ **Apply** Person $\sqsubseteq$ $\exists$hasFather to the atom $\mathsf{hasFather}(x, \_)$
$\mathsf{q}(x) \leftarrow \mathsf{Person}(x)$

Query answering in $DL\text{-}Lite_{\mathcal{F}}$

If we limit our attention to PIs, we can say that $DL\text{-}Lite_{\mathcal{F}}$ ontologies are $DL\text{-}Lite_{\mathcal{R}}$ ontologies of a special kind (i.e., with no PIs between roles).

As for NIs and functionality assertions, it is possible to prove that they can be disregarded in query answering over satisfiable $DL\text{-}Lite_{\mathcal{F}}$ ontologies.

From this the following result follows immediately.

Theorem

Let $\mathcal{T}$ be a $DL\text{-}Lite_{\mathcal{F}}$ TBox, $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$, q a CQ over $\mathcal{T}$, and let $r_{q,\mathcal{T}} = PerfectRef(q, \mathcal{T}_P)$. Then, for each ABox $\mathcal{A}$ such that $\langle \mathcal{T}, \mathcal{A} \rangle$ is satisfiable, we have that $cert(q, \langle \mathcal{T}, \mathcal{A} \rangle) = \mathbf{Eval}(\mathbf{SQL}(r_{q,\mathcal{T}}), \mathbf{DB}(\mathcal{A}))$.

In other words, query answering over a satisfiable $DL\text{-}Lite_{\mathcal{F}}$ ontology is FOL-rewritable.

Satisfiability of ontologies with only PIs

Let us now consider the problem of establishing whether an ontology is satisfiable.

Remember that solving this problem allow us to solve TBox reasoning and identify cases in which query answering is meaningless.

A first notable result tells us that PIs alone cannot cause an ontology to become unsatisfiable.

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be either a $DL\text{-}Lite_{\mathcal{R}}$ or a $DL\text{-}Lite_{\mathcal{F}}$ ontology, where $\mathcal{T}$ contains **only PIs**. Then, $\mathcal{O}$ is satisfiable.

DL-Lite$_{\mathcal{R}}$ ontologies

NIs, however, can make a *DL-Lite*$_{\mathcal{R}}$ ontology unsatisfiable.

Example

TBox $\mathcal{T}$: Professor $\sqsubseteq \neg$Student
$\exists$teaches $\sqsubseteq$ Professor

ABox $\mathcal{A}$: $\texttt{teaches}(\texttt{john}, \texttt{kbdb})$
$\texttt{Student}(\texttt{john})$

In what follows we provide a mechanism to establish, in an efficient way, whether a *DL-Lite*$_{\mathcal{R}}$ ontology is satisfiable.

Checking satisfiability of *DL-Lite*$_{\mathcal{R}}$ ontologies

Satisfiability of a *DL-Lite*$_{\mathcal{R}}$ ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ is reduced to evaluating a FOL-query (in fact a UCQ) over *DB*$(\mathcal{A})$.

We proceed as follows: Let $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$.

1. For each NI N between concepts (resp. roles) in $\mathcal{T}$, we ask $\langle \mathcal{T}_P, \mathcal{A} \rangle$ whether there exists some individual (resp. pair of individuals) that contradicts N, i.e., we construct over $\langle \mathcal{T}_P, \mathcal{A} \rangle$ a **boolean CQ** $q_N()$ such that
$$\langle \mathcal{T}_P, \mathcal{A} \rangle \models q_N() \quad \text{iff} \quad \langle \mathcal{T}_P \cup \{N\}, \mathcal{A} \rangle \text{ is unsatisfiable}$$
2. We exploit *PerfectRef* to verify whether $\langle \mathcal{T}_P, \mathcal{A} \rangle \models q_N()$, i.e., we compute *PerfectRef*$(q_N, \mathcal{T}_P)$, and evaluate it (in fact, its SQL encoding) over *DB*$(\mathcal{A})$.

Satisfiability of *DL-Lite*$_{\mathcal{R}}$ ontologies – Example

PIs $\mathcal{T}_P$: $\exists$teaches $\sqsubseteq$ Professor

NI N: **Professor $\sqsubseteq \neg$Student**

Query q_N: $q_N() \leftarrow \text{Student}(x), \text{Professor}(x)$

Perfect Reformulation: $q_N() \leftarrow \text{Student}(x), \text{Professor}(x)$
$q_N() \leftarrow \text{Student}(x), \text{teaches}(x, \_)$

ABox $\mathcal{A}$: $\texttt{teaches}(\texttt{john}, \texttt{kbdb})$
$\texttt{Student}(\texttt{john})$

It is easy to see that $\langle \mathcal{T}_P, \mathcal{A} \rangle \models q_N()$, and that the ontology $\langle \mathcal{T}_P \cup \{\text{Professor} \sqsubseteq \neg\text{Student}\},\ \mathcal{A} \rangle$ **is unsatisfiable**.

Queries for NIs

For each **NI** N in $\mathcal{T}$ we compute a boolean CQ $q_N()$ according to the following rules:

$$
\begin{array}{rcl}
A_1 \sqsubseteq \neg A_2 & \leadsto & q_N() \leftarrow A_1(x), A_2(x) \\
\exists P \sqsubseteq \neg A \ \text{ or } \ A \sqsubseteq \neg \exists P & \leadsto & q_N() \leftarrow P(x,y), A(x) \\
\exists P^- \sqsubseteq \neg A \ \text{ or } \ A \sqsubseteq \neg \exists P^- & \leadsto & q_N() \leftarrow P(y,x), A(x) \\
\exists P_1 \sqsubseteq \neg \exists P_2 & \leadsto & q_N() \leftarrow P_1(x,y), P_2(x,z) \\
\exists P_1 \sqsubseteq \neg \exists P_2^- & \leadsto & q_N() \leftarrow P_1(x,y), P_2(z,x) \\
\exists P_1^- \sqsubseteq \neg \exists P_2 & \leadsto & q_N() \leftarrow P_1(x,y), P_2(y,z) \\
\exists P_1^- \sqsubseteq \neg \exists P_2^- & \leadsto & q_N() \leftarrow P_1(x,y), P_2(z,y) \\
P_1 \sqsubseteq \neg P_2 \ \text{ or } \ P_1^- \sqsubseteq \neg P_2^- & \leadsto & q_N() \leftarrow P_1(x,y), P_2(x,y) \\
P_1^- \sqsubseteq \neg P_2 \ \text{ or } \ P_1 \sqsubseteq \neg P_2^- & \leadsto & q_N() \leftarrow P_1(x,y), P_2(y,x)
\end{array}
$$

DL-Lite$_\mathcal{R}$: From satisfiability to query answering

Lemma (Separation for *DL-Lite*$_\mathcal{R}$)

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a *DL-Lite*$_\mathcal{R}$ ontology, and $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$. Then, **$\mathcal{O}$ is unsatisfiable iff there exists a NI $N \in \mathcal{T}$ such that $\langle \mathcal{T}_P, \mathcal{A} \rangle \models q_N()$**.

The lemma relies on the following properties:

- NIs do not interact with each other.
- Interaction between NIs and PIs can be managed through *PerfectRef*.

Notably, each NI can be processed individually.

DL-Lite$_\mathcal{R}$: FOL-rewritability of satisfiability

From the previous lemma and the theorem on query answering for satisfiable *DL-Lite*$_\mathcal{R}$ ontologies, we get the following result.

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a *DL-Lite*$_\mathcal{R}$ ontology, and $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$. Then, **$\mathcal{O}$ is unsatisfiable iff there exists a NI $N \in \mathcal{T}$ such that Eval(SQL(PerfectRef($q_N, \mathcal{T}_P$)), DB($\mathcal{A}$)) returns true.**

In other words, satisfiability of a *DL-Lite*$_\mathcal{R}$ ontology can be reduced to FOL-query evaluation.

DL-Lite$_\mathcal{F}$ ontologies

Unsatisfiability in *DL-Lite*$_\mathcal{F}$ ontologies can be caused by **NIs** or by **functionality assertions**.

Example

TBox $\mathcal{T}$: Professor $\sqsubseteq$ ¬Student
$\exists$teaches $\sqsubseteq$ Professor
(**funct** teaches$^-$)

ABox $\mathcal{A}$: Student(john)
teaches(john, kbdb)
teaches(michael, kbdb)

In what follows we extend to *DL-Lite*$_\mathcal{F}$ ontologies the technique for *DL-Lite*$_\mathcal{R}$ ontology satisfiability given before.

Checking satisfiability of *DL-Lite*$_\mathcal{F}$ ontologies

Satisfiability of a *DL-Lite*$_\mathcal{F}$ ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ is reduced to evaluating a FOL-query over $DB(\mathcal{A})$.

We deal with NIs exactly as done in DL-Lite$_\mathcal{R}$ ontologies (indeed, limited to NIs, *DL-Lite*$_\mathcal{F}$ ontologies are *DL-Lite*$_\mathcal{R}$ ontologies of a special kind).

To deal with **functionality assertions**, we proceed as follows:

1. For each functionality assertion $F \in \mathcal{T}$, we ask if there exist two pairs of individuals in $\mathcal{A}$ that contradict F, i.e., we pose over $\mathcal{A}$ a boolean FOL query $q_F()$ such that
$$\mathcal{A} \models q_F() \quad \text{iff} \quad \langle \{F\}, \mathcal{A} \rangle \text{ is unsatisfiable.}$$
2. To verify if $\mathcal{A} \models q_F()$, we evaluate $SQL(q_F)$ over $DB(\mathcal{A})$.

Queries for functionality assertions

For each **functionality assertion** F in $\mathcal{T}$ we compute a boolean FOL query $q_F()$ according to the following rules:

$$\begin{array}{lcl}(\textbf{funct } P) & \rightsquigarrow & q_F() \leftarrow P(x,y), P(x,z), y \neq z \\ (\textbf{funct } P^-) & \rightsquigarrow & q_F() \leftarrow P(x,y), P(z,y), x \neq z\end{array}$$

Example

Functionality F: (**funct teaches**$^-$)

Query q_F: $q_F() \leftarrow \textbf{teaches}(x,y), \textbf{teaches}(z,y), x \neq z$

ABox $\mathcal{A}$: teaches(`john`, `kbdb`)
teaches(`michael`, `kbdb`)

It is easy to see that $\mathcal{A} \models q_F()$, and that $\langle\{(\textbf{funct teaches}^-)\}, \mathcal{A}\rangle$, **is unsatisfiable**.

DL-Lite$_{\mathcal{F}}$: From satisfiability to query answering

Lemma (Separation for *DL-Lite*$_{\mathcal{F}}$)

Let $\mathcal{O} = \langle\mathcal{T}, \mathcal{A}\rangle$ be a *DL-Lite*$_{\mathcal{F}}$ ontology, and $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$.
Then, $\mathcal{O}$ is unsatisfiable iff one of the following condition holds:

(a) There exists a NI $N \in \mathcal{T}$ such that $\langle\mathcal{T}_P, \mathcal{A}\rangle \models q_N()$.

(b) **There exists a functionality assertion $F \in \mathcal{T}$ such that $\mathcal{A} \models q_F()$.**

(a) relies on the properties that NIs do not interact with each other, and interaction between NIs and PIs can be managed through *PerfectRef*.

(b) exploits the property that **NIs and PIs do not interact with functionalities**: indeed, no functionality assertions are contradicted in a *DL-Lite*$_{\mathcal{F}}$ ontology $\mathcal{O}$, beyond those explicitly contradicted by the ABox.

Notably, the lemma asserts that to check ontology satisfiability, each NI and each functionality assertion can be processed individually.

DL-Lite$_{\mathcal{F}}$: FOL-rewritability of satisfiability

From the previous lemma and the theorem on query answering for satisfiable *DL-Lite*$_{\mathcal{F}}$ ontologies, we get the following result.

Theorem

Let $\mathcal{O} = \langle\mathcal{T}, \mathcal{A}\rangle$ be a *DL-Lite*$_{\mathcal{F}}$ ontology, and $\mathcal{T}_P$ the set of PIs in $\mathcal{T}$.
Then, $\mathcal{O}$ is unsatisfiable iff one of the following condition holds:

(a) There exists a NI $N \in \mathcal{T}$ such that *Eval*(*SQL*(*PerfectRef*($q_N, \mathcal{T}_P$)), *DB*($\mathcal{A}$)) returns *true*.

(b) **There exists a functionality assertion $F \in \mathcal{T}$ such that Eval(SQL(q_F), DB($\mathcal{A}$)) returns true.**

In other words, satisfiability of a *DL-Lite*$_{\mathcal{F}}$ ontology can be reduced to FOL-query evaluation.

Outline

Complexity of query answering over satisfiable ontologies

Theorem

Query answering over both *DL-Lite*$_{\mathcal{R}}$ and *DL-Lite*$_{\mathcal{F}}$ ontologies is

1. **NP-complete** in the size of **query and ontology** (combined comp.).
2. **PTime** in the size of the **ontology**.
3. **LogSpace** in the size of the **ABox** (data complexity).

Proof (sketch).

1. **Guess** the derivation of one of the CQs of the perfect reformulation, and an assignment to its existential variables. Checking the derivation and evaluating the guessed CQ over the ABox is then polynomial in combined complexity. NP-hardness follows from combined complexity of evaluating CQs over a database.
2. The number of CQs in the perfect reformulation is polynomial in the size of the TBox, and we can compute them in PTime.
3. Is the data complexity of evaluating FOL queries over a DB. □

Complexity of ontology satisfiability

Theorem

Checking satisfiability of both *DL-Lite*$_{\mathcal{R}}$ and *DL-Lite*$_{\mathcal{F}}$ ontologies is

1. **PTime** in the size of the **ontology** (combined complexity).
2. **LogSpace** in the size of the **ABox** (data complexity).

Proof (sketch).

Follows directly from the algorithm for ontology satisfiability and the complexity of query answering over satisfiable ontologies. □

Complexity of TBox reasoning

Theorem

TBox reasoning over both *DL-Lite*$_{\mathcal{R}}$ and *DL-Lite*$_{\mathcal{F}}$ ontologies is **PTime** in the size of the **TBox** (schema complexity).

Proof (sketch).

Follows from the previous theorem, and from the reduction of TBox reasoning to ontology satisfiability. Indeed, the size of the ontology constructed in the reduction is polynomial in the size of the input TBox. □

Beyond *DL-Lite*

Can we further extend these results to more expressive ontology languages?

Essentially NO!
(unless we take particular care)

Beyond *DL-Lite*

We now consider DL languages that allow for constructs not present in *DL-Lite* or for combinations of constructs that are not legal in *DL-Lite*.

We recall here syntax and semantics of constructs used in what follows.

Construct	**Syntax**	**Example**	**Semantics**
conjunction	$C_1 \sqcap C_2$	Doctor $\sqcap$ Male	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
disjunction	$C_1 \sqcup C_2$	Doctor $\sqcup$ Lawyer	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
qual. exist. restr.	$\exists Q.C$	$\exists$child.Male	$\{a \mid \exists b.\,(a,b) \in Q^{\mathcal{I}} \wedge b \in C^{\mathcal{I}}\}$
qual. univ. restr.	$\forall Q.C$	$\forall$child.Male	$\{a \mid \forall b.\,(a,b) \in Q^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\}$

Summary of results on data complexity

	Cl	Cr	$\mathcal{F}$	$\mathcal{R}$	Data complexity of query answering
1	*DL-Lite*$_{\mathcal{F}}$		$\surd$	–	in LogSpace
2	*DL-Lite*$_{\mathcal{R}}$		–	$\surd$	in LogSpace
3	$A \mid \exists P.A$	A	–	–	NLogSpace-hard
4	A	$A \mid \forall P.A$	–	–	NLogSpace-hard
5	A	$A \mid \exists P.A$	$\surd$	–	NLogSpace-hard
6	$A \mid \exists P.A \mid A_1 \sqcap A_2$	A	–	–	PTime-hard
7	$A \mid A_1 \sqcap A_2$	$A \mid \forall P.A$	–	–	PTime-hard
8	$A \mid A_1 \sqcap A_2$	$A \mid \exists P.A$	$\surd$	–	PTime-hard
9	$A \mid \exists P.A \mid \exists P^-.A$	$A \mid \exists P$	–	–	PTime-hard
10	$A \mid \exists P \mid \exists P^-$	$A \mid \exists P \mid \exists P^-$	$\surd$	$\surd$	PTime-hard
11	$A \mid \exists P.A$	$A \mid \exists P.A$	$\surd$	–	PTime-hard
12	$A \mid \neg A$	A	–	–	coNP-hard
13	A	$A \mid A_1 \sqcup A_2$	–	–	coNP-hard
14	$A \mid \forall P.A$	A	–	–	coNP-hard

All NLogSpace and PTime hardness results hold already for atomic queries.

Observations

- *DL-Lite*-family is FOL-rewritable, hence **LogSpace** – holds also with n-ary relations $\rightsquigarrow$ *DLR-Lite*$_{\mathcal{F}}$ and *DLR-Lite*$_{\mathcal{R}}$.
- RDFS is a subset of *DL-Lite*$_{\mathcal{R}}$ $\rightsquigarrow$ is FOL-rewritable, hence **LogSpace.**
- Horn-$\mathcal{SHIQ}$ [HMS05] is **PTime-hard** even for instance checking (line 11).
- DLP [GHVD03] is **PTime-hard** (line 6)
- $\mathcal{EL}$ [BBL05] is **PTime-hard** (line 6).

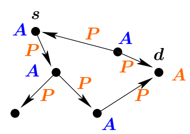
Qualified existential quantification in the lhs of inclusions

Adding **qualified existential on the lhs** of inclusions makes instance checking (and hence query answering) NLogSpace-hard:

	Cl	Cr	$\mathcal{F}$	$\mathcal{R}$	Data complexity
3	$A \mid \exists P.A$	A	–	–	NLogSpace-hard

Hardness proof is by a reduction from reachability in directed graphs:

- TBox $\mathcal{T}$: a single inclusion assertion $\exists P.A \sqsubseteq A$
- ABox $\mathcal{A}$: encodes graph using P and asserts $A(d)$



Result:
$\langle \mathcal{T}, \mathcal{A} \rangle \models A(s)$ iff d is reachable from s in the graph.

NLogSpace-hard cases

Instance checking (and hence query answering) is NLogSpace-hard in data complexity for:

	Cl	Cr	$\mathcal{F}$	$\mathcal{R}$	Data complexity
3	$A \mid \exists P.A$	A	−	−	NLogSpace-hard

By reduction from reachability in directed graphs

4	A	$A \mid \forall P.A$	−	−	NLogSpace-hard

Follows from 3 by replacing $\exists P.A_1 \sqsubseteq A_2$ with $A_1 \sqsubseteq \forall P^-.A_2$, and by replacing each occurrence of P^- with P', for a new role P'.

5	A	$A \mid \exists P.A$	$\surd$	−	NLogSpace-hard

Proved by simulating in the reduction $\exists P.A_1 \sqsubseteq A_2$ via $A_1 \sqsubseteq \exists P^-.A_2$ and (**funct** P^-), and by replacing again each occurrence of P^- with P', for a new role P'.

Path System Accessibility

Instance of Path System Accessibility: $PS = (N, E, S, t)$ with

- N a set of nodes
- $E \subseteq N \times N \times N$ an accessibility relation
- $S \subseteq N$ a set of source nodes
- $t \in N$ a terminal node

Accessibility of nodes is defined inductively:

- each $n \in S$ is accessible
- if $(n, n_1, n_2) \in E$ and n_1, n_2 are accessible, then also n is accessible

Given PS, checking whether t is accessible, is PTime-complete.

Reduction from Path System Accessibility

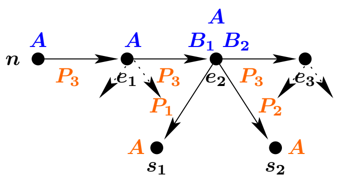
Given an instance $PS = (N, E, S, t)$, we construct

- TBox $\mathcal{T}$ consisting of the inclusion assertions

$$\begin{array}{rcl} \exists P_1.A & \sqsubseteq & B_1 \\ \exists P_2.A & \sqsubseteq & B_2 \end{array} \qquad \begin{array}{rcl} B_1 \sqcap B_2 & \sqsubseteq & A \\ \exists P_3.A & \sqsubseteq & A \end{array}$$

- ABox $\mathcal{A}$ encoding the accessibility relation using P_1, P_2, and P_3, and asserting $A(s)$ for each source node $s \in S$

$$e_1 = (n, \ .\ , \ .\ ) \qquad e_2 = (n, s_1, s_2) \qquad e_3 = (n, \ .\ , \ .\ )$$



Result:
$\langle \mathcal{T}, \mathcal{A} \rangle \models A(t)$ iff t is accessible in PS.

coNP-hard cases

Are obtained when we can use in the query **two concepts that cover another concept**. This forces **reasoning by cases** on the data.

Query answering is coNP-hard in data complexity for:

	Cl	Cr	$\mathcal{F}$	$\mathcal{R}$	Data complexity
11	$A \mid \neg A$	A	−	−	coNP-hard
12	A	$A \mid A_1 \sqcup A_2$	−	−	coNP-hard
13	$A \mid \forall P.A$	A	−	−	coNP-hard

All three cases are proved by adapting the proof of coNP-hardness of instance checking for $\mathcal{ALE}$ by [DLNS94].

2+2-SAT

2+2-SAT: satisfiability of a 2+2-CNF formula, i.e., a CNF formula where each clause has exactly 2 positive and 2 negative literals.

Example: $\varphi = c_1 \land c_2 \land c_3$, with

$$\begin{aligned} c_1 &= v_1 \lor v_2 \lor \neg v_3 \lor \neg v_4 \\ c_2 &= \mathit{false} \lor \mathit{false} \lor \neg v_1 \lor \neg v_4 \\ c_3 &= \mathit{false} \lor v_4 \lor \neg \mathit{true} \lor \neg v_2 \end{aligned}$$

2+2-SAT is NP-complete [DLNS94].

Reduction from 2+2-SAT

2+2-CNF formula $\varphi = c_1 \land \cdots \land c_k$ over variables $v_1, \ldots, v_n$, *true*, *false*

- Ontology is over concepts L, T, F, roles P_1, P_2, N_1, N_2 and individuals $\mathtt{v}_1, \ldots, \mathtt{v}_n$, $\mathtt{true}$, $\mathtt{false}$, $\mathtt{c}_1, \ldots \mathtt{c}_k$
- ABox $\mathcal{A}_\varphi$ constructed from φ:
  - for each propositional variable v_i: $L(\mathtt{v}_i)$
  - for each clause $c_j = v_{j_1} \lor v_{j_2} \lor \neg v_{j_3} \lor \neg v_{j_4}$:
    $P_1(\mathtt{c}_j, \mathtt{v}_{j_1})$, $P_2(\mathtt{c}_j, \mathtt{v}_{j_2})$, $N_1(\mathtt{c}_j, \mathtt{v}_{j_3})$, $N_2(\mathtt{c}_j, \mathtt{v}_{j_4})$
  - $T(\mathtt{true})$, $F(\mathtt{false})$
- TBox $\mathcal{T} = \{\ L \sqsubseteq T \sqcup F\ \}$
- $q() \leftarrow P_1(c, v_1), P_2(c, v_2), N_1(c, v_3), N_2(c, v_4),$
  $F(v_1), F(v_2), T(v_3), T(v_4)$

Note: the TBox $\mathcal{T}$ and the query q do not depend on φ, hence this reduction works for data complexity.

Reduction from 2+2-SAT (cont'd)

Lemma

$\langle \mathcal{T}, \mathcal{A}_\varphi \rangle \not\models q()$ iff φ is satisfiable.

Proof (sketch).

"$\Rightarrow$" If $\langle \mathcal{T}, \mathcal{A}_\varphi \rangle \not\models q()$, then there is a model $\mathcal{I}$ of $\langle \mathcal{T}, \mathcal{A}_\varphi \rangle$ s.t. $\mathcal{I} \not\models q()$. We define a truth assignment $\alpha_\mathcal{I}$ by setting $\alpha_\mathcal{I}(v_i) = \mathit{true}$ iff $\mathtt{v}_i^\mathcal{I} \in T^\mathcal{I}$. Notice that, since $L \sqsubseteq T \sqcup F$, if $\mathtt{v}_i^\mathcal{I} \notin T^\mathcal{I}$, then $\mathtt{v}_i^\mathcal{I} \in F^\mathcal{I}$.
It is easy to see that, since $q()$ asks for a false clause and $\mathcal{I} \not\models q()$, for each clause c_j, one of the literals in c_j evaluates to *true* in $\alpha_\mathcal{I}$.
"$\Leftarrow$" From a truth assignment α that satisfies φ, we construct an interpretation $\mathcal{I}_\alpha$ with $\Delta^{\mathcal{I}_\alpha} = \{c_1, \ldots, c_k, v_1, \ldots, v_n, t, f\}$, and:

- $\mathtt{c}_j^{\mathcal{I}_\alpha} = c_j$, $\mathtt{v}_i^{\mathcal{I}_\alpha} = v_i$, $\mathtt{true}^{\mathcal{I}_\alpha} = t$, $\mathtt{false}^{\mathcal{I}_\alpha} = f$
- $T^{\mathcal{I}_\alpha} = \{v_i \mid \alpha(v_i) = \mathit{true}\} \cup \{t\}$, $F^{\mathcal{I}_\alpha} = \{v_i \mid \alpha(v_i) = \mathit{false}\} \cup \{f\}$

It is easy to see that $\mathcal{I}_\alpha$ is a model of $\langle \mathcal{T}, \mathcal{A}_\varphi \rangle$ and that $\mathcal{I}_\alpha \not\models q()$. $\square$

Outline

Combining functionalities and role inclusions

We have seen till now that:

- By including in *DL-Lite* both functionality of roles and qualified existential quantification (i.e., $\exists P.A$), query answering becomes NLOGSPACE-hard (and PTIME-hard with also inverse roles) in data complexity (see Part 3).
- Qualified existential quantification can be simulated by using role inclusion assertions (see Part 2).
- When the data complexity of query answering is NLOGSPACE or above, the DL does not enjoy FOL-rewritability.

As a consequence of these results, we get:

To preserve FOL-rewritability, we need to restrict the interaction of functionality and role inclusions.

Let us analyze on an example the effect of an unrestricted interaction.

Combining functionalities and role inclusions – Example

TBox $\mathcal{T}$: $A \sqsubseteq \exists P$ $\quad$ $P \sqsubseteq S$
$\exists P^- \sqsubseteq A$ $\quad$ $(\textbf{funct } S)$

ABox $\mathcal{A}$: $A(c_1),\ S(c_1,c_2),\ S(c_2,c_3),\ \ldots,\ S(c_{n-1},c_n)$

$$\begin{array}{rrcl}
 & A(c_1),\quad A \sqsubseteq \exists P & \models & P(c_1,x), \text{ for some } x \\
 & P(c_1,x),\quad P \sqsubseteq S & \models & S(c_1,x) \\
S(c_1,x), & S(c_1,c_2),\quad (\textbf{funct } S) & \models & x = c_2 \\
 & P(c_1,c_2),\quad \exists P^- \sqsubseteq A & \models & A(c_2) \\
 & A(c_2),\quad A \sqsubseteq \exists P & \ldots & \\
 & & \models & A(c_n)
\end{array}$$

Hence, we get:

- If we add $B(c_n)$ and $B \sqsubseteq \neg A$, the ontology becomes inconsistent.
- Similarly, the answer to the following query over $\langle \mathcal{T}, \mathcal{A} \rangle$ is *true*:

$$q() \leftarrow A(z_1), S(z_1,z_2), S(z_2,z_3), \ldots, S(z_{n-1},z_n), A(z_n)$$

Interaction between functionalities and role inclusions

Note: The number of unification steps above **depends on the data**. Hence this kind of deduction cannot be mimicked by a FOL (or SQL) query, since it requires a form of **recursion**. As a consequence, we get:

Combining functionality and role inclusions is problematic.

It breaks **separability**, i.e., functionality assertions may force existentially quantified objects to be unified with existing objects.

Note: the problems are caused by the **interaction** among:

- an inclusion $P \sqsubseteq S$ between roles,
- a functionality assertion $(\textbf{funct } S)$ on the super-role, and
- a cycle of concept inclusion assertions $A \sqsubseteq \exists P$ and $\exists P^- \sqsubseteq A$.

Since we do not want to limit cycles of ISA, we pose suitable restrictions on the combination of functionality and role inclusions

Complexity of *DL-Lite* with funct. and role inclusions

Let *DL-Lite*$_{\mathcal{FR}}$ be the DL that is the union of *DL-Lite*$_\mathcal{F}$ and *DL-Lite*$_\mathcal{R}$, i.e., the *DL-Lite* logic that allows for using both role functionality and role inclusions without any restrictions.

Theorem [ACKZ09]

For *DL-Lite*$_{\mathcal{FR}}$ ontologies:

- Checking satisfiability of the ontology is
  - **ExpTime-complete** in the size of the **ontology** (combined complexity).
  - **PTime-complete** in the size of the **ABox** (data complexity).
- TBox reasoning is **ExpTime-complete** in the size of the **TBox**.
- Query answering is
  - **NP-complete** in the size of the query and the ontology (comb. com.).
  - **ExpTime-complete** in the size of the **ontology**.
  - **PTime-complete** in the size of the **ABox** (data complexity).

Restriction on TBox assertions in *DL-Lite*$_{\mathcal{A}}$ ontologies

To ensure FOL-rewritability, in *DL-Lite*$_{\mathcal{A}}$ we have imposed a **restriction** on the use of functionality and role/attribute inclusions.

Restriction on *DL-Lite*$_{\mathcal{A}}$ TBoxes

No functional role or attribute can be specialized by using it in the right-hand side of a role or attribute inclusion assertion.

Since qualified existentials in the right-hand side of concept inclusions are encoded using role inclusions, this restriction affects also qualified existentials.

Formally:

- If $\exists P.C$ or $\exists P^-.C$ appears in $\mathcal{T}$,
  then (**funct** P) and (**funct** P^-) are **not in** $\mathcal{T}$.
- If $Q \sqsubseteq P$ or $Q \sqsubseteq P^-$ is in $\mathcal{T}$,
  then (**funct** P) and (**funct** P^-) are **not in** $\mathcal{T}$.
- If $U_1 \sqsubseteq U_2$ is in $\mathcal{T}$,
  then (**funct** U_2) is **not in** $\mathcal{T}$.

Reasoning in *DL-Lite*$_{\mathcal{A}}$ – Separation

It is possible to show that, by virtue of the restriction on the use of role inclusion and functionality assertions, all nice properties of *DL-Lite*$_{\mathcal{F}}$ and *DL-Lite*$_{\mathcal{R}}$ continue to hold also for *DL-Lite*$_{\mathcal{A}}$.

In particular, w.r.t. **satisfiability of a DL-Lite$_{\mathcal{A}}$ ontology $\mathcal{O}$**, we have:

- NIs do not interact with each other.
- NIs and PIs do not interact with functionality assertions.

We obtain that for *DL-Lite*$_{\mathcal{A}}$ a separation result holds:

- Each NI and each functionality can be checked independently from the others.
- A functionality assertion is contradicted in an ontology $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ only if it is explicitly contradicted by its ABox $\mathcal{A}$.

Ontology satisfiability in *DL-Lite*$_{\mathcal{A}}$

Due to the separation property, we can associate

- to each NI N a boolean CQ $q_N()$, and
- to each functionality assertion F a boolean FOL query $q_F()$

and check satisfiability of $\mathcal{O}$ by suitably evaluating $q_N()$ and $q_F()$.

Theorem

Let $\mathcal{O} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a *DL-Lite*$_{\mathcal{A}}$ ontology, and $\mathcal{T}_P$ the set of PIs in $\mathcal{O}$. Then, $\mathcal{O}$ is unsatisfiable iff one of the following condition holds:

- There exists a NI $N \in \mathcal{T}$ such that
  Eval(*SQL*(*PerfectRef*($q_N, \mathcal{T}_P$)), *DB*($\mathcal{A}$)) returns *true*.
- There exists a functionality assertion $F \in \mathcal{T}$ such that
  Eval(*SQL*(q_F), *DB*($\mathcal{A}$)) returns *true*.

Query answering in *DL-Lite*$_{\mathcal{A}}$

- Queries over *DL-Lite*$_{\mathcal{A}}$ ontologies are analogous to those over *DL-Lite*$_{\mathcal{R}}$ and *DL-Lite*$_{\mathcal{F}}$ ontologies, except that they can also make use of attribute and domain atoms.
- Exploiting the previous result, the query answering algorithm of *DL-Lite*$_{\mathcal{R}}$ can be easily extended to deal with *DL-Lite*$_{\mathcal{A}}$ ontologies:
  - Assertions involving attribute domain and range can be dealt with as for role domain and range assertions.
  - $\exists Q.C$ in the right hand-side of concept inclusion assertions can be eliminated by making use of role inclusion assertions.
  - Disjointness of roles and attributes can be checked similarly as for disjointness of concepts, and does not interact further with the other assertions.

Complexity of reasoning in $DL\text{-}Lite_{\mathcal{A}}$

As for ontology satisfiability, $DL\text{-}Lite_{\mathcal{A}}$ maintains the nice computational properties of $DL\text{-}Lite_{\mathcal{R}}$ and $DL\text{-}Lite_{\mathcal{F}}$ also w.r.t. query answering. Hence, we get the same characterization of computational complexity.

Theorem [PLC+08, ACKZ09]

For $DL\text{-}Lite_{\mathcal{A}}$ ontologies:

- Checking satisfiability of the ontology is
  - **NLogSpace-complete** in the size of the **ontology** (combined complexity).
  - **LogSpace** in the size of the **ABox** (data complexity).
- TBox reasoning is **NLogSpace-complete** in the size of the **TBox**.
- Query answering is
  - **NP-complete** in the size of the query and the ontology (comb. com.).
  - **NLogSpace-complete** in the size of the **ontology**.
  - **LogSpace** in the size of the **ABox** (data complexity).

Outline

References I

[ACK+07] A. Artale, D. Calvanese, R. Kontchakov, V. Ryzhikov, and M. Zakharyaschev.
Reasoning over extended ER models.
In *Proc. of the 26th Int. Conf. on Conceptual Modeling (ER 2007)*, volume 4801 of *Lecture Notes in Computer Science*, pages 277–292. Springer, 2007.

[ACKZ09] A. Artale, D. Calvanese, R. Kontchakov, and M. Zakharyaschev.
The *DL-Lite* family and relations.
Technical Report BBKCS-09-03, School of Computer Science and Information Systems, Birbeck College, London, 2009.
Available at http://www.dcs.bbk.ac.uk/research/techreps/2009/bbkcs-09-03.pdf.

[BBL05] F. Baader, S. Brandt, and C. Lutz.
Pushing the $\mathcal{EL}$ envelope.
In *Proc. of the 19th Int. Joint Conf. on Artificial Intelligence (IJCAI 2005)*, pages 364–369, 2005.

[BCDG05] D. Berardi, D. Calvanese, and G. De Giacomo.
Reasoning on UML class diagrams.
Artificial Intelligence, 168(1–2):70–118, 2005.

References II

[BCM+03] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. F. Patel-Schneider, editors.
The Description Logic Handbook: Theory, Implementation and Applications.
Cambridge University Press, 2003.

[CDGL98] D. Calvanese, G. De Giacomo, and M. Lenzerini.
On the decidability of query containment under constraints.
In *Proc. of the 17th ACM SIGACT SIGMOD SIGART Symp. on Principles of Database Systems (PODS'98)*, pages 149–158, 1998.

[CDGL+05a] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati.
Tailoring OWL for data intensive ontologies.
In *Proc. of the 1st Int. Workshop on OWL: Experiences and Directions (OWLED 2005)*, volume 188 of *CEUR Electronic Workshop Proceedings*, http://ceur-ws.org/, 2005.

[CDGL+05b] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati.
DL-Lite: Tractable description logics for ontologies.
In *Proc. of the 20th Nat. Conf. on Artificial Intelligence (AAAI 2005)*, pages 602–607, 2005.

References III

[CDGL+06a] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, A. Poggi, and R. Rosati.
Linking data to ontologies: The description logic *DL-Lite*$_\mathcal{A}$.
In *Proc. of the 2nd Int. Workshop on OWL: Experiences and Directions (OWLED 2006)*, volume 216 of *CEUR Electronic Workshop Proceedings*, `http://ceur-ws.org/`, 2006.

[CDGL+06b] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati.
Data complexity of query answering in description logics.
In *Proc. of the 10th Int. Conf. on the Principles of Knowledge Representation and Reasoning (KR 2006)*, pages 260–270, 2006.

[CDGL+07] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati.
Tractable reasoning and efficient query answering in description logics: The *DL-Lite* family.
J. of Automated Reasoning, 39(3):385–429, 2007.

References IV

[CK06] A. Calì and M. Kifer.
Containment of conjunctive object meta-queries.
In *Proc. of the 32nd Int. Conf. on Very Large Data Bases (VLDB 2006)*, pages 942–952, 2006.

[DLNN97] F. M. Donini, M. Lenzerini, D. Nardi, and W. Nutt.
The complexity of concept languages.
Information and Computation, 134:1–58, 1997.

[DLNS94] F. M. Donini, M. Lenzerini, D. Nardi, and A. Schaerf.
Deduction in concept languages: From subsumption to instance checking.
J. of Logic and Computation, 4(4):423–452, 1994.

[Don03] F. M. Donini.
Complexity of reasoning.
In Baader et al. [BCM+03], chapter 3, pages 96–136.

References V

[GHLS07] B. Glimm, I. Horrocks, C. Lutz, and U. Sattler.
Conjunctive query answering for the description logic $\mathcal{SHIQ}$.
In *Proc. of the 20th Int. Joint Conf. on Artificial Intelligence (IJCAI 2007)*, pages 399–404, 2007.

[GHVD03] B. N. Grosof, I. Horrocks, R. Volz, and S. Decker.
Description logic programs: Combining logic programs with description logic.
In *Proc. of the 12th Int. World Wide Web Conf. (WWW 2003)*, pages 48–57, 2003.

[HMS05] U. Hustadt, B. Motik, and U. Sattler.
Data complexity of reasoning in very expressive description logics.
In *Proc. of the 19th Int. Joint Conf. on Artificial Intelligence (IJCAI 2005)*, pages 466–471, 2005.

[LR98] A. Y. Levy and M.-C. Rousset.
Combining Horn rules and description logics in CARIN.
Artificial Intelligence, 104(1–2):165–209, 1998.

References VI

[Lut07] C. Lutz.
Inverse roles make conjunctive queries hard.
In *Proc. of the 2007 Description Logic Workshop (DL 2007)*, volume 250 of *CEUR Electronic Workshop Proceedings*, `http://ceur-ws.org/`, pages 100–111, 2007.

[MH03] R. Möller and V. Haarslev.
Description logic systems.
In Baader et al. [BCM+03], chapter 8, pages 282–305.

[OCE06] M. M. Ortiz, D. Calvanese, and T. Eiter.
Characterizing data complexity for conjunctive query answering in expressive description logics.
In *Proc. of the 21st Nat. Conf. on Artificial Intelligence (AAAI 2006)*, pages 275–280, 2006.

[PLC+08] A. Poggi, D. Lembo, D. Calvanese, G. De Giacomo, M. Lenzerini, and R. Rosati.
Linking data to ontologies.
J. on Data Semantics, X:133–173, 2008.

References VII

[Sch93] A. Schaerf.
On the complexity of the instance checking problem in concept languages with existential quantification.
J. of Intelligent Information Systems, 2:265–278, 1993.