E6.1

Exercise 1 (product construction)

Show that if L and M are regular languages then so is $L \cap M$ (without using the De Morgan law $L \cap M = \overline{\overline{L} \cup \overline{M}}$).

Solution:

Let L and M be the languages of the automata $A_L = (Q_L, \Sigma_L, \delta_L, q_L, F_L)$ and $A_M = (Q_M, \Sigma_M, \delta_M, q_M, F_M)$. We assume: (a) $\Sigma_L = \Sigma_M = \Sigma$, (b) A_L and A_M are deterministic (the construction also works for NFA's).
We construct an automaton A that simulates A_L and A_M. The states of A are pairs of states (p, q) where $p \in Q_L$ and $q \in Q_M$.
If a is an input symbol and A is in state (p, q) then A goes in state (s, t) where $s = \delta_L(p, a)$ and $t = \delta_M(q, a)$.
The start state of A is the pair of start states of A_L and A_M. The accepting states of A are all those pairs (p, q) where $p \in F_L$ and $q \in F_M$. We have that

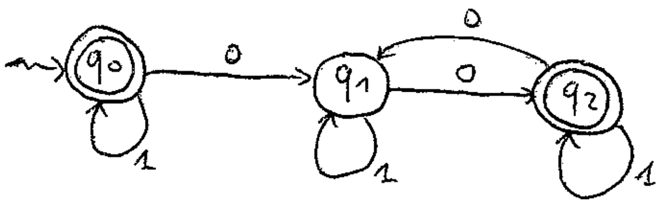
$$A = (Q_L \times Q_M, \Sigma, \delta, (q_L, q_M), F_L \times F_M)$$

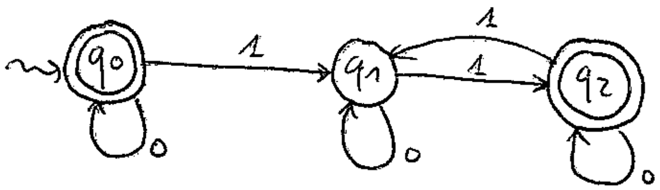
where $\delta((p, q), a) = (\delta_L(p, a), \delta_M(q, a))$.

Note that A is constructed in such a way that w is accepted by A (i.e. $w \in L(A)$) if and only if w is accepted by A_L and A_M (i.e. $w \in L(A_L)$ and $w \in L(A_M)$), hence $w \in L(A_L) \cap L(A_M)$.

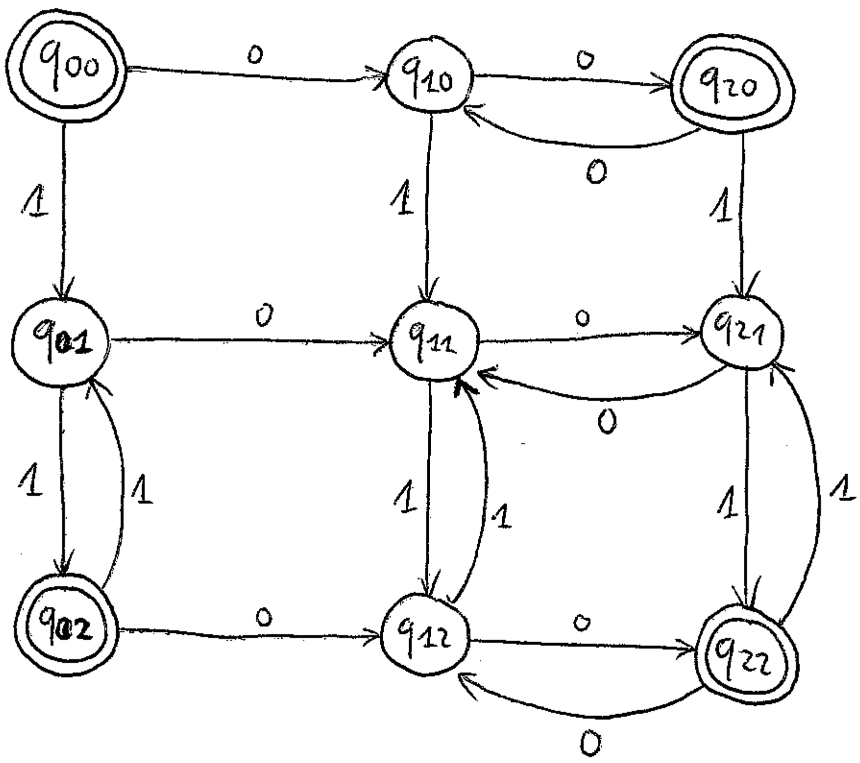
Exercise 2

Apply the product construction to the following automata :

$A_L =$ 

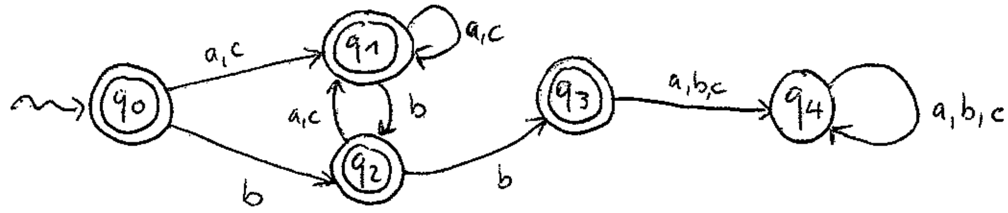
$A_M =$ 

Solution :



Exercise 3

Minimize the following DFA:



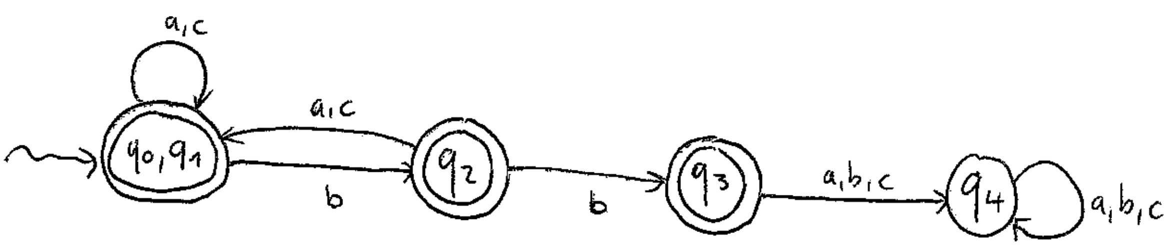
Solution:

We use the table-filling algorithm:

	q_0	q_1	q_2	q_3
q_1				
q_2	2	2		
q_3	1	1	1	
q_4	0	0	0	0

- (p_1, p_2) marked with 0: either $p_1 \in F$ and $p_2 \notin F$ or $p_1 \notin F$ and $p_2 \in F$
- (p_1, p_2) marked with 1: there is $a \in \Sigma$ s.t. $(\delta(p_1,a), \delta(p_2,a))$ is marked with 0
- (p_1, p_2) marked with 2: there is $a \in \Sigma$ s.t. $(\delta(p_1,a), \delta(p_2,a))$ is marked with 1

We have that the states q_0 and q_1 are equivalent and the minimized DFA is thus:



Exercise 4

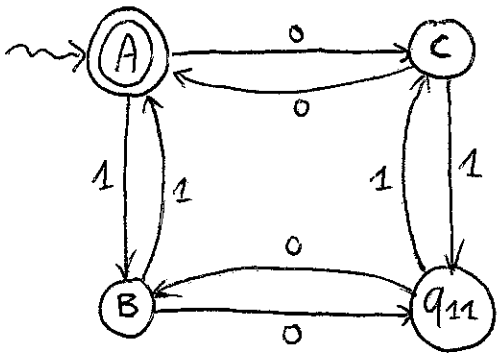
Minimize the DFA of exercise 2.

Solution:

We use the table-filling algorithm:

	00	01	02	10	11	12	20	21
01	0							
02		0						
10	0	1	0					
11	0	1	0	1				
12	0	1	0		1			
20		0		0	0	0		
21	0		0	1	1	1	0	
22		0		0	0	0		0

If $A = \{00, 02, 20, 22\}$, $B = \{01, 21\}$ and $C = \{10, 12\}$ then the minimized automaton looks as follows:



HW: Verify that you get the same result if you first minimize the automata A_L and A_M and then apply the product construction.