E4.1

Exercise 1

Write regular expressions for the following languages:

a) The set of strings over $\{0,1\}$ that either begin or end (or both) with 01;

b) The set of strings over $\{x,y,z\}$ such that the number of y's is divisible by three;

c) The set of strings over $\{0,1,\dots,9\}$ such that the final digit has appeared before;

d) The set of strings over $\{0,1,\dots,9\}$ such that the final digit has <u>not</u> appeared before.

Solution:

a) $(01)(0+1)^* + (0+1)^*(01)$

b) $((x+z)^* y (x+z)^* y (x+z)^* y (x+z)^*)^*$

c) Let $E = 0+1+\dots+9$

$$E^*0E^*0 + E^*1E^*1 + \dots + E^*9E^*9$$

d) Let $E_0 = 1+2+\dots+9$

$E_i = 0+\dots+(i-1)+(i+1)+\dots+9 \quad (1 \leq i \leq 8)$

$E_9 = 0+1+\dots+8$

$E = 0+1+\dots+9$

$$E + E_0^+0 + E_1^+1 + \dots + E_9^+9$$

Exercise 2

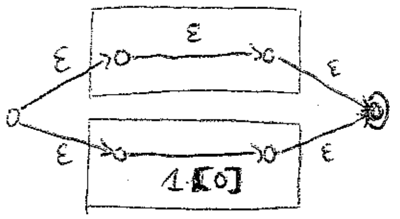
Convert the following regular expression to a ε-NFA:

$$(\varepsilon + 1)(01)^*(\varepsilon + 0)$$

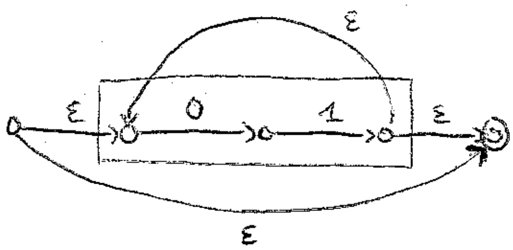
Solution:

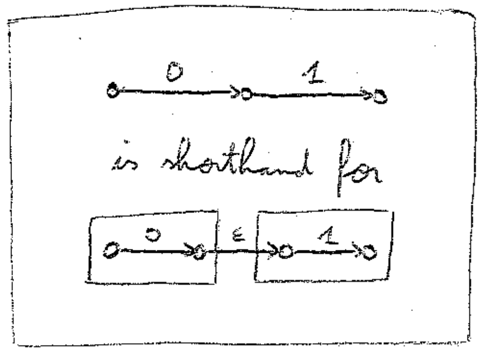
ε-NFA's for $\varepsilon+1$, $\varepsilon+0$, and $(01)^*$ are as follows:

a) $\varepsilon + 1$
[$\varepsilon + 0$]

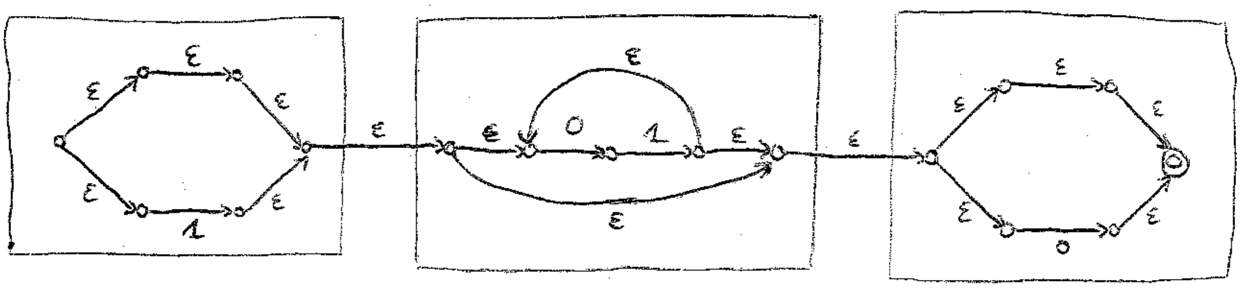


b) $(01)^*$

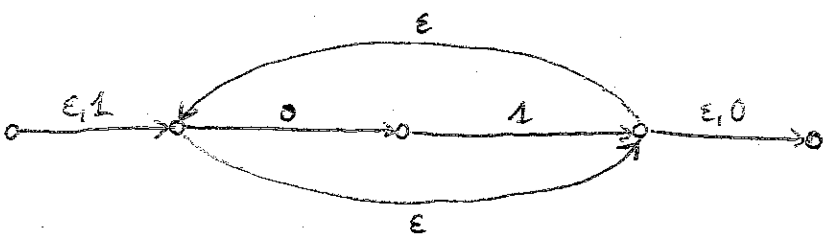




Composition of the above ε-NFA's yields:



or:



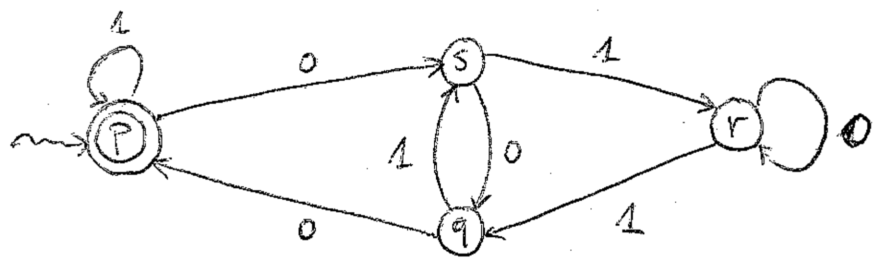
Exercise 3 (3.2.3 from textbook)

Convert the following DFA to a regular expression, using the state-elimination technique.

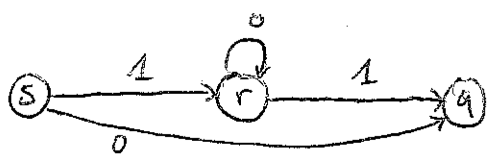
	0	1
$\rightarrow$ * p	s	p
q	p	s
r	r	q
s	q	r

Solution:

The DFA looks as follows

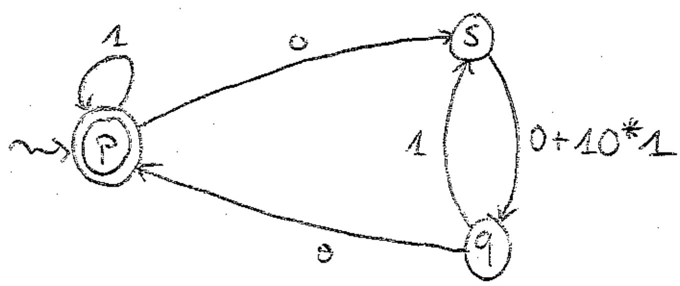


First, we eliminate the state r

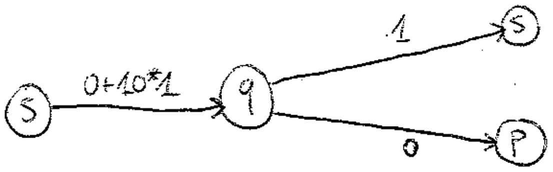


($0 + 10^*1$ for $s \rightarrow q$)

and obtain

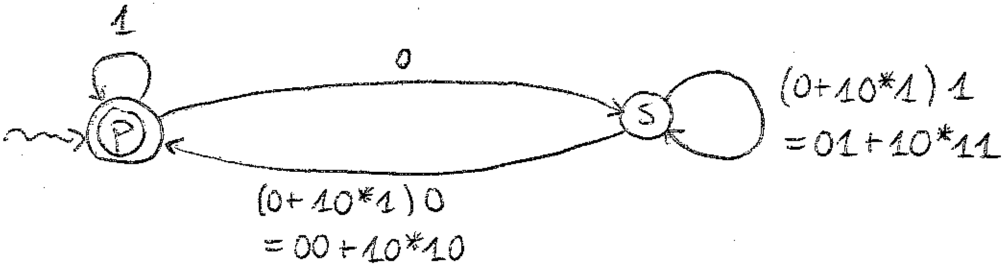


Second, we eliminate the state q

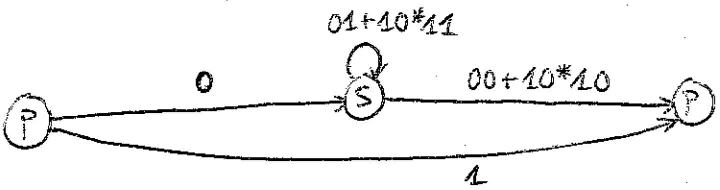


$$\begin{pmatrix} \emptyset + (0+10^*1)1 & \text{for } s \to s \\ \emptyset + (0+10^*1)0 & \text{for } s \to p \end{pmatrix}$$

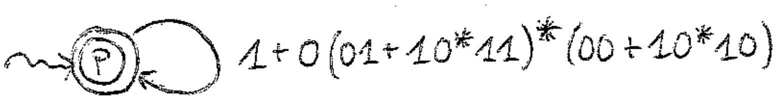
and obtain



Third, we eliminate the state s



and obtain



The regular expression is therefore

$$(1 + 0(01+10^*11)^*(00+10^*10))^*$$

Exercise 4

If we eliminate first the state r, but then s before q, the solution to the previous exercise is the regular expression $(1 + (00+010^*1)(10+110^*1)^*0)^*$. Verify that the following strings accepted by the DFA of exercise 3 are in the languages of both regular expressions.

a) 00101100101

b) 010001101110

Solution:

Let $E_1 = (1+0(01+10^*11)^*(00+10^*10))^*$

$E_2 = (1+(00+010^*1)(10+110^*1)^*0)^*$

a) • $00101100101 \in \mathcal{L}(E_1)$:

$$\big(0\underbrace{(01)(01)}_{\text{from } (01+10^*11)^*}\underbrace{(10010)}_{\text{from } (00+10^*10)}\big)1$$

• $00101100101 \in \mathcal{L}(E_2)$:

$$\big(\underbrace{(00)}_{\text{from } (00+010^*1)}\underbrace{(10)(11001)}_{\text{from } (10+110^*1)^*}0\big)1$$

b) • $0\underbrace{(100011)(01)}_{\text{from } (01+10^*11)^*}\underbrace{(110)}_{\text{from } (00+10^*10)} \in \mathcal{L}(E_1)$

• $\underbrace{(010001)}_{\text{from } (00+010^*1)}\underbrace{(10)(111)}_{\text{from } (10+110^*1)^*}0 \in \mathcal{L}(E_2)$

HW: Verify that E_2 is obtained by eliminating first r, then s, and then q.