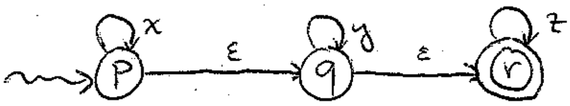
# ε-NFA AND REGULAR EXPRESSIONS

19/11/2007

E3.1

## Exercise 1

Convert the following ε-NFA to a DFA.

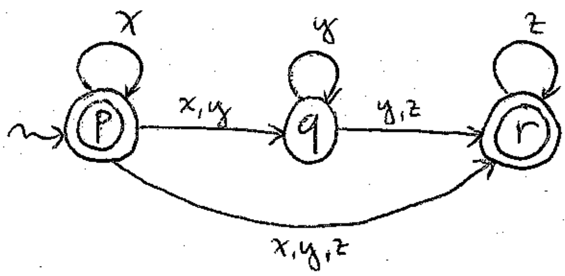



Solution:

- From ε-NFA to NFA

| $\hat{\delta}_\varepsilon$ | x | y | z |
|---|---|---|---|
| *→ p | {p,q,r} | {q,r} | {r} |
| q | ∅ | {q,r} | {r} |
| * r | ∅ | ∅ | {r} |

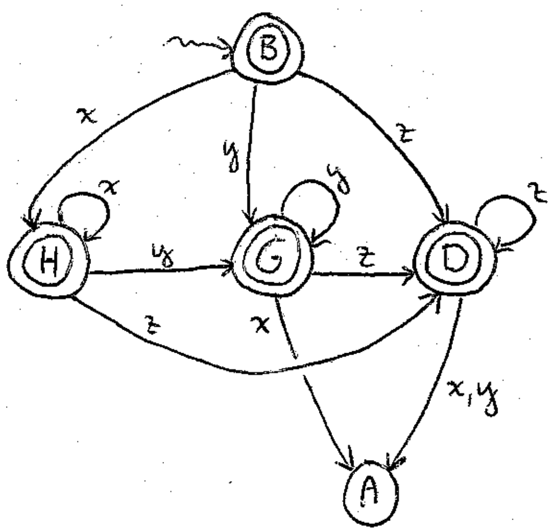
The NFA looks as follows:




- From NFA to DFA

| subset const. | x | y | z |
|---|---|---|---|
| A = ∅ | A | A | A |
| *→ B = {p} | H | G | D |
| C = {q} | A | G | D |
| * D = {r} | A | A | D |
| * E = {p,q} | H | G | D |
| * F = {p,r} | H | G | D |
| * G = {q,r} | A | G | D |
| * H = {p,q,r} | H | G | D |

The DFA looks as follows:



# Exercise 2

Give English descriptions of the languages over the alphabet $\{a, b, c\}$ of the following regular expressions:

a) $(a+b)(a+b)(a+b)$

b) $(\varepsilon+a)\,b\,(\varepsilon+c)$

c) $(cb)^* + b(cb)^* + (cb)^*c + b(cb)^*c$

Solution:

a) The set of all strings of length three that do not contain the symbol c:
$\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$

b) The set of all strings with one b, eventually proceeded by an a and/or followed by a c:
$\{b, ab, bc, abc\}$

c) The set of all strings consisting of alternating b's and c's

[alternative regular expressions for the language are: $(\varepsilon+c)(bc)^*(\varepsilon+b)$
$(bc)^* + (cb)^* + c(bc)^* + b(cb)^*$]

## Exercise 3

Write regular expressions for the following languages:

a) The set of all strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

c) The set of strings of 0's and 1's such that at least one of the last ten positions is a 1.

Solution:

a) $a^* b^* c^*$

b) $\underbrace{(01)(01)^*}_{(01)^+} + \underbrace{(010)(010)^*}_{(010)^+}$

c) $(0+1)^* (E_0 + E_1 + \ldots + E_9)$

where: $E_i = \underbrace{(0+1) \cdots (0+1)}_{i \text{ times}} 1 \underbrace{(0+1) \cdots (0+1)}_{(9-i) \text{ times}}$

for all $i \in \{0, 1, \ldots, 9\}$

## Exercise 4

Show that for every regular language $L$ we have $(L^*)^* = L^*$.

Solution:

We need to show both $L^* \subseteq (L^*)^*$ and $(L^*)^* \subseteq L^*$.

- $L^* \subseteq (L^*)^*$

  Trivial, since $(L^*)^* =_{def} \{\varepsilon\} \cup L^* \cup L^*L^* \cup L^*L^*L^* \cup \ldots$

- $(L^*)^* \subseteq L^*$

  Given $w \in (L^*)^*$ we have to show that $w \in L^*$.
  We know that there exists $n \in \mathbb{N}$ such that
  $w = w_1 w_2 \cdots w_n$, where $w_i \in L^*$ $(1 \leq i \leq n)$.
  On the other hand, for all $i \in \{1, 2, \ldots n\}$
  $w_i = w_{i_1} w_{i_2} \cdots w_{i m_i}$, where $w_{ij} \in L$ $(1 \leq j \leq m_i, m_i \in \mathbb{N})$.
  Therefore $w = (w_{11} w_{12} \cdots w_{1m_1})(w_{21} w_{22} \cdots w_{2m_2}) \cdots (w_{n1} w_{n2} \cdots w_{nm_n})$.
  Since $w$ is a concatenation of strings of $L$, the thesis ($w \in L^*$) follows.

## Exercise 5



Show that for regular languages $L$ and $M$ we have $(L^* M^*)^* = (L \cup M)^*$. [ Note: $(L \cup M)^* = \mathcal{L}((L+M)^*)$ ]

Solution:

We need to show both $(L^*M^*)^* \subseteq (L \cup M)^*$ and $(L \cup M)^* \subseteq (L^* M^*)^*$.

- $(L^*M^*)^* \subseteq (L \cup M)^*$

  Given $w \in (L^*M^*)^*$ we have to show that $w \in (L \cup M)^*$.
  We know that there exists $n \in \mathbb{N}$ such that $w = w_1 w_2 \cdots w_n$, where $w_i \in L^*M^*$ $(1 \leq i \leq n)$.
  On the other hand, for all $i \in \{1, 2, \ldots, n\}$ $w_i = u_{i_1} u_{i_2} \cdots u_{i_{k_i}} v_{i_1} v_{i_2} \cdots v_{i_{l_i}}$, where $u_{i_j} \in L$ $(1 \leq j \leq k_i,\ k_i \in \mathbb{N})$ and $v_{i_j} \in M$ $(1 \leq j \leq l_i,\ l_i \in \mathbb{N})$.

  Therefore $w = (u_{1_1} \cdots u_{1_{k_1}} v_{1_1} \cdots v_{1_{l_1}}) \cdots (u_{n_1} \cdots u_{n_{k_n}} v_{n_1} \cdots v_{n_{l_n}})$ and the thesis $(w \in (L \cup M)^*)$ follows.

- $(L \cup M)^* \subseteq (L^* M^*)^*$

  Given $w \in (L \cup M)^*$ we have to show that $w \in (L^*M^*)^*$.
  We know that $w = w_1 w_2 \cdots w_n$ for some $n$, where each $w_i$ is in either $L$ or $M$. If $w_i$ is in $L$ then $w_i$ is also in $L^*$ and, since $\varepsilon$ is in $M^*$, $w_i = w_i \varepsilon$ is in $L^*M^*$. Similarly, if $w_i$ is in $M$ then $w_i$ is in $L^*M^*$.
  Since every $w_i$ is in $L^*M^*$ we have that the thesis $(w \in (L^*M^*)^*)$ follows.