

Exercise 1 (2.5.1 from textbook)

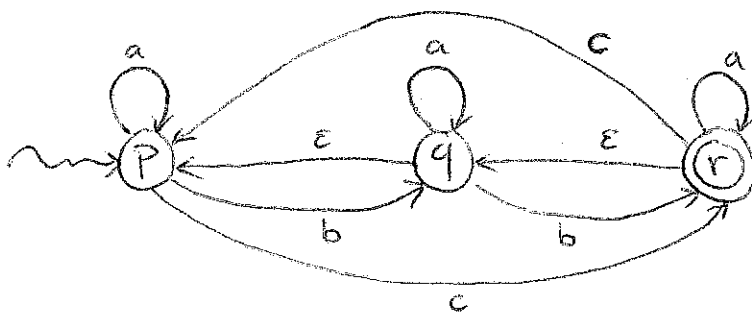
Consider the following ϵ -NFA.

	ϵ	a	b	c
$\rightarrow p$	$\emptyset$	$\{p\}$	$\{q\}$	$\{r\}$
q	$\{p\}$	$\{q\}$	$\{r\}$	$\emptyset$
* r	$\{q\}$	$\{r\}$	$\emptyset$	$\{p\}$

- Compute the ϵ -closure of each state.
- Give all the strings of length three or less accepted by the automaton.
- Convert the automaton to a DFA.

Solution:

The ϵ -NFA looks as follows



a) $ECLOSE(p) = \{p\}$ $ECLOSE(q) = \{p, q\}$ $ECLOSE(r) = \{p, q, r\}$

$q \in ECLOSE(q)$; if $p \in ECLOSE(q)$ and $(p \xrightarrow{\epsilon} r)$ then $r \in ECLOSE(q)$

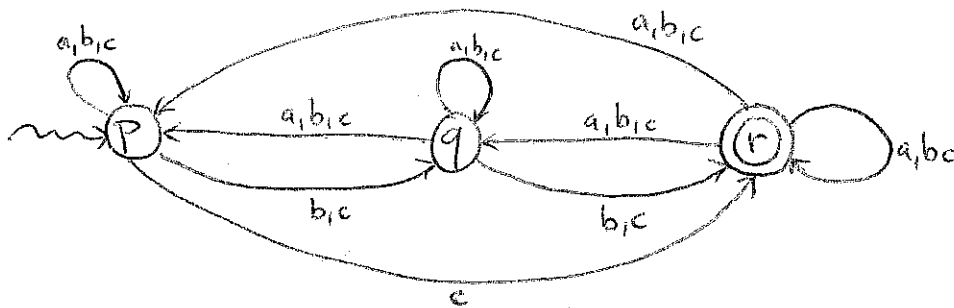
b) All strings except for: $\epsilon, a, b, aa, ab, ba, aaa, aab, aba, baa$

c) From E-NFA to NFA:

$\hat{\delta}_E$	a	b	c
P	{P}	{P, q}	{P, q, r}
q	{P, q}	{P, q, r}	{P, q, r}
r	{P, q, r}	{P, q, r}	{P, q, r}

$$\hat{\delta}_E(q, a) = \text{ECLOSE} \left(\bigcup_{p \in \text{ECLOSE}(q)} \delta(p, a) \right)$$

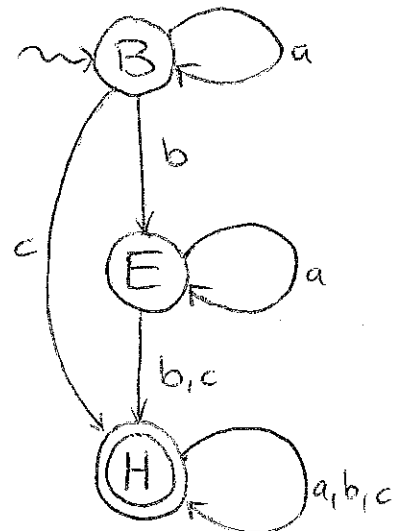
The NFA looks as follows



From NFA to DFA:

subset construction	a	b	c
$A = \emptyset$	A	A	A
$\rightarrow B = \{P\}$	B	E	H
$C = \{q\}$	E	H	H
$* D = \{r\}$	H	H	H
$E = \{P, q\}$	E	H	H
$* F = \{P, r\}$	H	H	H
$* G = \{q, r\}$	H	H	H
$* H = \{P, q, r\}$	H	H	H

The DFA looks as follows



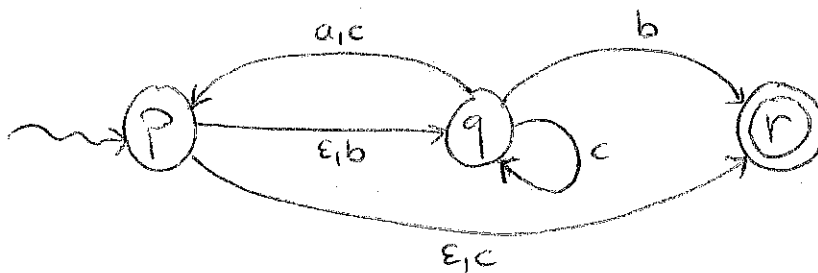
Exercise 2 (2.5.2 from textbook)

Repeat the previous exercise for the following ϵ -NFA.

	ϵ	a	b	c
$\rightarrow P$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
q	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
* r	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Solution:

The ϵ -NFA looks as follows



a) $\text{ECLOSE}(P) = \{p, q, r\}$ $\text{ECLOSE}(q) = \{q\}$ $\text{ECLOSE}(r) = \{r\}$

b) All strings except for: bba, bbb, bbc

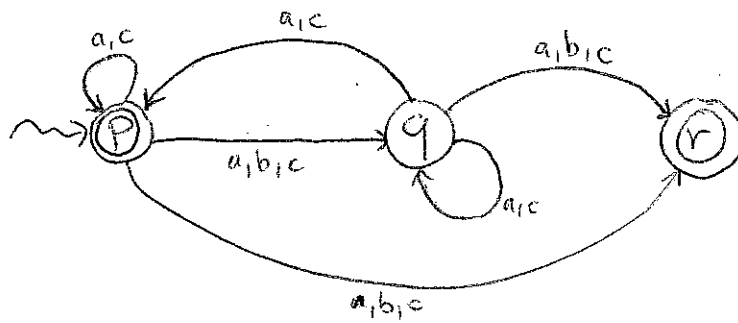
c) From ϵ -NFA to NFA:

$\hat{S}_\epsilon$	a	b	c
P	$\{p, q, r\}$	$\{q, r\}$	$\{p, q, r\}$
q	$\{p, q, r\}$	$\{r\}$	$\{p, q, r\}$
r	$\emptyset$	$\emptyset$	$\emptyset$

$$\begin{aligned}
 \text{Ex.: } \hat{S}_\epsilon(p, a) &= \text{ECLOSE} \left(\bigcup_{s \in \text{ECLOSE}(p)} S(s, a) \right) \\
 &= \text{ECLOSE} (S(p, a) \cup S(q, a) \cup S(r, a)) = \text{ECLOSE} (\emptyset \cup \{p\} \cup \emptyset) = \{p, q, r\}
 \end{aligned}$$

The NFA looks as follows

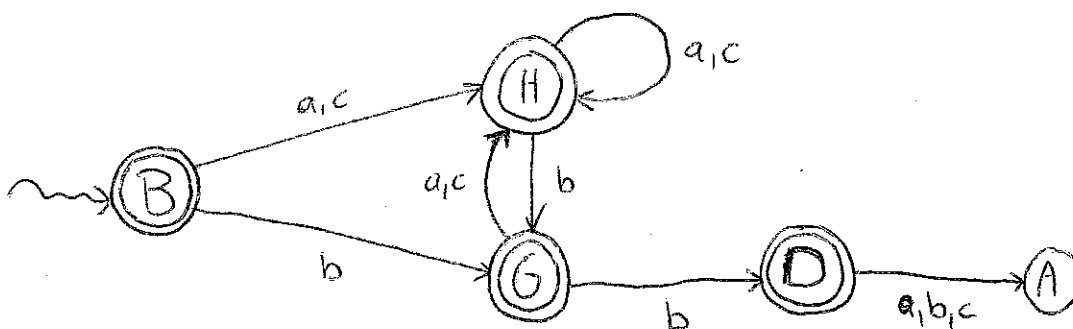
E2.4



From NFA to DFA :

subset construction	a	b	c
$A = \emptyset$	A	A	A
* $\rightarrow B = \{p\}$	H	G	H
$C = \{q\}$	H	D	H
* $D = \{r\}$	A	A	A
$E = \{p, q\}$	H	G	H
* $F = \{p, r\}$	H	G	H
* $G = \{q, r\}$	H	D	H
* $H = \{p, q, r\}$	H	G	H

The DFA looks as follows



Exercise 3 (2.5.3 from textbook)

E2.5

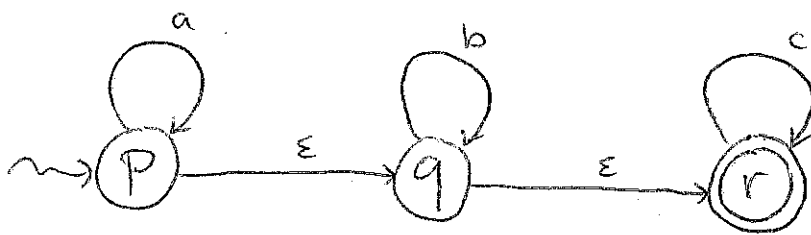
Design ϵ -NFA's for the following languages.

a) The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

Solution:

a)



b)

