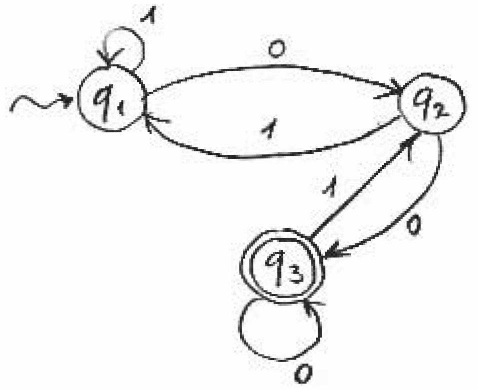
Exercise (3.2.1 from textbook)

Given the following DFA:



(a) give all regular expressions $R_{ij}^{(0)}$

(b) " " " " $R_{ij}^{(1)}$, symplifying as much as possible

(c) " " " " $R_{ij}^{(2)}$, " " " " "

(d) give a regular expression for the language of the automaton

(e) starting from the diagram above, give a regular expression for the language by eliminating state q_2.

solution (a)

$R_{11}^{(0)} = 1 + \epsilon$

$R_{12}^{(0)} = 0$

$R_{13}^{(0)} = \emptyset$

$R_{21}^{(0)} = 1$

$R_{22}^{(0)} = \epsilon$

$R_{23}^{(0)} = 0$

$R_{31}^{(0)} = \emptyset$

$R_{32}^{(0)} = 1$

$R_{33}^{(0)} = 0 + \epsilon$

Remember:

$$\boxed{R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} \left(R_{kk}^{(k-1)}\right)^* R_{kj}^{(k-1)}}$$

E 3.2

solution (b)

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} \cdot \left(R_{11}^{(0)}\right)^* \cdot R_{11}^{(0)} + R_{11}^{(0)} = (1+\epsilon) + (1+\epsilon)(1+\epsilon)^*(1+\epsilon) =$$
$$= 1+\epsilon + 1^* = 1^*$$

$$R_{12}^{(1)} = R_{12}^{(0)} + R_{11}^{(0)} \cdot \left(R_{11}^{(0)}\right)^* \cdot R_{12}^{(0)} = 0 + (1+\epsilon)\cdot(1+\epsilon)^* \cdot 0 =$$
$$= 0 + 1^*0 = 1^*0$$

$$R_{13}^{(1)} = R_{13}^{(0)} + R_{11}^{(0)} \cdot \left(R_{11}^{(0)}\right)^* \cdot R_{13}^{(0)} = \emptyset$$

$$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} \left(R_{11}^{(0)}\right)^* \cdot R_{11}^{(0)} = 1 + 1(1+\epsilon)^*(1+\epsilon) = 11^*$$

$$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} \left(R_{11}^{(0)}\right)^* R_{12}^{(0)} = \epsilon + 1(1+\epsilon)^* \cdot 0 = \epsilon + 11^*0$$

$$R_{23}^{(1)} = R_{23}^{(0)} + R_{21}^{(0)} \left(R_{11}^{(0)}\right)^* R_{13}^{(0)} = 0 + 1\cdot(1+\epsilon)^* \cdot \emptyset = 0$$

$$R_{31}^{(1)} = R_{31}^{(0)} + R_{31}^{(0)} \left(R_{11}^{(0)}\right)^* R_{11}^{(0)} = \emptyset + \emptyset = \emptyset$$

$$R_{32}^{(1)} = R_{32}^{(0)} + R_{31}^{(0)} \left(R_{11}^{(0)}\right)^* R_{12}^{(0)} = 1 + \emptyset = 1$$

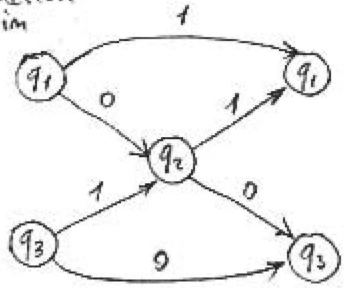
$$R_{33}^{(1)} = R_{33}^{(0)} + R_{31}^{(0)} \left(R_{11}^{(0)}\right)^* R_{13}^{(0)} = (0+\epsilon) + \emptyset = 0+\epsilon$$

(c),(d)

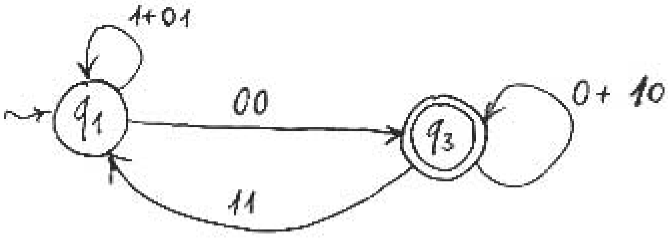
[THE REST IS LEFT AS HOMEWORK]

solution (e)

construction that helps in exercise:



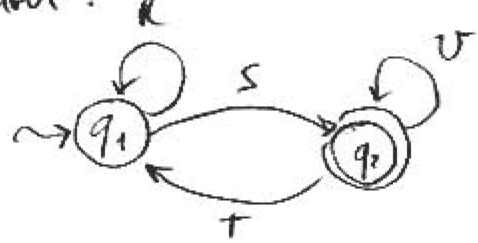
Elimination of state q_2:



$$E = \left(\left(1+01\right) 00 \left(0+10\right)^* 11\right)^* 00 \left(0+10\right)^*$$

final regular expression

Remember:



leads to expression

$$\left(R^* + S U^* T\right)^* S U^*$$

Exercise (3.2.4 from textbook)

E 3.4

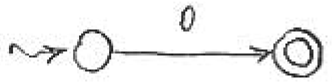
Convert the following regular expressions to ϵ-NFAs :

(a) 01^*

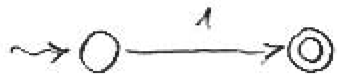
(b) $(0+1)01$

(c) $00(0+1)^*$

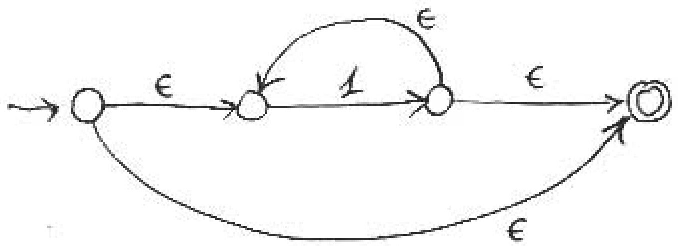
solution (a)



accepts 0

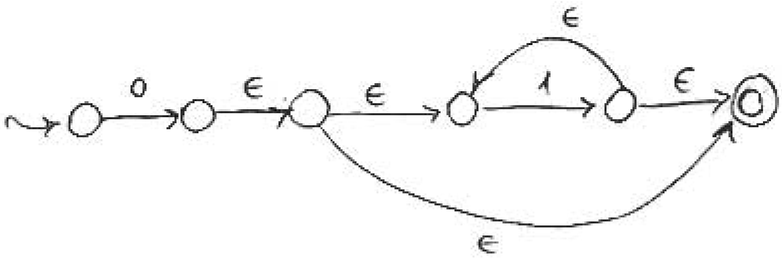


accepts 1

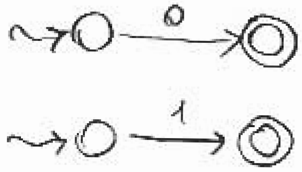


accepts 1^*

Final automaton :

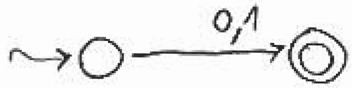


solution (b)



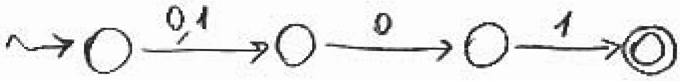
accept 0 and 1 respectively

Merging starting and accepting states:



accepts $0+1$

Merging final state of the first automaton and initial state of the second (see Exercise 3.2.7 from textbook) for concatenation:

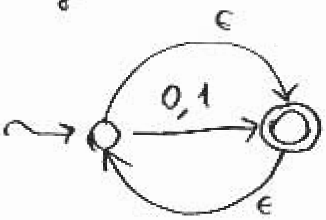


solution (c)

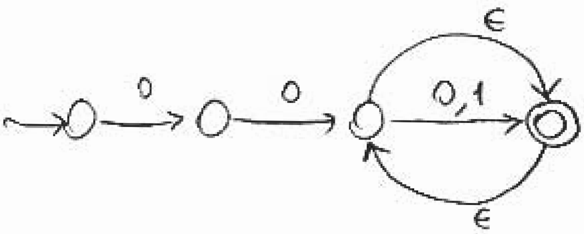
accepts 00

accepts $0+1$

From the latter, we add ϵ-transitions from starting state to accepting state and vice-versa (exercise 3.2.7 from textbook):

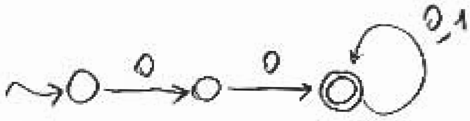


accepts $(0+1)^*$



accepts $00(0+1)^*$

We can make the two rightmost states (with respect to the figure) collapse, thus eliminating the residual ϵ-transitions:



Exercise (3.2.5 from textbook)

E 3.6

Eliminate ϵ-transitions from automata constructed in Exercise 3.2.4 from textbook.

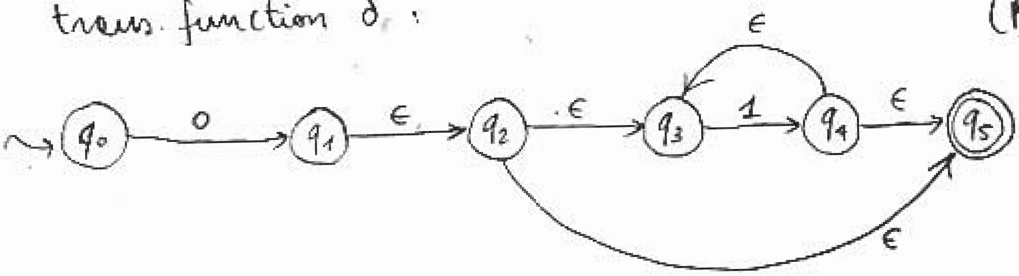
Remember:

$$\delta_N(q,a) = \text{Eclose}\left(\bigcup_{p \in \text{Eclose}(q)} \{\delta(p,a)\} \right)$$

(plus considering initial state as accepting, if needed)

Solution (a)

Initial automaton with trans. function δ:



DEF.
$\text{Eclose}(\{q_1, \dots, q_m\}) = \bigcup_{1 \le i \le m} \text{Eclose}(q_i)$

$$\delta_N(q_0,0) = \text{Eclose}\left(\bigcup_{p \in \{q_0\}} \delta(p,0) \right) = \text{Eclose}(\delta(q_0,0)) =$$

$$= \text{Eclose}(\{q_1\}) = \{q_1, q_2, q_3, q_5\}$$

$$\delta_N(q_0,1) = \text{Eclose}(\delta(q_0,1)) = \emptyset$$

$$\delta_N(q_1,0) = \text{Eclose}\left(\bigcup_{p \in \{q_1,q_2,q_3,q_5\}} \delta(p,0) \right) =$$

$$= \text{Eclose}(\delta(q_1,0) \cup \delta(q_2,0) \cup \delta(q_3,0) \cup \delta(q_5,0)) =$$

$$= \text{Eclose}(\emptyset \cup \emptyset \cup \emptyset \cup \emptyset) = \emptyset$$

$$\delta_N(q_1,1) = \text{Eclose}(\delta(q_1,1) \cup \delta(q_2,1) \cup \delta(q_3,1) \cup \delta(q_5,1)) =$$

$$= \text{Eclose}(\emptyset \cup \emptyset \cup \{q_4\} \cup \emptyset) = \{q_4, q_5, q_3\}$$

$$\delta_N(q_2,0) = \text{Eclose}(\delta(q_2,0) \cup \delta(q_3,0) \cup \delta(q_5,0)) =$$

$$= \text{Eclose}(\emptyset \cup \emptyset \cup \emptyset) = \emptyset$$

$$\delta_N(q_2,1) = \text{Eclose}(\delta(q_2,1) \cup \delta(q_3,1) \cup \delta(q_5,1)) =$$

$$= \text{Eclose}(\emptyset \cup \{q_4\} \cup \emptyset) = \{q_4, q_5, q_3\}$$

(E 3.7)

$\delta_N(q_3,0) = \text{Eclose}(\delta(q_3,0)) = \emptyset$

$\delta_N(q_3,1) = \text{Eclose}(\delta(q_3,1)) = \text{Eclose}(q_4) = \{q_4, q_5, q_3\}$

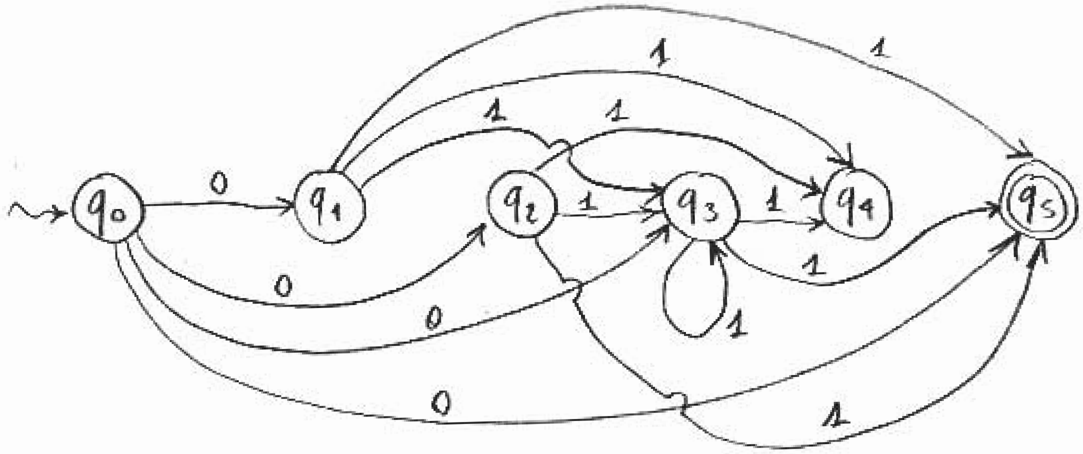
$\delta_N(q_4,0) = \text{Eclose}(\delta(q_4,0) \cup \delta(q_5,0)) =$

$= \text{Eclose}(\emptyset \cup \emptyset) = \emptyset$

$\delta_N(q_4,1) = \emptyset$ (as in previous step)

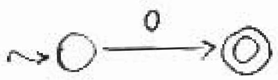
$\delta_N(q_5,0) = \emptyset$ (trivial)

$\delta_N(q_5,1) = \emptyset$ (trivial)



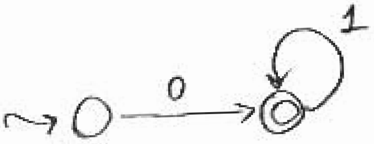
Notice that it is extremely redundant.

By applying merging of states as in cases (b) and (c), we would have immediately obtained the following:



accepts 0

accepts 1^*



accepts 01^* (no ϵ-transitions)