Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2008/2009
Final exam – 15/6/2009 – Part 2

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE (or believed to be so, under certain assumptions, which you should state). You must give an explanation of your answer to receive full credit.

(a) $\mathrm{P}^{\mathrm{coNP}} = \mathrm{coNP}^{\mathrm{P}}$.

(b) Let L_1 and L_2 be languages. If $L_1 <_{poly} L_2$ and $L_2 \in \mathrm{NP}$, then $L_1 \in \mathrm{P}$.

(c) $\mathrm{P} \subsetneq \textsc{Size}(n^{O(1)})$.

Problem 2.2 [6 points] For a string $x \in \{0,1\}^*$, let $\overline{x}$ denote the *complement* of x, i.e., the string obtained from x by substituting each 0 with a 1 and each 1 with a 0. Let L be a language over $\Sigma = \{0,1\}$, and let

$$L' = \{\ w \in \Sigma^* \mid w \text{ has a substring } x \text{ such that either } x \in L \text{ or } \overline{x} \in L\}.$$

Show that if L is in NP, then also L' is in NP.

Problem 2.3 [6 points] Consider the proof of Cook's theorem that CSAT is NP-hard. Describe how in that proof the computation of a non-deterministic TM with running time $p(n)$, where $p(n)$ is a polynomial in n, is represented using propositional variables. Consider then *only* the conditions holding between such propositional variables that depend on the actual transitions of the TM, and provide the CNF-formulas that encode such conditions. How many clauses of which length are necessary to encode such conditions?

Problem 2.4 [6 points] Consider the simulation of TMs by uniform circuits. Illustrate the main ideas regarding this simulation to show that if a language L is accepted by a TM M with running time $t(n)$, then $L \in \textsc{Size}(O(t(n)^2))$ and $L \in \textsc{Depth}(O(t(n)))$.
[*Hint*:] Show how to simulate the run of M on inputs of length n by a circuit of size $O(t(n)^2)$.

Problem 2.5 [6 points]

(a) Provide the definition of a Binary Decision Diagram (BDD) for n boolean variables. Consider the function $f_v(x_1, \ldots, x_n)$ associated to a node v of a BDD. Describe how, given binary values $a_1, \ldots, a_n$, the value $f_v(a_1, \ldots, a_n)$ can be computed. Define the complexity measures for BDDs.

(b) Construct a BDD that computes the even and odd parity bits for 3 inputs x_1, x_2, x_3. An even (resp., odd) parity bit is set to 1 if the number of 1s in a given set of bits is odd (resp., even), thus making the total number of 1s, including the parity bit, even (resp., odd).

(c) Give the values of the complexity measures for the BDD you have constructed.

(d) Illustrate how the even parity bit is computed on the example of $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and on the example of $x_1 = 1$, $x_2 = 1$, $x_3 = 0$.