Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2008/2009
Final exam – 15/6/2009 – Part 1

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] You must give a brief explanation of your answer to receive full credit.

(a) Decide whether the following statement is TRUE or FALSE: The complement of a recursive language is recursively-enumerable.

(b) Let N be a non-deterministic TM, and let D be the result of converting N into a deterministic TM. Sketch using two to three sentences the main idea underlying the conversion. How are the running times of N and D related to each other?

(c) Decide whether the following statement is TRUE or FALSE: There exist two languages L_1 and L_2 such that there exists a reduction from L_1 to L_2, but there exists no reduction from $\overline{L_1}$ to $\overline{L_2}$ (where $\overline{L_i}$ denotes the complement of L_i).

Problem 1.2 [6 points] Consider the language $L_c = \{w\#\overline{w} \mid w \in \{0,1\}^*\}$, where $\overline{w}$ denotes the *complement* of w, i.e., the string obtained from w by substituting each 0 with a 1 and each 1 with a 0.
E.g.: $\# \in L_c$, `100#011` $\in L_c$, `10#1` $\notin L_c$, `100#100` $\notin L_c$, `100#001` $\notin L_c$.

(a) Construct a TM M that accepts L_c.

(b) Show the sequence of IDs of M on the input strings "`10#1`" and "`10#01`".

Problem 1.3 [6 points] Consider a language L over $\{0,1\}$ for which there exists a TM M_e over $\{0,1,\texttt{\#}\}$ that outputs on its tape all strings of L *in lexicographic order*, separating each string from the next by a `#`. A string $w \in L$ is considered to be output by M_e, as soon as the `#` following w is written, and from that moment onward w is not touched anymore by M_e. Show that L is recursive.

Problem 1.4 [6 points] Let f and g be primitive recursive functions. Show that the following functions are μ-recursive:

(a) $h_1(x) = \begin{cases} 1, & \text{if } f(z) = x + g(z), \text{ for some } z \geq 0 \\ \uparrow, & \text{otherwise.} \end{cases}$

(b) $h_2(x) = \begin{cases} \uparrow, & \text{if } f(z) + g(z) < x, \text{ for } \textbf{all } z \geq 0 \\ 0, & \text{otherwise.} \end{cases}$

(c) $h_3(x) = \begin{cases} 1, & \text{if } f(y) + g(z) = x, \text{ for some } y \geq 0,\ z \geq 0 \\ \uparrow, & \text{otherwise.} \end{cases}$

Problem 1.5 [6 points] Let p be a total predicate with $n+1$ arguments, and $f(\vec{x}, y)$ the $n+1$-argument function defined from $p(\vec{x}, z)$ by *bounded minimization*.

(a) Provide the formal definition of $f(\vec{x}, y)$. Describe the intuitive meaning of $f(\vec{x}, y)$.

(b) Show that, when p is a primitive recursive predicate, then f is a primitive recursive function.
[*Hint*: Exploit the fact that a bounded product and a bounded sum of a primitive recursive function is primitive recursive.]