Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2008/2009

Midterm exam – 2/12/2008

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) There exist two languages L_1 and L_2 that are both non recursively enumerable, and such that $L_1 \cup L_2$ is recursive.

(b) There exists a language L such that both L and $\overline{L}$ are recursively enumerable.

(c) According to Rice's Theorem, the following problem is undecidable: Given the encoding $\mathcal{E}(M)$ of a TM M, decide whether M has a transition whose effect is to move right after having written symbol a on the tape.

Problem 1.2 [6 points] Consider the language $L_p = \{ww^R \mid w \in \{0,1\}^*\}$, where w^R denotes the *reverse* of w, i.e., the string obtained from w by reversing the order of its symbols.
E.g.: $\varepsilon \in L_p$, $\texttt{1001} \in L_p$, $\texttt{101} \notin L_p$, $\texttt{1010} \notin L_p$.

(a) Construct a TM M that accepts L_p.

(b) Show the sequence of IDs of M on the input strings "`101`" and "`1001`".

Problem 1.3 [6 points] The *quotient* $L_1 \backslash L_2$ of two languages L_1 and L_2 is defined as

$$L_1 \backslash L_2 = \{w \mid \text{ there exists } w_2 \in L_2 \text{ such that } w \cdot w_2 \in L_1\}.$$

Show that the class of recursively enumerable languages is closed under the quotient operation, i.e., that if L_1 and L_2 are recursively enumerable, then so is $L_1 \backslash L_2$.
[*Hint*: Show how to construct, from two (deterministic) TMs M_1 accepting L_1 and M_2 accepting L_2, a (possibly multi-tape) non-deterministic TM N accepting $L_1 \backslash L_2$. You need not detail completely the construction of N, but you should provide sufficient information to make clear that you would be able to detail the construction, if you were given enough time to do so.]

Problem 1.4 [6 points]

(a) Let f and g be primitive recursive functions. Show that the following predicate p is primitive recursive:

$$p(x) = \begin{cases} 1 & \text{if } f(i) \neq g(i), \text{ for some } 0 \leq i \leq x \\ 0 & \text{otherwise} \end{cases}$$

(b) Show that the following function f is primitive recursive:

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \text{ or } x = 1 \\ 1 + 3 \cdot f(x-2) + 2 \cdot f(x-1) & \text{if } x \geq 2 \end{cases}$$

Problem 1.5 [6 points] Let $h : \mathbb{N}^n \to \mathbb{N}$ and $g_i : \mathbb{N}^k \to \mathbb{N}$, for $1 \leq i \leq n$, be partial functions.

(a) Define the function f obtained from h and $g_1, \ldots, g_n$ by *composition*, i.e., $f = h \circ (g_1, \ldots, g_n)$. Give a simple example of a function defined by composition.

(b) State the conditions under which $f(x_1, \ldots, x_k)$ is undefined.