Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2007/2008
Final exam – 30/9/2008 – Part 1

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] You must give a brief explanation of your answer to receive full credit.

(a) Decide whether the following statement is TRUE or FALSE: The complement of a recursive language is recursively-enumerable.

(b) Determine whether the following problem is decidable: Given a pair $\langle M, w\rangle$ constituted by the encoding $\mathcal{E}(M)$ of a TM M followed by an input string w, decide whether for each state q of M and each input alphabet symbol a appearing in w, M contains a transition $\delta(q,a) = (q', a', d)$ (for some state q' of M, tape alphabet symbol a', and direction d).

(c) Decide whether the following statement is TRUE or FALSE: For all languages L_1, L_2, and L_3, if there exist a reduction from L_1 to L_2 and a reduction from L_2 to L_3, then there exists a reduction from L_1 to L_3.

Problem 1.2 [6 points] Construct a TM M that accepts the language $L = \{n\#w \mid n$ is a number represented in binary with the least significant digit on the *right*, and $w \in \{\mathtt{a},\mathtt{b}\}^*$ with $|w|_\mathtt{a} = n\}$, where $|w|_\mathtt{a}$ denotes the number of occurrences of a in w.

E.g.: `10#abbab` $\in L$, `0#` $\in L$, `10#abbaab` $\notin L$ `10#bbab` $\notin L$.

Show the sequence of IDs of M on the input strings "`10#abab`" and "`10#ab`".

Problem 1.3 [6 points] We define the *shuffle* $L_1 \& L_2$ of two languages L_1 and L_2 as follows:

$L_1 \& L_2 = \{v_1 w_1 \cdots v_n w_n \mid n > 0,\ v_1 \cdots v_n \in L_1,\ w_1 \cdots w_n \in L_2,\ v_i \in \Sigma^*, w_i \in \Sigma^*, \text{ for } 1 \le i \le n\}$

Show that the class of recursively enumerable languages is closed under the *shuffle* operation, i.e., that if L_1 and L_2 are recursively enumerable, then so is $L_1 \& L_2$.

[*Hint*: Show how to construct, from two (deterministic) TMs M_1 accepting L_1 and M_2 accepting L_2, a (possibly multi-tape) non-deterministic TM N accepting $L_1 \& L_2$. You need not detail completely the construction of N, but you should provide sufficient information to make clear that you would be able to detail the construction, if you were given enough time to do so.]

Problem 1.4 [6 points]

- Let f and g be primitive recursive functions. Show that the following predicate p is primitive recursive:

$$p(x) = \begin{cases} 1 & \text{if } f(i) \neq g(i), \text{ for all } 0 \le i \le x \\ 0 & \text{otherwise} \end{cases}$$

- Show that the following function f is primitive recursive:

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \text{ or } x = 1 \\ 2 \cdot f(x-1) - f(x-2) & \text{if } x \ge 2 \end{cases}$$

Problem 1.5 [6 points] Let g and h be partial number-theoretic functions with n and $n+2$ variables, respectively.

(a) Provide the definition of the $n+1$-variable function f obtained from g and h by *primitive recursion*.

(b) State the conditions under which $f(x_1, \ldots, x_n, y)$ is undefined, where y is the recursive variable.