Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2007/2008
Final exam – 14/2/2008 – Part 2

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE (or believed to be so, under certain assumptions, which you should state). You must give an explanation of your answer to receive full credit.

(a) $\Sigma^p_{k+1} = \mathrm{NP}^{\Pi^p_k}$.

[Recall the following definitions: $\Sigma^p_{k+1} = \mathrm{NP}^{\Sigma^p_k}$, and $\Pi^p_k = co\Sigma^p_k$.]

(b) Let L_1, L_2, and L_3 be languages. If $L_1 <_{poly} L_2$ and $L_2 <_{poly} L_3$, then $L_1 <_{poly} L_3$.

(c) $\mathrm{P} = \textsc{Size}(n^{O(1)})$.

Problem 2.2 [6 points] For a language L, let

- $L_1 = \{\ w \mid w \text{ has a suffix } x \text{ in } L\ \}$
- $L_2 = \{\ w\#0^n \mid w \text{ is a suffix of some } x \text{ in } L \text{ with } |x| = n\ \}$

Show that if L is in NP, then also L_1 and L_2 are in NP.

Problem 2.3 [6 points] Consider the proof of Cook's theorem that CSAT is NP-hard. Describe how in that proof the computation of a non-deterministic TM with running time $p(n)$, where $p(n)$ is a polynomial in n, is represented using propositional variables. Provide the CNF-formulas that encode those conditions holding between such propositional variables that do not depend on the actual transitions of the TM. How many clauses of which length are necessary to encode such conditions?

Problem 2.4 [6 points]

(a) Give a sketch of the proof of the following result: If M is a TM with space bound $p(n)$, and $w \in \mathcal{L}(M)$, then w is accepted by M within $c^{1+p(n)}$ steps, for some constant c.

(b) Use the result in (a) to sketch the proof of the following theorem: Every language $L \in \textsc{Pspace}$ is accepted by a polynomial-space bounded TM that makes at most $c^{q(n)}$ steps, for some constant $c > 1$ and polynomial $q(n)$.

Problem 2.5 [6 points]

(a) Provide the definition of a Binary Decision Diagram (BDD) for n boolean variables. Define the complexity measures for BDDs.
Give an example of a BDD for 3 variables x_1, x_2, x_3, of size at least 8.

(b) Consider the function $f_v(x_1, \ldots, x_n)$ associated to a node v of a BDD. Describe how the value $f_v(a_1, \ldots, a_n)$ can be computed.
Choose a node of the BDD of your example and illustrate this computation for two sets of values for x_1, x_2, x_3 for which the function $f_v(x_1, x_2, x_3)$ has different values.