Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2007/2008
Final exam – 14/2/2008 – Part 1

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1.1 [6 points] You must give a brief explanation of your answer to receive full credit.

(a) Decide whether the following statement is TRUE or FALSE: The complement of a recursively enumerable language is non-recursively enumerable.

(b) Decide whether the following statement is TRUE or FALSE: There exist two languages L_1 and L_2 such that there exists a reduction from L_1 to L_2, but there is no reduction from $\overline{L_1}$ to $\overline{L_2}$.

(c) What is the complexity of transforming a 2-track (deterministic) TM with a 5 symbol storage in the state into a (1-tape deterministic) TM?

(d) Determine whether the following problem is decidable: Given a pair $\langle M, w\rangle$ constituted by the encoding $\mathcal{E}(M)$ of a TM M followed by an input string w, decide whether for each input alphabet symbol a appearing in w, M contains a transition $\delta(q, a) = (q', a', d)$ (for some states q, q', tape alphabet symbol a', and direction d).

Problem 1.2 [6 points] Construct a TM M that accepts the language $L = \{n\#w \mid n$ is a number represented in binary with the least significant digit on the right, and $w \in \{\mathtt{a}\}^*$ with $|w| > n\}$.

E.g.: $\mathtt{10\#aaa} \in L$, $\mathtt{10\#aa} \notin L$, $\mathtt{0\#} \notin L$.

Show the sequence of IDs of M on the input strings "10#aaa" and "10#aa".

Problem 1.3 [6 points] Show that the class of recursively enumerable languages is closed under the closure operation, i.e., that if L is recursively enumerable, then so is L^*.

Recall that $L^* = \{\varepsilon\} \cup \{w_1 w_2 \cdots w_n \mid n \geq 1$, and $w_i \in L$ for $1 \leq i \leq n\}$.

[*Hint*: Show how to construct, from a (deterministic) TM M accepting L, a (possibly multi-tape) non-deterministic TM N accepting L^*. You need not detail completely the construction of N, but you should provide sufficient information to make clear that you would be able to detail the construction, if you were given enough time to do so.]

Problem 1.4 [6 points] Let g_1 and g_2 be one-variable primitive recursive functions, and h_1 and h_2 be four-variable primitive recursive functions. The two functions f_1 and f_2 defined by:

$$\begin{aligned}
f_1(x, 0) &= g_1(x)\\
f_2(x, 0) &= g_2(x)\\
f_1(x, y+1) &= h_1(x, y, f_1(x, y), f_2(x, y))\\
f_2(x, y+1) &= h_2(x, y, f_1(x, y), f_2(x, y))
\end{aligned}$$

are said to be constructed by simultaneous recursion from g_1, g_2, h_1, and h_2.

Show that f_1 and f_2 are primitive recursive. You may make use of auxiliary functions that have already shown to be primitive recursive.

Problem 1.5 [6 points] Provide the *complete* inductive definition of the family of μ-recursive functions. "Complete" means that you should detail all cases of the inductive definition.