Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2005/2006
Midterm exam – 23/11/2005
Time: 90 minutes

Problem 1.1 [4.5 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $(L_1 \cdot L_2)^* = (L_1 \cup L_2)^*$.

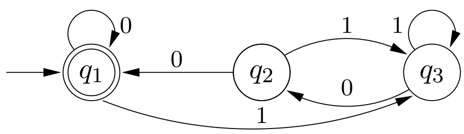
(b) If L_1 is non-regular and L_2 is non-regular, then $L_1 \cup L_2$ must be non-regular.

(c) There exists a language L such that $L = L \cdot L$.

Problem 1.2 [1.5 points] Find a regular expression for the set of binary strings that have both 00 and 11 as substrings.

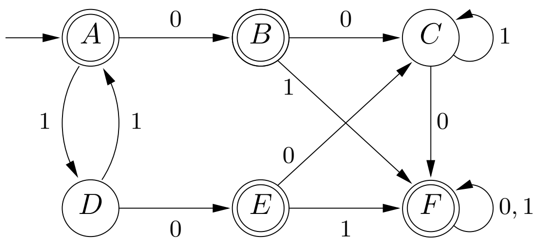
Problem 1.3 [2 points] Explain what is wrong in the following argument: "Let L be a language that is not regular. Since regular languages are closed under the $*$ operator, we have that also L^* is not regular."

Problem 1.4 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct E.

Problem 1.5 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct A_m.

Problem 1.6 [4 points] Show that the language $\{a^m b^n c^p d^q \mid m + n = p + q\}$ is context free by exhibiting a context free grammar that generates it.

Problem 1.7 [6 points] Consider the grammar $G = (\{S, A, B\}, \{0, 1\}, P, S)$, where P consists of the following productions

$$\begin{array}{lcl} S & \longrightarrow & A \mid B \\ A & \longrightarrow & 0A \mid AA1 \mid 0 \\ B & \longrightarrow & B1 \mid 0BB \mid 1 \end{array}$$

Prove that in every word of the language $\mathcal{L}(G)$ the number of 0's and the number of 1's are different.