Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2004/2005

Midterm exam – 24/11/2004

Duration: 90 minutes

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $L_1^* \cup L_2^* = (L_1 \cup L_2)^*$.

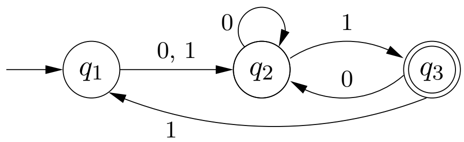
(b) If L_1 is regular and L_2 is non-regular, then $L_1 \cap L_2$ must be regular.

(c) If L is not of type 2 (i.e., not context-free), then it is not of type 3 (i.e., not regular).

(d) If the language L^* is regular, then L must be regular.

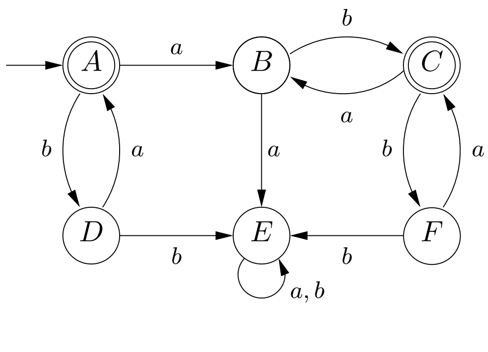
Problem 2 [3 points] Consider the regular expression $E = 1 \cdot (0^* + 0 \cdot 1)^*$. Construct an ϵ-NFA A such that $\mathcal{L}(A) = \mathcal{L}(E)$. Illustrate the steps of the algorithm you have followed to construct A.

Problem 3 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct E.

Problem 4 [6 points] Consider the following DFA A over $\{a, b\}$:



Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct A_m.

Problem 5 [4 points] Show that the language $L = \{a^i b^j \mid i, j \geqslant 0, i \neq j\}$ is not regular. [*Hint*: Exploit in your argument closure properties of regular languages and the known facts that the language $L_1 = \{a^i b^j \mid i, j \geqslant 0\}$ is regular and the language $L_2 = \{a^n b^n \mid n \geqslant 0\}$ is not regular.]

Problem 6 [5 points] Consider the grammar $G = (\{S\}, \{a, b\}, P, S)$, where P consists of the following productions

$$S \longrightarrow Sa \mid bSS \mid SSb \mid SbS \mid a$$

Prove that every string in $\mathcal{L}(G)$ has more a's than b's.