Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Applied Computer Science
Formal Languages – A.Y. 2007/2008
Final Exam – 26/9/2008

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if $L_1^* \subseteq L_2^*$, then $L_1 \subseteq L_2$.

(b) For all languages L_1 and L_2, if $L_1 = \overline{L_2}$, then $L_1 \cap L_2 = \emptyset$ and $L_1 \cup L_2 = \Sigma^*$.

(c) If L_1 and L_2 are regular languages, then $(L_1 \cup L_2)^* \subseteq L_1^* \cup L_2^*$.

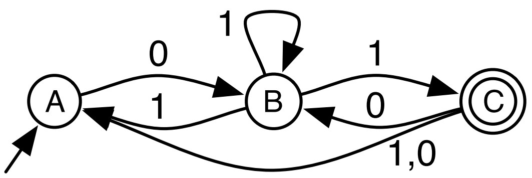
(d) If $L \setminus \{\varepsilon\}$ is a regular language, then L is a regular language.

Problem 2 [6 points]

(a) Construct a **deterministic** finite automaton (DFA) A that accepts the language over the alphabet $\{0, 1, 2\}$ constituted by all strings in which each 2 is immediately preceded by a 1.
E.g., $001201121201 \in \mathcal{L}(A)$, while $0012011021201 \notin \mathcal{L}(A)$.

(b) Construct a regular expression E that generates the language over the alphabet $\{x, y, z\}$ constituted by all strings that contain an odd number of y.
E.g., $xxyyzyz \in \mathcal{L}(E)$, $xxzzxx \notin \mathcal{L}(E)$, and $xyxzzyyzy \notin \mathcal{L}(E)$.

Problem 3 [6 points] Consider the following NFA N over $\{0, 1\}$:



(a) Construct a DFA D such that $\mathcal{L}(D) = \mathcal{L}(N)$. The algorithm you have followed to construct D should become evident in your construction.

(b) Show **all** sequences of states of N and of D that lead to acceptance of 01101.

Problem 4 [6 points] Consider the regular expression $E = 0^* \cdot ((1 \cdot 0) + (0 \cdot 1))^*$. Construct an ε-NFA A_ε such that $\mathcal{L}(A_\varepsilon) = \mathcal{L}(E)$. Simplify intermediate results whenever possible. Then, by eliminating ε-transitions from A_ε, construct an NFA A such that $\mathcal{L}(A) = \mathcal{L}(A_\varepsilon)$. The algorithms you have followed to construct A_ε and A should become evident in your construction.

Problem 5 [6 points]

(a) Describe the algorithm to eliminate the ε-productions from a context free grammar.

(b) Describe the algorithm to eliminate the non-reachable symbols from a context free grammar.

Apply first algorithm (a) and then algorithm (b) to the grammar $G = (\{S, A, B, C, D\}, \{a, b, c\}, P, S)$, where P consists of the following productions:

$$
\begin{array}{rcl}
S & \longrightarrow & B \mid BaBb \mid DbB \\
A & \longrightarrow & DC \mid Aac \mid Bbc \\
B & \longrightarrow & Bb \mid DC \mid \varepsilon \\
C & \longrightarrow & DC \mid CB \mid Ba \\
D & \longrightarrow & Db \mid DC \mid \varepsilon
\end{array}
$$