Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Applied Computer Science
Formal Languages – A.Y. 2007/2008
Final Exam – 27/6/2008

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if $L_1 \subsetneq L_2$, then $L_1^* \subsetneq L_2^*$. [N.B. $A \subsetneq B$ means $A \subseteq B$ and $A \neq B$]

(b) For all languages L_1 and L_2, if $L_1 \cap L_2 = \emptyset$ and $L_1 \cup L_2 = \Sigma^*$, then $L_1 = \overline{L_2}$.

(c) If L_1 and L_2 are regular languages, then $(L_1 \cap L_2)^* \subseteq L_1^* \cap L_2^*$.

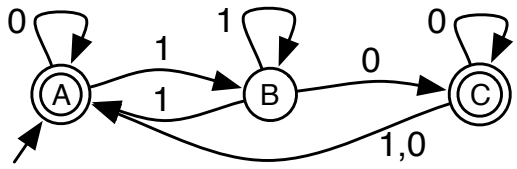
(d) If L is a context-free language, then $L \setminus \{\varepsilon\}$ is a context-free language.

Problem 2 [6 points]

(a) Construct a **deterministic** finite automaton (DFA) A that accepts the language over the alphabet $\{x, y, z\}$ constituted by all strings in which each x is immediately preceded or immediately followed (or both) by y. E.g., $zzyxxyyzzyxyzxy \in \mathcal{L}(A)$, while $zzyxxyyxxzyxyzxy \notin \mathcal{L}(A)$.

(b) Construct a regular expression E that generates the language over the alphabet $\{0, 1, 2\}$ constituted by all strings that contain an even number of 2.
E.g., $0010 \in \mathcal{L}(E)$, $0221021121 \in \mathcal{L}(E)$, while $02210211221 \notin \mathcal{L}(E)$.

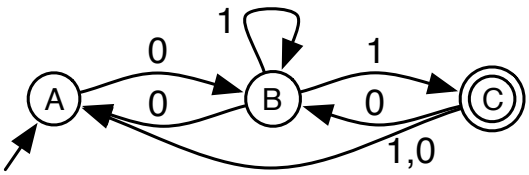
Problem 3 [6 points] Consider the following NFA N_1 over $\{0, 1\}$:



(a) Construct a DFA D such that $\mathcal{L}(D) = \mathcal{L}(N_1)$. The algorithm you have followed to construct D should become evident in your construction.

(b) Show **all** sequences of states of N_1 and of D that lead to acceptance of 11100.

Problem 4 [6 points] Consider the following NFA N_2 over $\{0, 1\}$:



(a) Construct a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(N_2)$. The algorithm you have followed to construct E should become evident in your construction.

(b) Give 3 strings (of length at least 4) that are in $\mathcal{L}(N_2)$ and 3 strings (of length at least 4) that are not in $\mathcal{L}(N_2)$.

Problem 5 [6 points]

(a) Describe the algorithm to eliminate the ε-transitions from a context free grammar.

(b) Describe the algorithm to eliminate non-generating symbols from a context free grammar.

Apply first algorithm (a) and then algorithm (b) to the grammar $G = (\{S, A, B, C\}, \{a, b\}, P, S)$, where P consists of the following productions:

$$\begin{array}{lcl} S & \longrightarrow & C \mid bAAa \mid CaA \\ A & \longrightarrow & Aa \mid CB \mid \varepsilon \\ B & \longrightarrow & CB \mid BA \mid Ba \\ C & \longrightarrow & Ca \mid CB \mid \varepsilon \end{array}$$