Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Applied Computer Science
Formal Languages – A.Y. 2007/2008
Final Exam – 12/2/2008
Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if $L_1^* = L_2^*$, then $L_1 = L_2$.

(b) If L_1 and L_2 are regular languages, then $L_1 \setminus L_2$ is a regular language.

(c) If L_1 and L_2 are regular languages, then $L_1^* \cup L_2^* = (L_1 \cup L_2)^*$.

(d) For all languages L_1 and L_2, if $L_1 \cap L_2 = \emptyset$, then either $L_1 = \emptyset$ or $L_2 = \emptyset$.

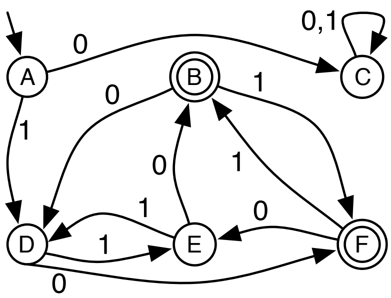
Problem 2 [6 points]

(a) Construct a **deterministic** finite automaton (DFA) A that accepts the language over the alphabet $\{0, 1, 2\}$ constituted by all strings in which each 1 is immediately followed by a 2.
E.g., $0012122012 \in \mathcal{L}(A)$, while $001122012 \notin \mathcal{L}(A)$.

(b) Construct a regular expression E that generates the language over the alphabet $\{x, y, z\}$ constituted by all strings that begin with z and end with a sequence of two or more y's.
E.g., $zzyyxxxyyy \in \mathcal{L}(E)$, while $zzyyxxxy \notin \mathcal{L}(E)$.

Problem 3 [6 points] Consider the regular expression $E = ((0 \cdot 1) + (1 \cdot 0))^* \cdot 1^*$. Construct an ε-NFA A_ε such that $\mathcal{L}(A_\varepsilon) = \mathcal{L}(E)$. Simplify intermediate results whenever possible. Then, by eliminating ε-transitions from A_ε, construct an NFA A such that $\mathcal{L}(A) = \mathcal{L}(A_\varepsilon)$. The algorithm you have followed to construct A_ε and A should become evident in your construction.

Problem 4 [6 points] Consider the following DFA A over $\{0, 1\}$:



(a) Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. The algorithm you have followed to construct A_m should become evident in your construction.

(b) Give 3 strings (of length at least 4) that are in $\mathcal{L}(A)$ and 3 strings (of length at least 4) that are not in $\mathcal{L}(A)$. Provide a description of $\mathcal{L}(A)$ in plain English.

Problem 5 [6 points]

(a) Describe the steps to follow, in the correct order, to convert a context free grammar into Chomsky Normal Form.

(b) Apply these steps to the grammar $G = (\{S, A, B, C\}, \{a, b\}, P, S)$, where P consists of the following productions:

$$\begin{array}{rcl} S & \longrightarrow & B \mid BaBb \mid AbB \\ A & \longrightarrow & Ab \mid AC \\ B & \longrightarrow & Bb \mid AC \mid \varepsilon \\ C & \longrightarrow & AC \mid CB \mid Ba \end{array}$$