Writing Mathematical Papers —a Few Tips

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1 Some generalities

• It is very easy to publish a paper these days—you can just put it on the internet. This has led to a huge rise in the number of publications. An author, especially a novice, cannot be sure that anybody will read his/her paper. Some indirect "advertising" is therefore necessary, the main "selling points" of a paper being the abstract and the introduction.

• The dramatically increasing production scale has forced publishers to seek cost reductions, mainly by outsourcing. This has resulted in a deterioration in the quality of much published work. As a consequence, authors are now often required to undertake tasks previously performed by publishers: files must be prepared with extreme care and submitted in absolutely final form (as regards mathematical content, notation, language and sometimes even TEX typesetting). One must not forget that the page proofs prepared by the publisher also require a very thorough check.

• The electronic version of an article has become increasingly more important; it is not impossible that in the near future it will become the *only* published form of a mathematical article. Already today, it is unreasonable to submit a paper to a journal that does not publish electronic versions of articles.

• The publication process itself is more computerized than ever before—for some journals, authors can only contact the editorial office via an internet website, without ever contacting a person. In this case, papers must be prepared meticulously, because automated procedures cannot handle exceptional situations, like additional corrections or the withdrawal of a paper at a late stage of production.

2 Your reader

Everybody wants their paper to be read by someone—but who is going to read it?

Mathematical papers are read by specialists in a given domain.

Tip: think of a (specific) reader—someone whom you'd like to see reading your paper. This should be a mathematician who can understand your text completely, but does not work on exactly the same subject (and is not necessarily a Fields' medallist; your choice should be realistic!). When writing your paper, try to interest your reader; give

Thanks to Daniel Davies for his corrections.

the definitions he/she does not (or may not) know; explain why the next fragment is important or worth reading—your reader has no time to waste; leave out all that the reader considers trivial. Finally, prepare your paper carefully, so that your reader does not become bored or irritated.

3 Language

Mathematical papers ought to be written in grammatical (but not necessarily idiomatical) English. You should not think that a text without articles or with many language mistakes is still understandable—there are many examples to the contrary. Caring about the language of your paper is also a form of respecting your reader.

Tips:

• Gather a collection of books and articles by British and American mathematical writers in your area of mathematics; look them up as often as possible when writing your paper; they should be your main source of information on what is and what is not correct mathematical English.

• When writing, if at all possible, do not translate from your native tongue, but write in English at once, borrowing phrases and sentences from English-speaking authors. It is much better to copy a definition or theorem from a text by a native speaker than write it in your own words, with your own mistakes.

• Do not copy fragments of your previous papers—with each consecutive article, your command of English improves (with high probability). Also, do not follow the style of your thesis adviser (if he is not a native speaker of English) or other non-native speakers.

• If you find a new expression or word in some text, and you'd like to use it, do not rely on your intuition: first check its meaning in a (thick) dictionary.

• Do not use fancy and/or witty words, complicated expressions, inverted commas etc. It is best to use simple and direct language, otherwise you can open yourself up to ridicule.

4 How to write and how not to write in LaTeX

A file for yourself and a file for the publisher

The (source) file you send to the publisher must—by definition—be prepared in the form required by the publisher (and not for yourself).

Tip: for each article, you should have two source files: one for your archive and another one, a final one, for the publisher.

When starting your work, do not adopt the style of any specific journal (you may well end up publishing the paper in another one); instead, use one of the standard styles (`amsart` or `article`), and reformat the article only at the very last stage, when all the mathematical and language changes have been implemented.

The advantage of `amsart` is that it contains the `\address`, `\keywords` and `\subjclass` commands. If you prefer `article`, add `\usepackage{amsmath,amssymb}`.

Your archival file may (and even should) contain comments, omitted fragments of proofs, etc. in lines marked %; it may also have any number of macros. The file for the publisher should be "cleaned up": leave only the macros needed in the article, and remove all the commented-out fragments.

Simplest is best

• Do not re-invent LaTeX; before using your own construction or creating a new symbol look up Grätzer [G]—most probably, your intended construction or symbol is already there, and in a better version.

• A simple formula should have a simple code: if you want to write $ab(x+y)^2$, typeset

`a(x+y)^2`, and not e.g. `a\,b\,{\left(x+y\right)}^{2}`;

code a fraction $\frac{1}{2}$ as `\frac 12`; only when writing $\frac{1}{2p}$, you need braces: `\frac 1{2p}`; etc. Add small spaces `\,` only exceptionally, e.g. before differentials.

• For the same reason, use standard LaTeX constructions, defined in the preamble, instead of your own ones.

If you want, for example, all your theorems, definitions etc. to be numbered together and separately in each section (this makes them easy to find for the reader), write:

```
\newtheorem{thm}{Theorem}[section]
\newtheorem{cor}[thm]{Corollary}
\newtheorem{lem}[thm]{Lemma}
\newtheorem{prob}[thm]{Problem}
```

All these objects will be set in italics, i.e., in theorem style.

• Definitions, remarks, examples etc. should not be italicized, because this leads to emphasizing large parts of text ("if everything is emphasized, nothing is emphasized"). Instead, declare these objects like this:

```
\theoremstyle{definition}
\newtheorem{defin}[thm]{Definition}
\newtheorem{rem}[thm]{Remark}
\newtheorem{exa}[thm]{Example}
```

Thanks to these preparations you will not have to bother about typefaces, vertical spacing etc.

If you want your equations to be also numbered by section, typeset (in the preamble)

```
\numberwithin{equation}{section}
```

• Objects that are not referred to in the paper need not be numbered; declare e.g.

```
\newtheorem*{xrem}{Remark}
```

Similarly, do not number displayed formulas unless they are referred to later; for non-numbered formulas, typeset

```
\[
...
\]
```

• The principle "write it simple" also concerns typesetting complicated multiline formulas. Use spacing and line breaks to make your code as clear as possible. The `eqnarray` construction leads to well-known mistakes—if you have learnt it, just forget it.

If you want e.g. to write

$$\begin{aligned} A &= B \\ &\quad + C \\ &= D \end{aligned}$$

just typeset

```
\begin{align*}
A &= B\\
  &\quad + C\\
  &= D
\end{align*}
```

(lots of other variants can be found in the files available at `http://journals.impan.pl`).

Macros

• If a complicated symbol, e.g. `\widetilde\mathcal{S}`, appears often in your paper, invent an abbreviation for it, say `\tcS`, and add its definition to the preamble:

```
\newcommand{\tcS}{\widetilde\mathcal{S}}
```

• Introducing an abbreviation is even more advisable if you often use formulas built according to some scheme that depends on parameters, e.g. $i_1, \dots, i_n$, $k_1, \dots, k_m$; these depend on two parameters and you could write them briefly, say, `\row in` and `\row km`, provided you set

```
\newcommand{\row}[2]{#1_1,\ldots,#1_{#2}}
```

• Using a macro is almost mandatory if you introduce a new piece of notation, which may be changed later in the writing process for various reasons. For example, if you define new spaces depending on three parameters and at first the notation of the type $H^{p,q}_r$ seems suitable, introduce the macro

```
\newcommand{\msp}[3]{H^{#1,#2}_{#3}}
```

("msp" for "my space"). If, at some later date, you wish to change H to some other letter or rearrange the indices, it will require modifying a single line in the whole file.

• The so-called "delimited macros" are often useful. For example, if you use many scalar products and do not want to write `\langle a,b\rangle` all the time, you can write e.g. `\<a,b>`, provided you set

```
\def\<#1>{\langle#1\rangle}
```

This is the only case where you have to use the TEX command `\def`, and not `\newcommand`. Note that `\<` is the name of the operator, while `>` limits its range. (It follows, in particular, that if you have nested scalar products, the macro will not work.)

• In the file for the publisher, do not use macros for LATEX commands, e.g. do not abbreviate `\begin{equation}` to `\be`—the former is instantly understandable to every LATEX user, while the latter is not.

Labels

For obvious reasons never write in the file

```
By (5), (12) and Theorem 3.1 we have
```

—these numbers may change with every new version of the manuscript; instead, use "labels". But how to choose them without getting lost?

• Tip: objects of different kinds should have different types of labels, e.g.

theorems: `\label{T:Pythagoras}`
lemmas: `\label{L:Kurat-Zorn}`
propositions: `\label{P:compact}`
equations: `\label{E:tr.ineq.}` etc.

If you follow this convention, it will be more difficult to make the (often encountered) mistake of writing "Theorem 2.3" instead of e.g. "Proposition 2.3".

• It is often difficult to find satisfactory names for equations; you can then just take numbers as labels, e.g. `\label{E:5}`.

• In the course of writing, you can use the `showkeys` package, preferably in the form

```
\usepackage[nocite]{showkeys}
```

Then you will see the labels on your screen and on the printout. The optional parameter `nocite` prevents the labels of reference items from showing up—for these, everybody can easily invent a system of abbreviations of their own.

Mathematics and all the rest

A page of mathematical text consists of displayed formulas and undisplayed text, which also in general contains formulas.

Long passages without displayed formulas are hard to read; on the other hand, not every formula ought to be displayed.

• What should be displayed?

– a formula longer than 3/4 of text width;
– a formula with high elements, like fractions, matrices, sums, integrals (unless they are very simple);
– a formula containing a definition that will be used after a few pages;
– two analogous formulas in the proof.

• What should not be displayed?

– short formulas, to which the reader will not have to return later;
– twice the same formula, especially not far apart.

• In undisplayed text, put only mathematics within the $ signs; leave (non-mathematical) punctuation marks and spaces outside.

Instead of writing

```
the polynomials $x+1, x^2+1, x^3-2$
```

typeset

```
the polynomials $x+1$, $x^2+1$, $x^3-2$
```

In the former case, TEX views the commas as mathematical symbols, like the commas in $f(x, y, z)$; hence they are followed by thin spaces (thinner than those between words). Also, line breaks after these commas are not allowed.

• In undisplayed text, formulas should not "touch" other formulas too often: put words in between, wherever appropriate.

Here are a few (formally correct) sentences, which, however, do not look nice:

– Since $x = 2$, $x^2 = 4$.
– Since $x = 2$, $y = 3$, $z = 1$, $(x + y + z)^2 = 36$.
– Then for all $f \in X$, $f(0) = 0$.
– Then for all $f \in X$, $f(0) = 0$, A_f is compact.

These sentences are easily improved:

– Since $x = 2$, it follows that $x^2 = 4$.
– Since $x = 2$ and $y = 3$, and since $z = 1$, we have $(x + y + z)^2 = 36$.
– Then for all $f \in X$, we have $f(0) = 0$. Or:
Then $f(0) = 0$ for all $f \in X$.
– Then for all $f \in X$ with $f(0) = 0$, the set A_f is compact.

Note that by adding words between dollar signs, the reader is spared from having to frequently pause and wonder whether a given formula is a premise, a conclusion or something else. Consequently, a slightly longer sentence can make your text easier to understand and faster to read.

Items

For lists of statements of various kinds, you can use

```
\begin{itemize}
\item ...
\item ...
\end{itemize}
```

or, if the items have to be labelled, `\begin{enumerate}` – `\end{enumerate}`.

• Numbering style (1), (2) etc. may conflict with formula numbers. Numbering style 1., 2. is inconvenient to cite ("by 1"). You can force a numbering style e.g. by writing

```
\item[\textrm{(a)}]
```

You can also use the `enumerate` package, which enables you to write e.g.

```
\begin{enumerate}[\upshape (i)]
```

to get Roman numbering automatically.

• Using itemized structures may help you write sentences having a complicated logical structure, e.g.

– Since either
  - A or $B(x)$ for some x, or
  - $C(x)$ for all x, and D,

it follows that
  - E,
  - F.

A sentence like this, written "continuously", may be unintelligible.

Cut and paste

Copying a large fragment of text and repeating it somewhere else with minor modifications may be convenient for the author, but not always for the reader, who—apart from an unpleasant "déjà vu" feeling—may overlook the differences.

• If two very similar formulations appear close to each other, you can write e.g.:

– The statement of (i) remains true with "bounded" replaced by "unbounded", and "convex" by "concave".

• If you feel you must repeat a very similar proof, it is advisable to warn the reader beforehand:

– We now mimic the proof of Lemma 2.3 with appropriate adjustments in the constants.

Assigning (proper) names

When writing names in different languages, always add all the appropriate accents (see [G] for details and tables). For example, to code characters of the Polish alphabet, use

```
\usepackage[T1]{fontenc}
```

Then e.g. "Żołądź" is coded as `\.Zo{\l}\k{a}d\'z`.

5 How to entice your reader to your text

Title

Your typical reader will find out your text by browsing the internet. Here the role of the title cannot be overestimated.

• Here are some conditions a good title has to satisfy:

– Should indicate the branch of mathematics ("On a theorem of Kuratowski" is no good).
– Should not be too long.
– Should not be too general.
– Should not contain abbreviations or complicated symbols, especially in special typefaces (such titles are often misquoted); when typesetting a title, do not use macros: the title is often extracted from the paper by the publisher for various purposes.

• Stylistic devices in titles:

– Question: "Can $B(L^p)$ ever be amenable?"
– Complete sentence, statement of the main result: "Every weak L^p space has the Radon–Nikodym property"
– Verbal element (gerund or participle): "Computing the eigenvalues of M-matrices", "The complemented subspace problem revisited"

• Instead of decorating the author's name or the title with asterisks, it is preferable to place all thanks, grant acknowledgements etc. at the end of the paper, just before "References", and put them into `\subsection*{Acknowledgements}`.

This also saves space on the first and second pages of the paper—and these pages will decide whether the reader will want to read more.

Abstract

Unfortunately, too many abstracts contain sentences like "We prove some properties of some objects." Such an abstract can hardly encourage anybody to read further, and nobody will remember it (as a matter of fact, there is no point in trying, due to lack of information).

The principal aim of the abstract is to *present the main results of the paper*, that is, theorems (use the word "Theorem" sparingly). Hence the key to writing a good abstract is in *formulating your theorems.*

• An ideal formulation of a theorem does not need a blackboard or a piece of paper; in particular, it contains only simple symbols and everything has been put into words. If your theorem is formulated like that, you can just repeat it in the abstract.

• If this ideal situation is unattainable, at least try to present your theorems in the abstract *in the least obscure way possible.* Moreover, your abstract can contain advertising elements, e.g. "We improve a result of Kowalski [Studia Math. 187 (2006)]"—and in principle nothing else.

• Other conditions your abstract ought to satisfy:

– Should be short (as a rule, one paragraph).
– Should not contain empty phrases; instead of "In this article we prove, among other results, that" just write "We prove that".
– Should be as independent of the paper as possible, e.g. without references to theorem numbers, because it will also be used separately: on web pages and in data bases (MathSciNet, Zentralblatt). In particular, it should be possible to process it independently of the article, so it should not contain macros and `\cite`'s.
– Should not contain complicated formulas (it may be represented in the html language or as a TEX code).

Introduction

Amusingly, this is the LAST part of the paper that your typical reader will read (statistically speaking); the rest will be read by very few individuals. So what you write here will determine the impression the paper will make on most readers.

• What are the criteria for including something in (or omitting something from) the introduction?

– Write only what you consider *INTERESTING.*
– Two elements should always appear: *your theorems* and *discussion of the relevant literature.*

• All the main results of the paper should be "transportable" to the introduction, after possibly adding a few (but not many!) definitions (for obvious reasons, this does not concern minor, "local" results).

The theorems presented in the introduction may be (but do not have to be) repeated later—literally or with some modifications; with the same numbering (e.g. as Theorems 1, 2, 3) or with other labels (e.g. Theorem 5.1 etc.).

• You have to convince your reader that you know what other people have written on similar subjects; the reader is interested in comparisons of your results with other results in the literature. Here you can (and should) advertise your achievements. References to the literature should be closely connected with your work—do not start "where the world began". If you cannot comment on the results of some paper, then maybe it should not

be cited in the introduction (sentences like "For other related results see" plus a long list of numbers in square brackets are only appropriate in survey papers).

• How to begin? One possibility is to formulate the fundamental problem your paper is concerned with. Alternatively, you might provide some historical information. First target: interest your reader with the first paragraph.

• How not to begin? Whatever else you do, *do not* start by giving a long and precise list of definitions and notations. There will be time for precision later; first, arrest the reader's attention.

• Besides your results and historical comments, you can put into the introduction any other INTERESTING elements, like:

– comments about proofs, or even sketches and/or heuristical proofs;
– schemes of logical dependence of sections;
– suggestions for further research, etc.

• In most cases, it is of no interest to the reader what appears in which section; besides, that is easy to find out by looking at section titles. Describing the contents of each section is not necessary. In a very long paper you can give a table of contents instead (generated by an appropriate LaTeX command). The information that "in Section 2 we shall give some preliminaries" is a useless statement, in *any* paper.

6 Details

References

Many people will look into this part—to see what literature the paper is related to, but also e.g. to find their name. It is therefore of importance to present all data involving other people's names (and achievements) with extreme care.

• Extract information about papers and books from those items themselves; if they are not directly available, use MathSciNet and Zentralblatt (in Zentralblatt, accents in names are often missing). Copying data from other articles is risky. Do not rely on your memory when citing your own articles—copy data from an offprint instead.

• Information about each reference item should be complete (e.g. including the page numbers of an article and the publisher of a book) and up-to-date—before qualifying an item as "preprint", check whether it has appeared.

Papers in contributed volumes are especially difficult to locate; for such papers, give the exact title of the volume (which, in most cases, is NOT "Proceedings of the Conference" etc.), the place and the date of the conference, publisher, and possibly the editors' names. Note that the publication date is, in general, different from the date of the conference; both should be given.

• As a rule, list a paper only if it is cited in the text (in some journals, this is mandatory). If your own papers outnumber all the others in the reference list, the reader may not be impressed (maybe no one else is interested in the subject...)

Theorem

• Here are some examples of theorem formulations:

– Every finite group is...

– If... then...
– Let... Suppose that... Moreover, suppose that... Then...

Using the last style, you do not have to repeat the assumptions of the theorem at the beginning of the proof. Note the imperative mood. By convention, do not put fragments of the proof into the formulation ("by [15]").

• A theorem is a sentence without free variables—it is either true, or not a theorem. So instead of writing: "we shall prove that the theorem holds for $k = 1$", write: "we shall prove that the conclusion of the theorem holds for $k = 1$."

Definition

• In definitions, emphasize the term being defined, using `\emph{...}`:

– We define the *convex hull* of E to be the smallest convex set containing E.

The word order is different if "call" is used:

– We call the smallest convex set containing E the *convex hull* of E.

• When defining symbols, the notation := or =: is useful, as it shows which side of the equality is being defined (the one next to the colon):

– Then $F = abcde + fghi =: A + B$.

Proof

When writing a proof, do think about your (intended) reader. A person who is reading a proof has to be 1) competent and 2) concentrated; quite a lot of things are obvious to him/her, and not every argument has to be given. Long lists of arguments ("by (1), (3), (5), (6) and (19)") may obscure the situation.

• Do not give arguments that are obvious to every mathematician. Instead of

– Then, by the triangle inequality, we have $|a| \leq |a - b| + |b - c|$

just write

– Then $|a| \leq |a - b| + |b - c|$.

• If the proof requires analysing several cases, maybe some of them can be omitted:

– The analysis of case (b) is similar and left to the reader.

You can also refer the reader to your web page:

– The complete details of the calculations are available on the author's web site (http://...)

• Make sure that, for each statement you write, the reader knows whether it has already been proved, will be proved, or whether the reader is supposed to supply the proof. Writing

– By (12), we have $A = B$

suggests that the given equality is (considered to be) proved; the combination "By (12), we have $A = B$. To see this,..." may be misleading. It is then preferable to announce the proof before the statement:

– We now prove that (12) implies $A = B$. To see this,...

References

[G] G. Grätzer, *More Math into LaTeX*, 4th ed., Springer, 2007.

[H] N. Higham, *Handbook of Writing for the Mathematical Sciences*, 2nd ed., SIAM, 1998.

[T] J. Trzeciak, *Writing Mathematical Papers in English. A Practical Guide*, 2nd ed., Eur. Math. Soc., 2005.