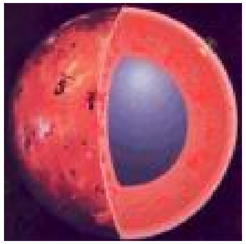
Computing Cores for Data Exchange
New Algorithms and Practical Solutions



Georg Gottlob
TU Wien

Talk Structure

Brief introduction

General core computation (no dependencies)

- Core computation via hypertree decompositions $O(n^{b+3}) \rightarrow O(n^{\underline{b/2}+2})$
- Fixed-Parameter Intractability w.r.t. blocksize b

Computing cores in presence of dependencies **(TGDs & EGDs)**

- A tractable class: Simple TGDs + arbitrary EGDs
- Another tractable class: Full TGDs + arbitrary EGDs
- NP-complete problem variations

Cores

Instance:

{ p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

Logical meaning

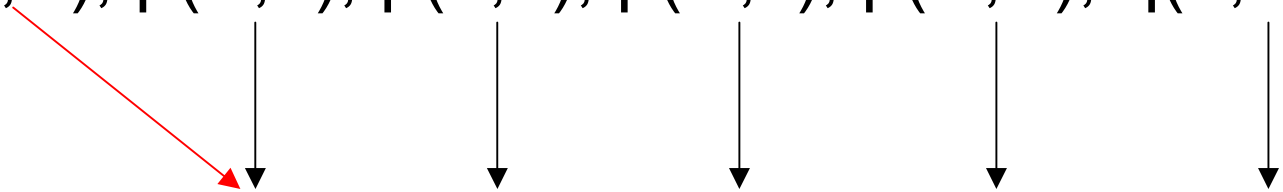
∃ X, Y, U, V:

p(X,Y) **&** p(X,b) **&** p(a,b) **&** p(U,c) **&** p(U,V) **&** q(a,c,d)

Cores

endomorphism h: {Y → b}

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }



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{ p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

REDUNDANT!

∃X,Y p(X,Y) & P(X,b)

⇑⇓

∃X p(X,b)

Cores

endomorphism h: {Y → b}

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

{ ~~p(X,Y)~~, p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

REDUNDANT!

$\exists X,Y\ p(X,Y)$

$\Uparrow$

$\exists X\ p(X,b)$

Cores

endomorphism h: {Y → b}

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

⇔

h(I) = { p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

Cores

endomorphism h: {Y → b}

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

⇔

h(I) = { p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

X → a

V → c

h(I) can be further reduced by endomorphism g: {X → a, V→ c}

Cores

endomorphism h: {Y → b}

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Cores

$I = \{ p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) \}$

$\Leftrightarrow$

$h(I) = \{ p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) \}$

$\Leftrightarrow$

$f(I) = g(h(I)) = g \circ h(I) = \{ p(a,b), p(U,c), q(a,c,d) \}$

endomorphism f: {X → a, Y→b, V→ c}

Cores

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⇔

f(I)= g(h(I)) = g○h(I)= { p(a,b), p(U,c), q(a,c,d) }

no refinement by endomorphisms possible !

endomorphism f: {X → a, Y→b, V→ c}

Cores

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Core(I)

unique up to variable-renaming!

endomorphism f: {X → a, Y→b, V→ c}

Blocks

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

Blocks: Connected components in the variable-graph

Atom-Blocks: corresponding sets of atoms

Blocks

I = { p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) }

{X,Y}   {U,V}   blocksize(I)=2

Blocks: Connected components in the variable-graph

Atom-Blocks: corresponding sets of atoms

blocksize(I) = size of largest block of I

Computing core(I) is NP-hard in general.

It is tractable for bounded blocksize
[Fagin, Kolaitis, Popa]

We can use a suitable adaptation of
the DC algorithm [G.,Leitsch 85]

Core Computation

$I \; = \; \{ p(X,Y), p(X,b), p(a,b), p(U,c), p(U,V), q(a,c,d) \}$

[Fagin et al]: Compute Core(I) by successively
finding and applying
useful local endomorphisms

useful: at least 1 variable disappears
local: identity, except for 1 block

Can be regarded as a series of query-containment checks
I contains J, I ► J iff ∃ homomorphism h: h(I) ⊆ J

Algorithm CORECOMP

INPUT: An instance I
OUTPUT: Core(I)

1. Identify the blocks of I; B(X) denotes block of X in I;

2. K := I;

3. FOR each X $\in$ var(I) DO
   IF X $\in$ var(K) AND K[B(X)] ► (K – atoms (X,K))
   THEN K := K – atoms(X,K);

4. Output K.

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blocksize=b

≤ **b variables**

Essentially |var(I)| block containment tests

Each takes $O(n^{b+2})$ steps. ➔ runtime $O(n^{b+3})$

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we improve this bound

THEOREM:

Let Q and Q´ be conjunctive queries.

Checking Q ► Q´ can be done in time

$O(|Q|\, n^{b/2+1})$ where $b = |var(Q)|$

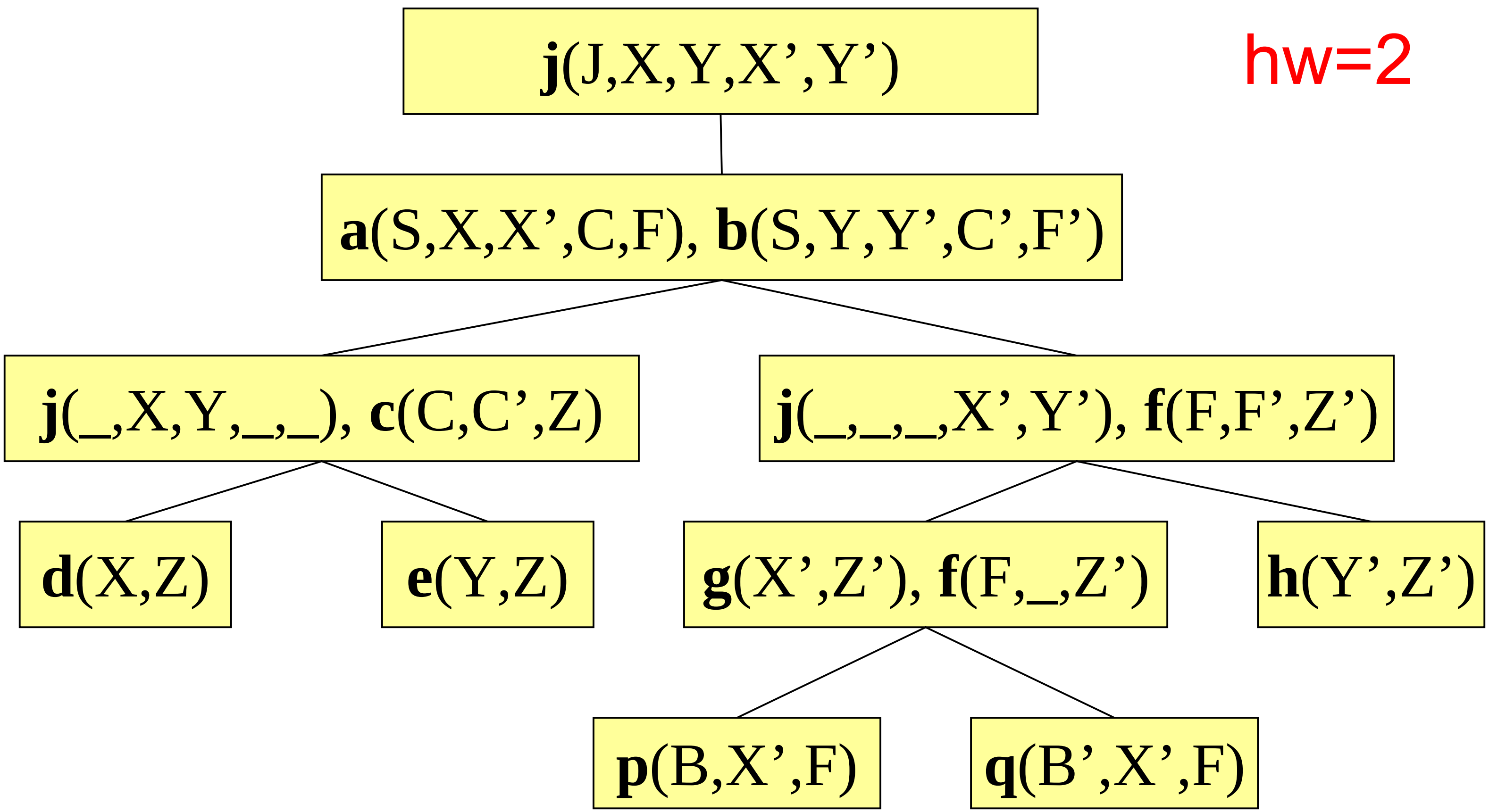
It is well-known that checking whether Q ► Q´ is equivalent to evaluate the Boolean query Q over can(Q´)

Proof (idea).

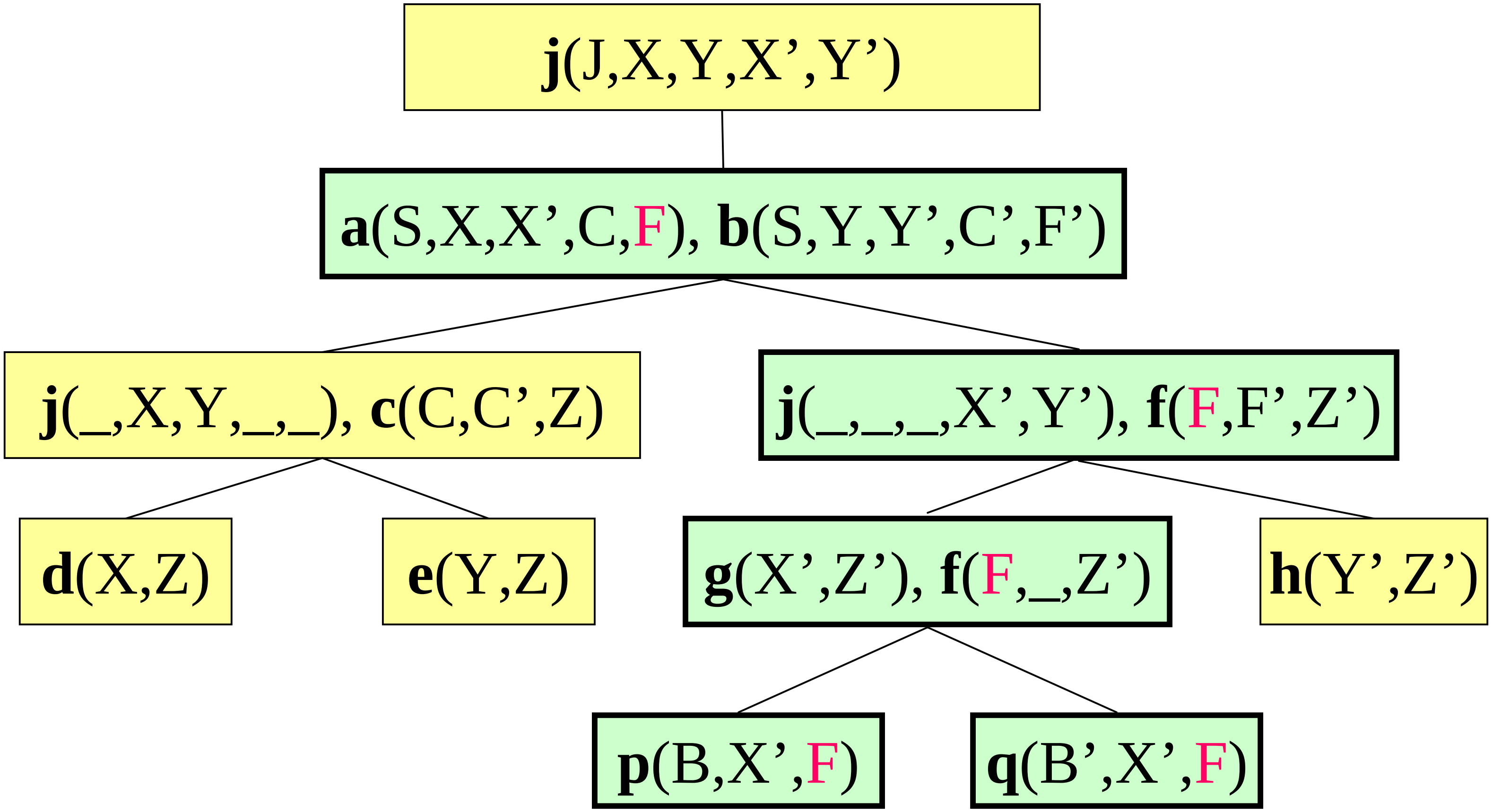
Use hypertree decompositions.

[G., Leone, Scarcello 98]

$$ans \leftarrow a(S,X,X',C,F) \wedge b(S,Y,Y',C',F') \wedge c(C,C',Z) \wedge d(X,Z) \wedge$$
$$e(Y,Z) \wedge f(F,F',Z') \wedge g(X',Z') \wedge h(Y',Z') \wedge$$
$$j(J,X,Y,X',Y') \wedge p(B,X',F) \wedge q(B',X',F)$$



connectedness condition



Proof (idea). Use hypertree decompositions. [G., Leone, Scarcello 98]

We show that each query Q with b variables has hypertree-width ≤ b/2 + 1

It can be transformed into an equivalent acyclic query and evaluated in time $O(n^{b/2+c})$ using Yannakakis´ algorithm for Boolean acyclic queries (i.e. the bottom-up phase only).

Proof (continued).

q(x,y,z),s(z,r), q(u,v,w), s(t,u), s(r,w),q(w,s,t), q(t,r,y),s(r,z),q(r,y,z)

Add atoms with 2 new variables as long as Possible.

Proof (continued).

q(x,y,z),s(z,r), q(u,v,w), s(t,u), s(r,w),q(w,s,t), q(t,r,y),s(r,x),q(r,y,z)

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q(x,y,z), q(u,v,w), q(w,s,t)

root

Proof (continued).

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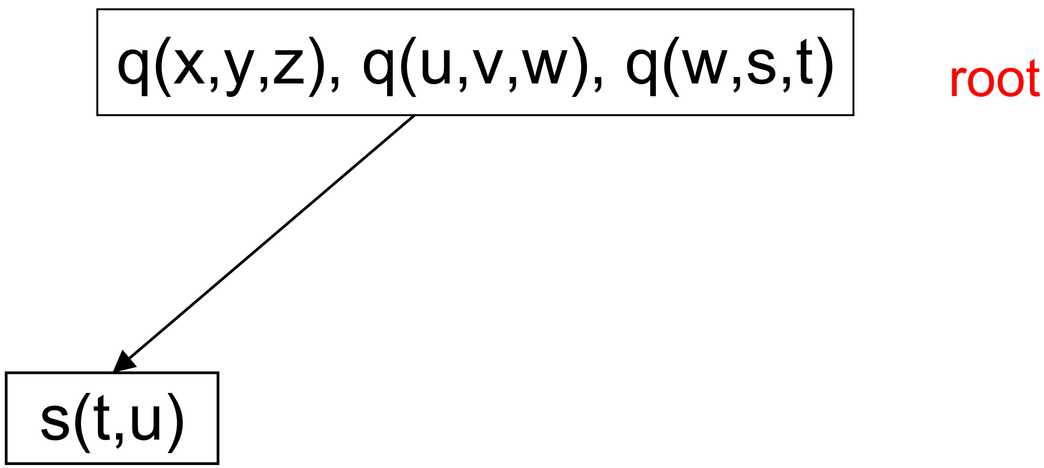

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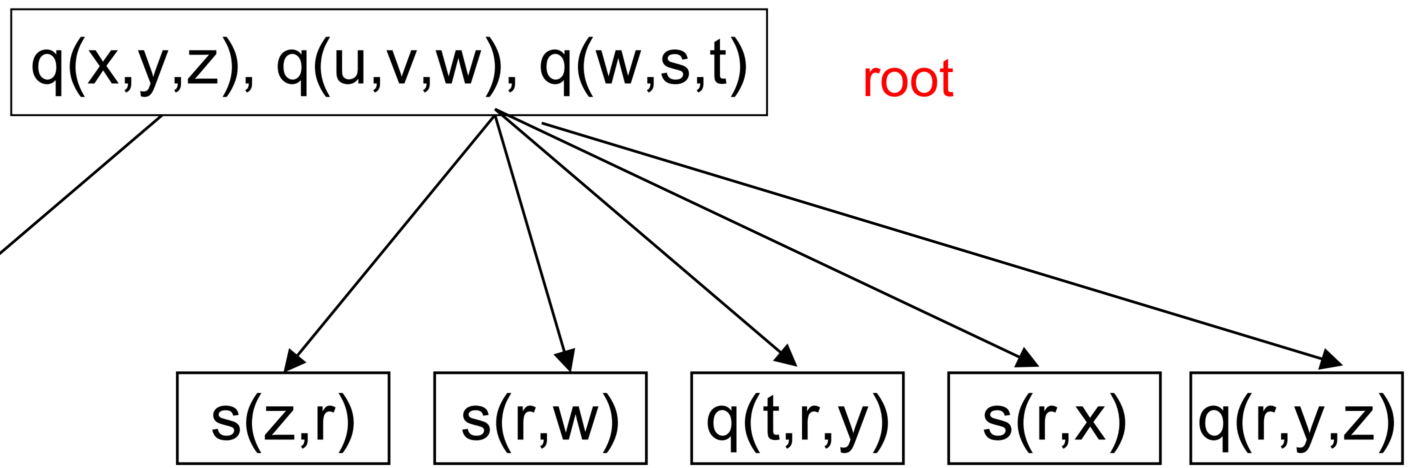

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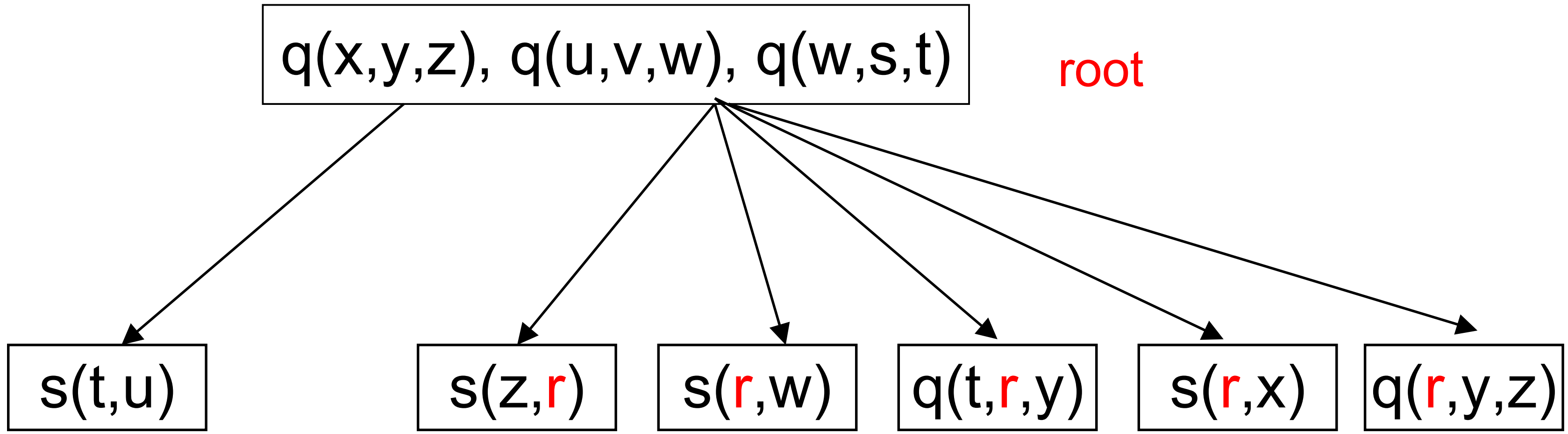
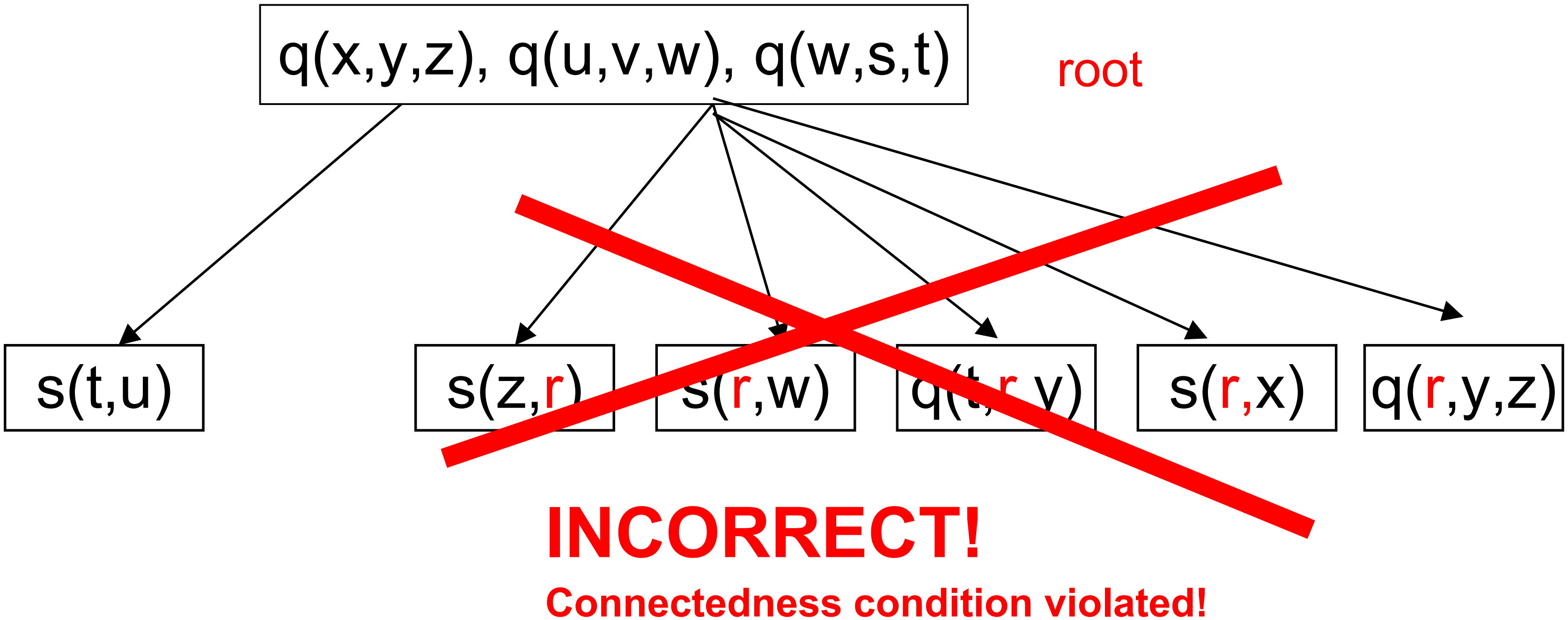


INCORRECT!

Connectedness condition violated!

Proof (continued).

q(x,y,z),s(z,r), q(u,v,w), s(t,u), s(r,w),q(w,s,t), q(t,r,y),s(r,x),q(r,y,z)



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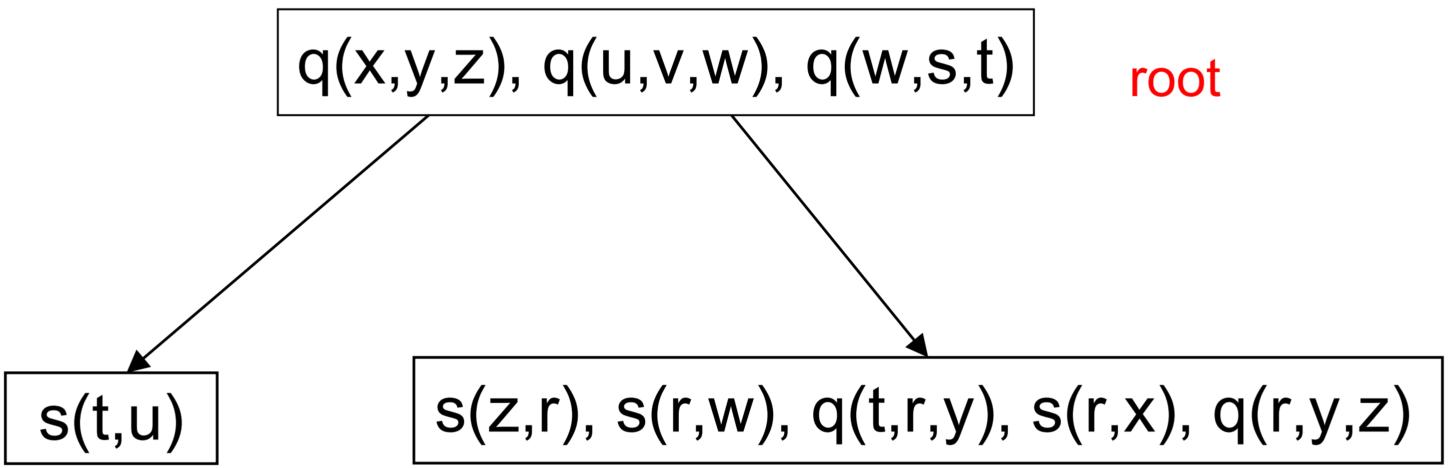

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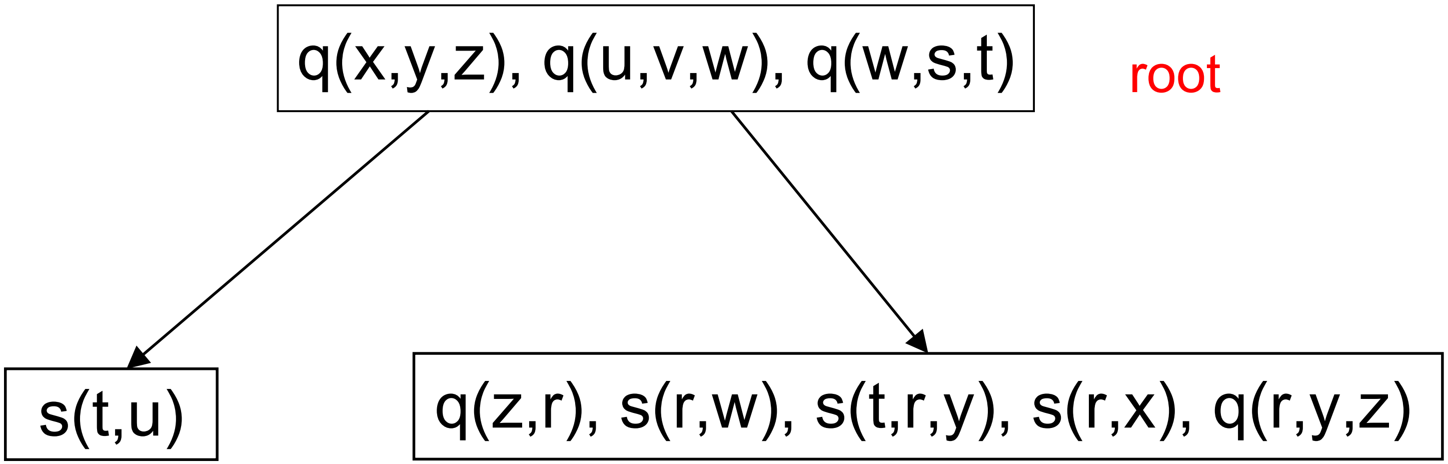

Too large!

Width would increase with each additional r-atom

Proof (continued).

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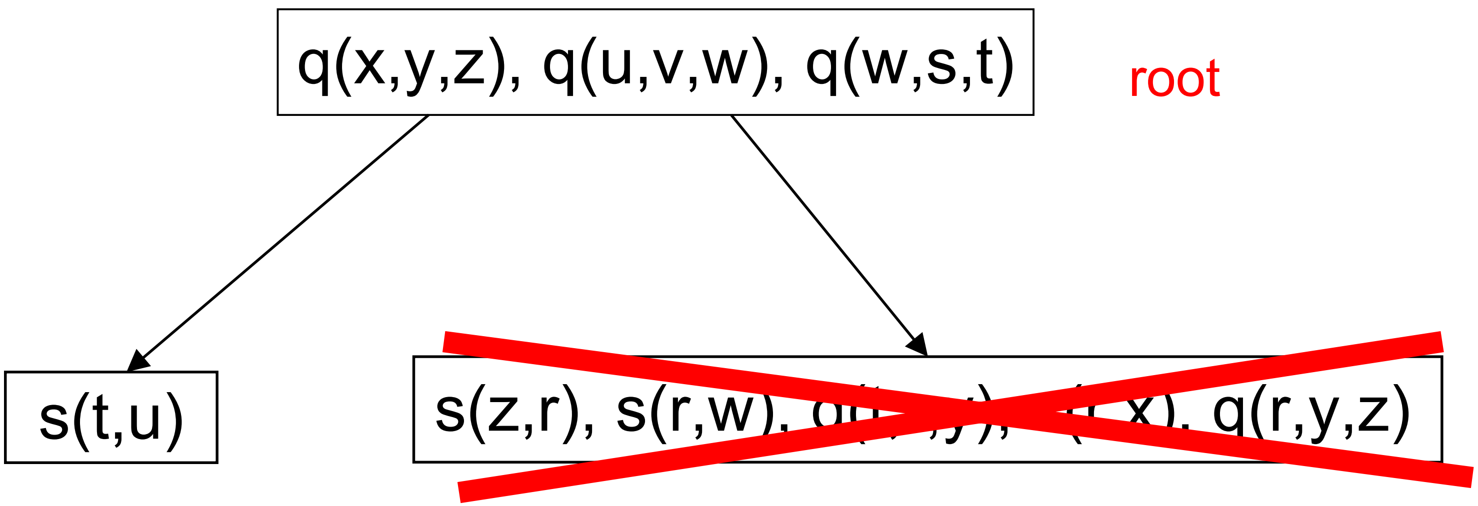

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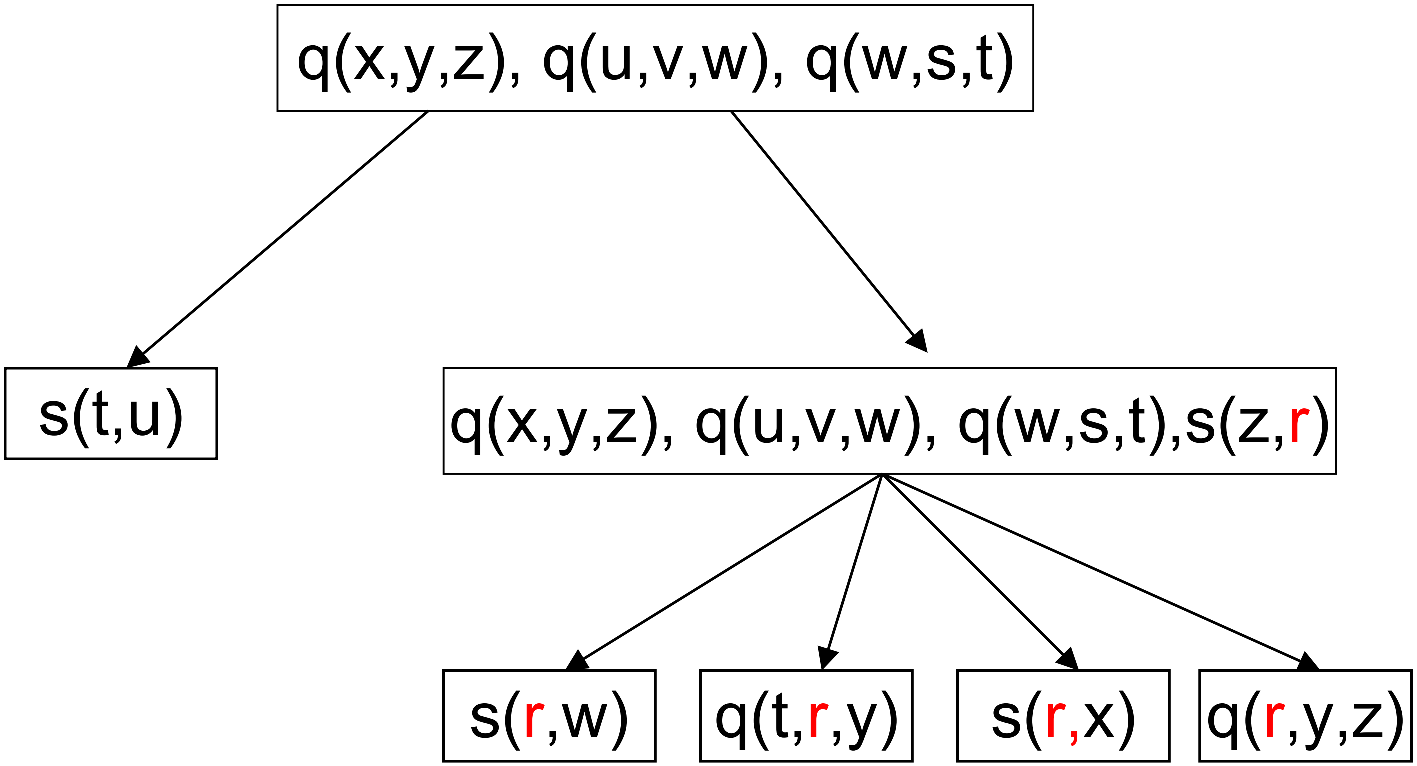
The solution:



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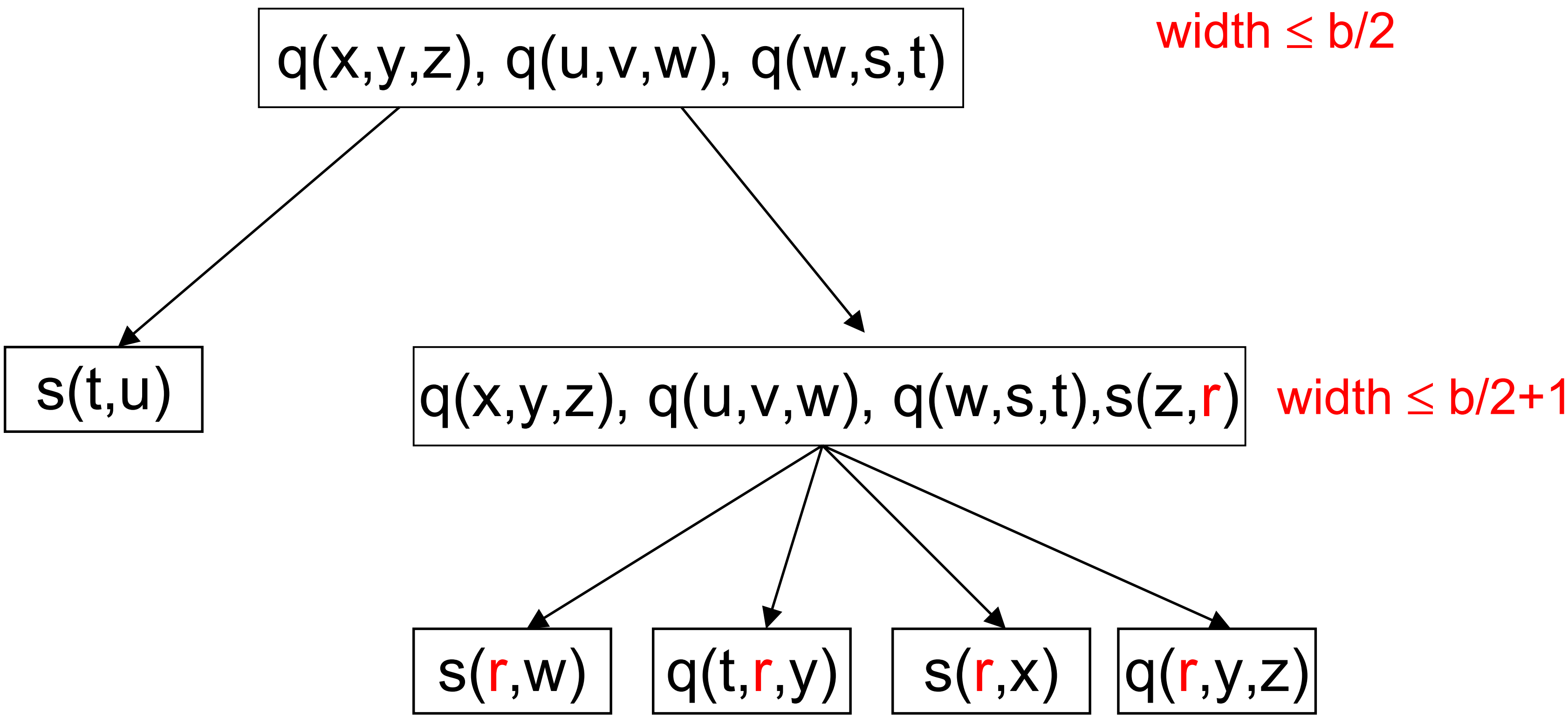

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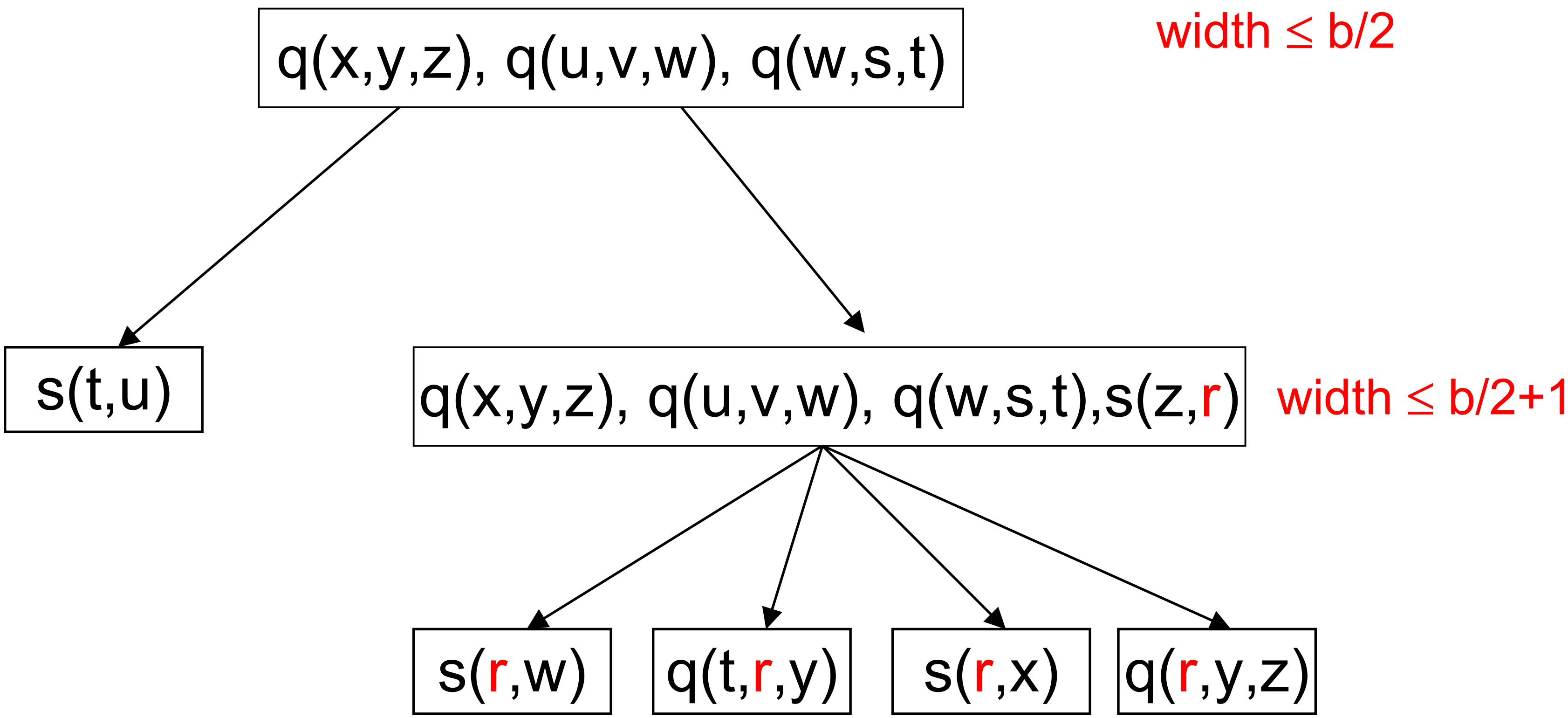

Note: Treewidth instead does not help!

q.e.d

Corollary:

Bound for CORE computation: $O(n^{b/2+2})$.

Question: Can we eliminate the parameter b in the exponent?

Answer: Most likely not !

THEOREM:

Core computation is fixed-parameter intractable w.r.t. the blocksize b.

Dependencies

Tuple generating dependencies   TGDs:

$$p(X,Y)\ \&\ q(Y,Z) \rightarrow \exists\, U,V\ \ r(X,U)\ \&\ p(Z,V)$$

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Simple TGD: $r(X,Y,Z) \rightarrow \exists\, U,V \;\; s(X,Y,U,V) \;\&\; p(X,Z,V)$

Dependencies

Tuple generating dependencies TGDs:

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Note: Inclusion dependencies are simple TGDs!

Schemas: R(A,B) , S(C,D,E)

IND: $R[A] \subseteq S[C]$

$$r(X,Y) \rightarrow \exists\, U,V\;\; s(X,U,V)$$

Dependencies

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Note: INDs are simple TGDs

Weakly acyclic TGDs   (defined for sets of TGDs)

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Weakly acyclic TGDs   (defined for sets of TGDs)

Full TGDs: no $\exists$ quantifiers e.g. $p(X,Y) \,\&\, p(Y,Z) \rightarrow p(X,Z)$

Dependencies

Tuple generating dependencies TGDs:

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Full TGDs: no ∃ quantifiers e.g. $p(X,Y) \& p(Y,Z) \rightarrow p(X,Z)$

Equality generating dependencies EGDs:

$$p(X,Y) \& p(X,Z) \rightarrow Y=Z$$

Dependencies

Tuple generating dependencies   TGDs:

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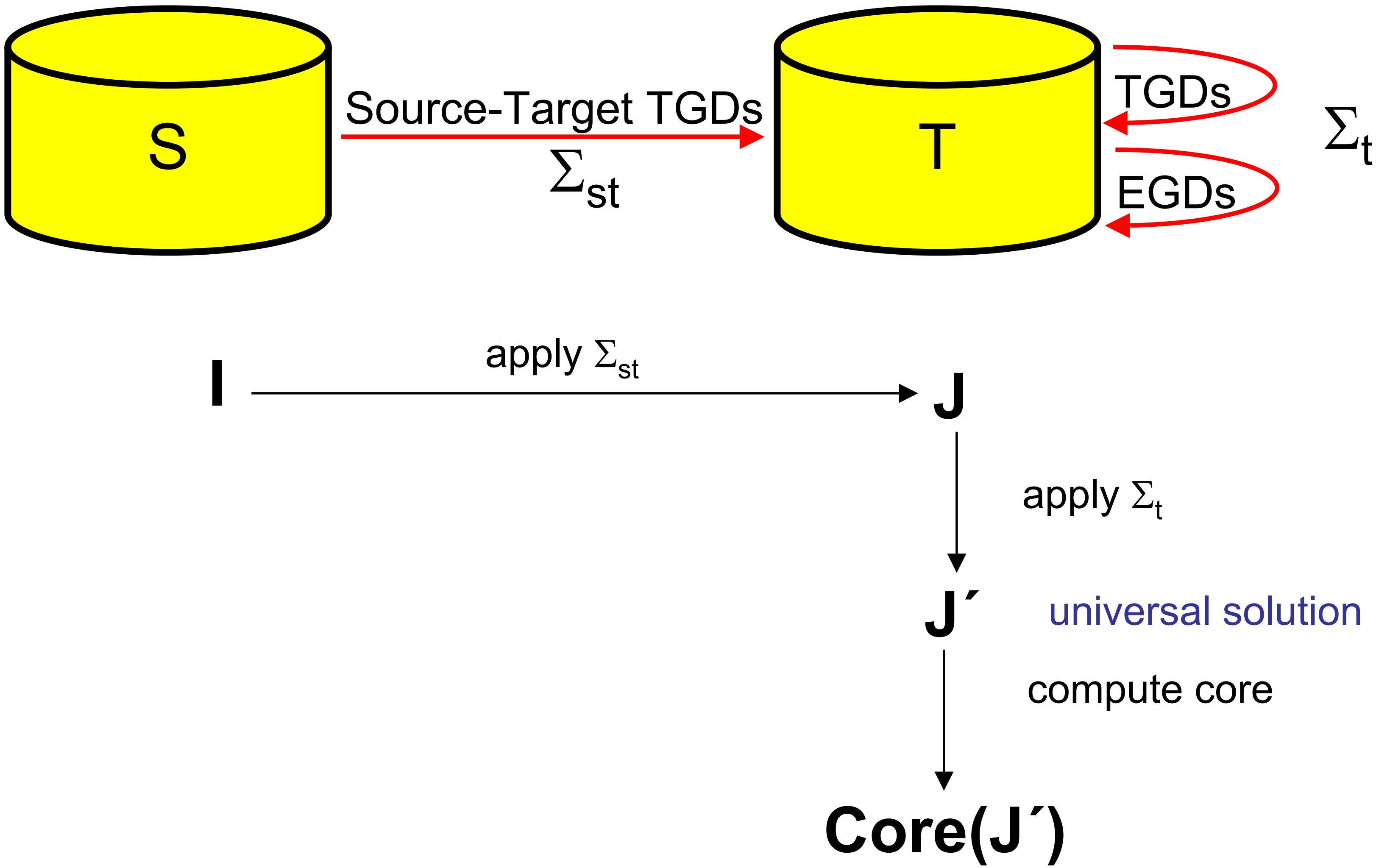
Note: FDs are EGDs

Data Exchange



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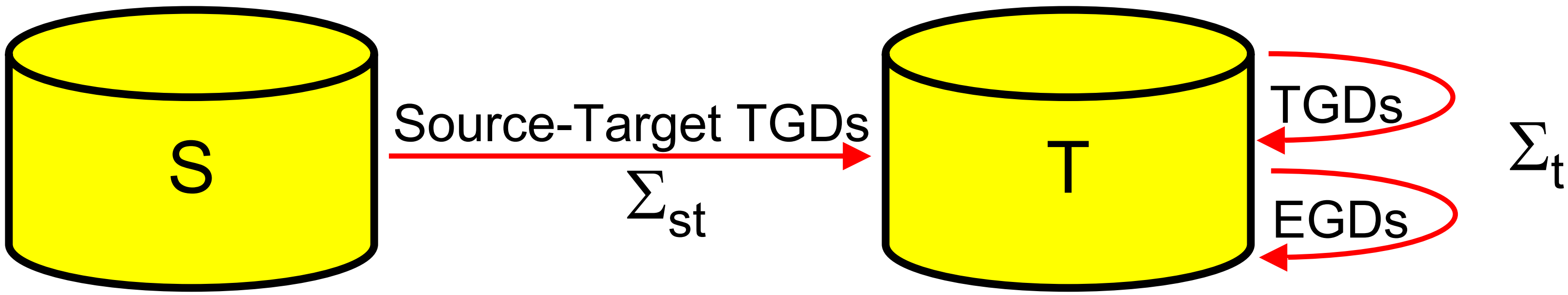
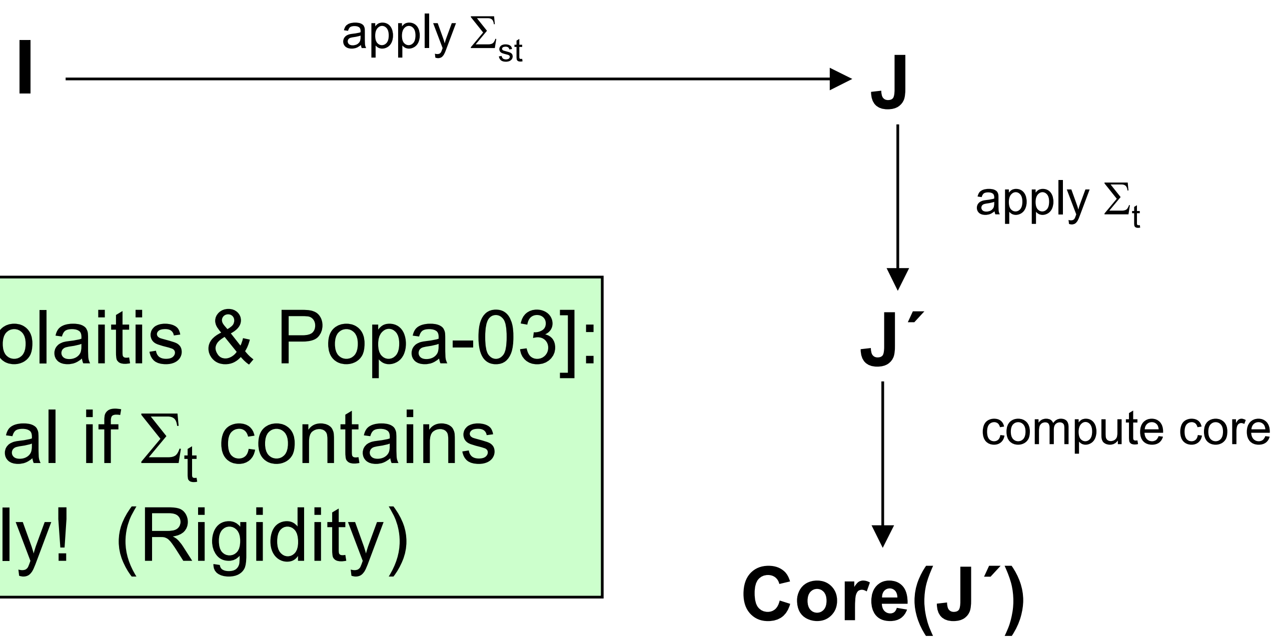
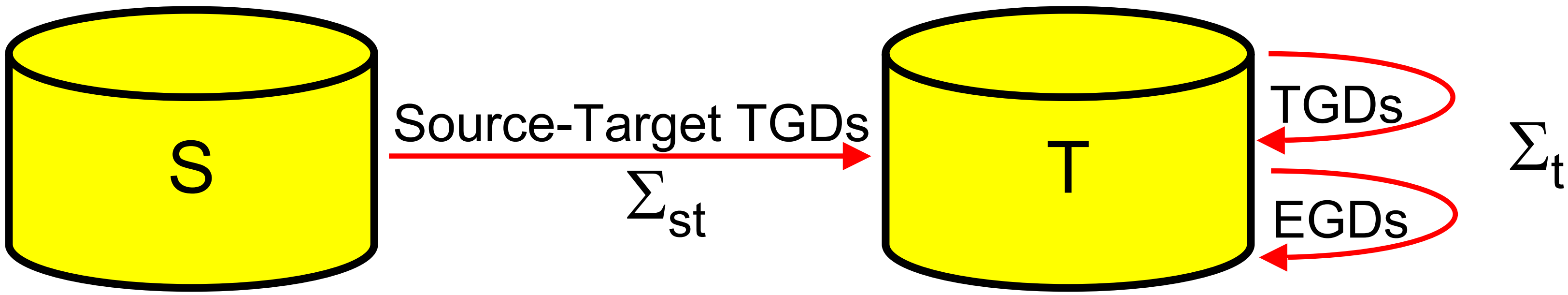
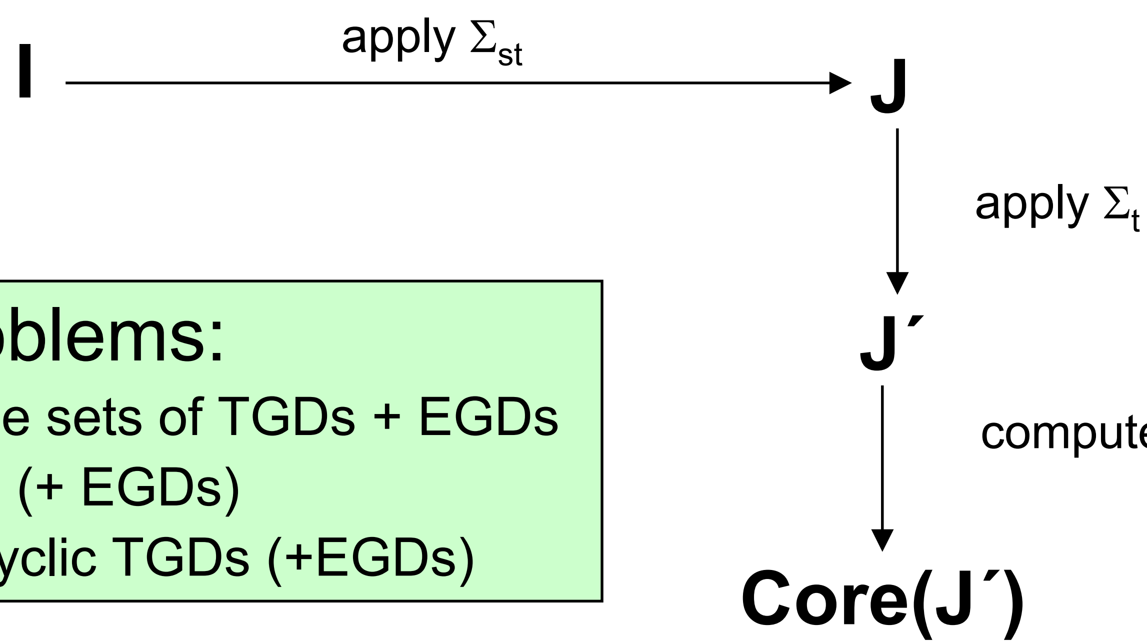




[Fagin, Kolaitis & Popa-03]:
Polynomial if Σ_t contains EGDs only! (Rigidity)

Data Exchange





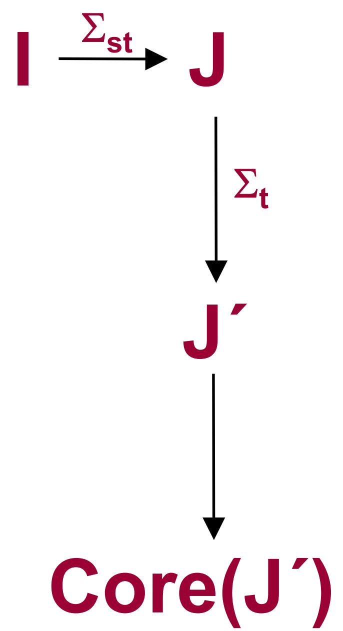
Open problems:

- →Reasonable sets of TGDs + EGDs
- →Full TGDs (+ EGDs)
- →Weakly acyclic TGDs (+EGDs)

Tractability Result 1

THEOREM: Data exchange with EGDs +
weakly acyclic simple TGDs is tractable.

PROOF IDEA:



- J has bounded hypertree width (hw)
- simple TGDs preserve hw
  - ➔ J' has bounded hw
- Use [Fagin et. al.]'s rigidity argument for dealing with EGDs.

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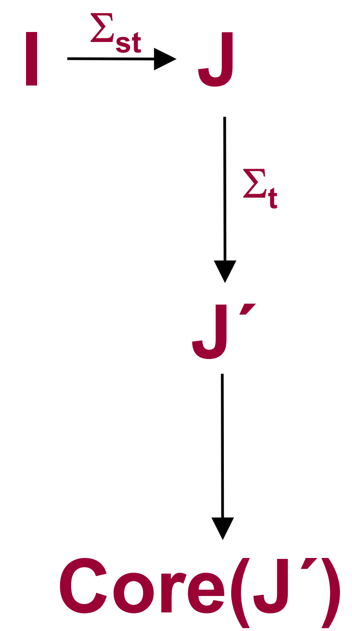
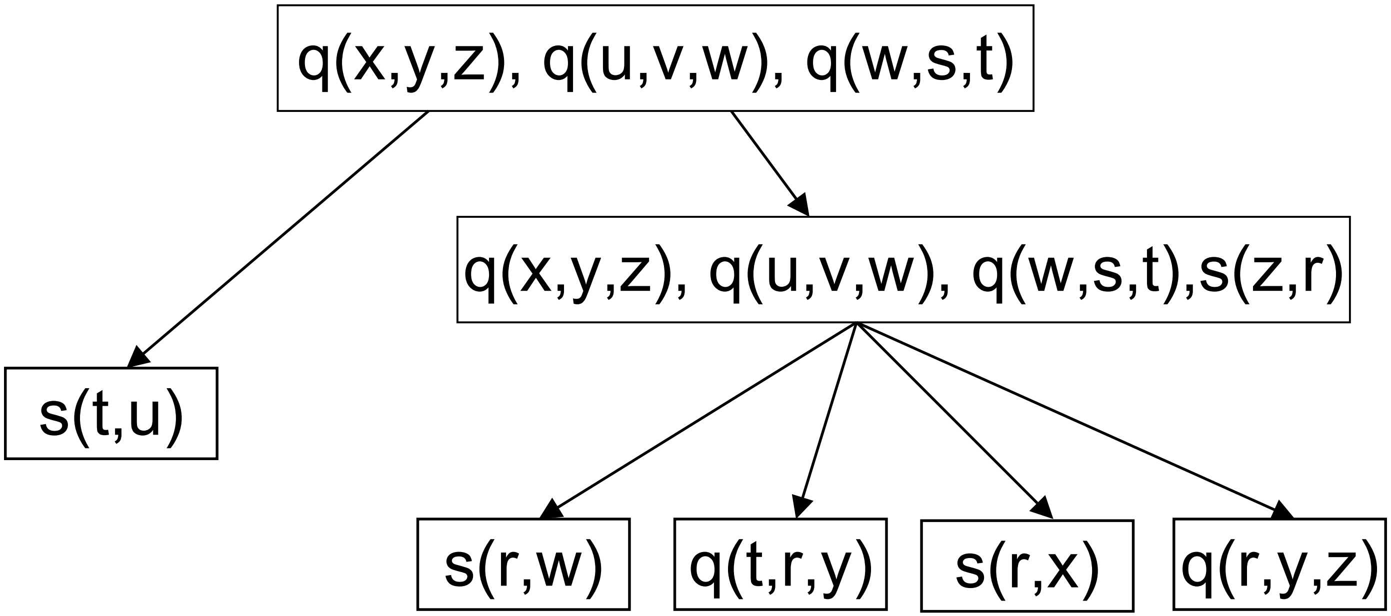


Proof Sketch (continued).

simple TGDs preserve hw $s(x_1,x_2) \rightarrow \exists x_3\ q(x_1,x_2,x_3)$



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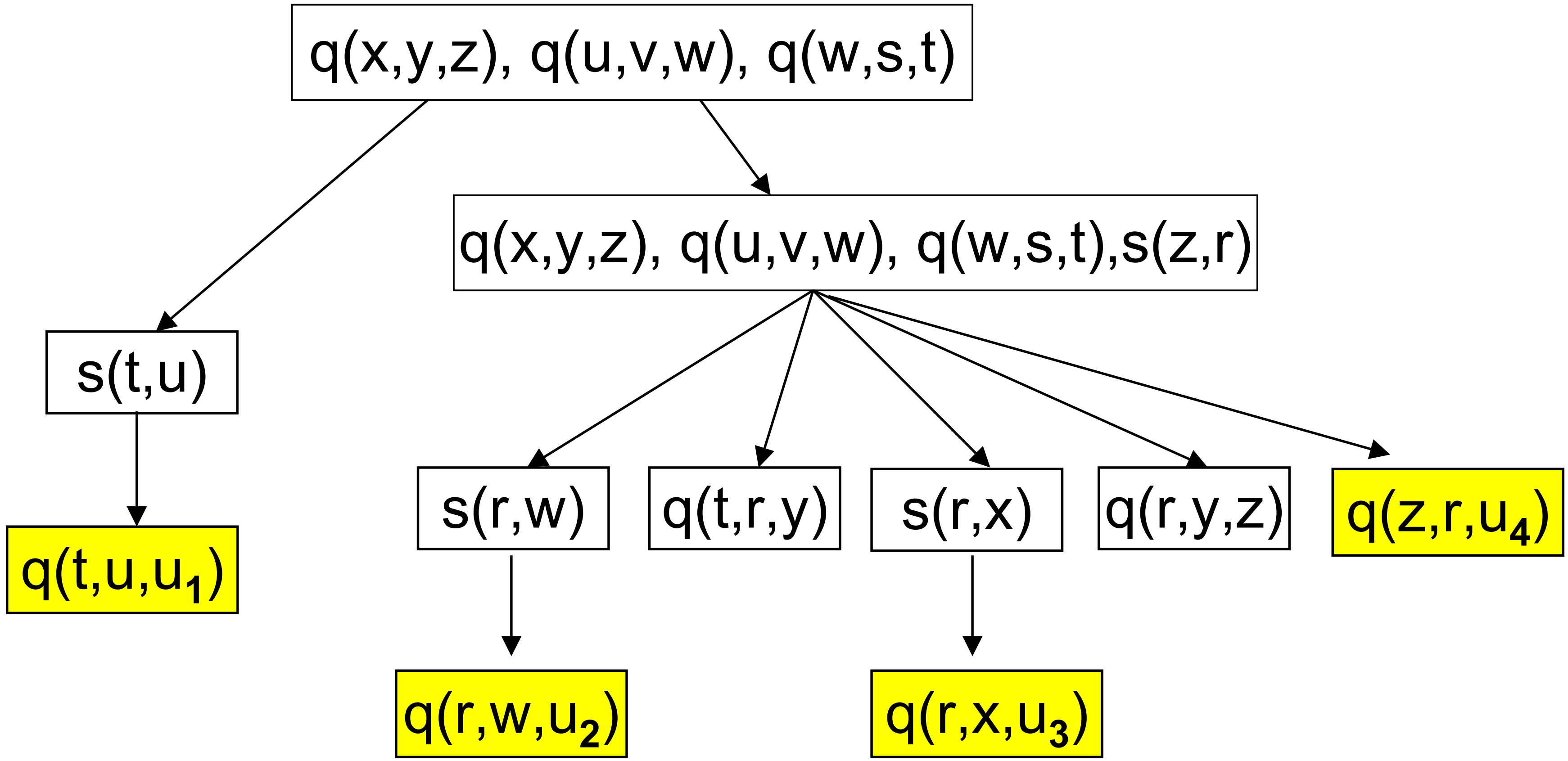

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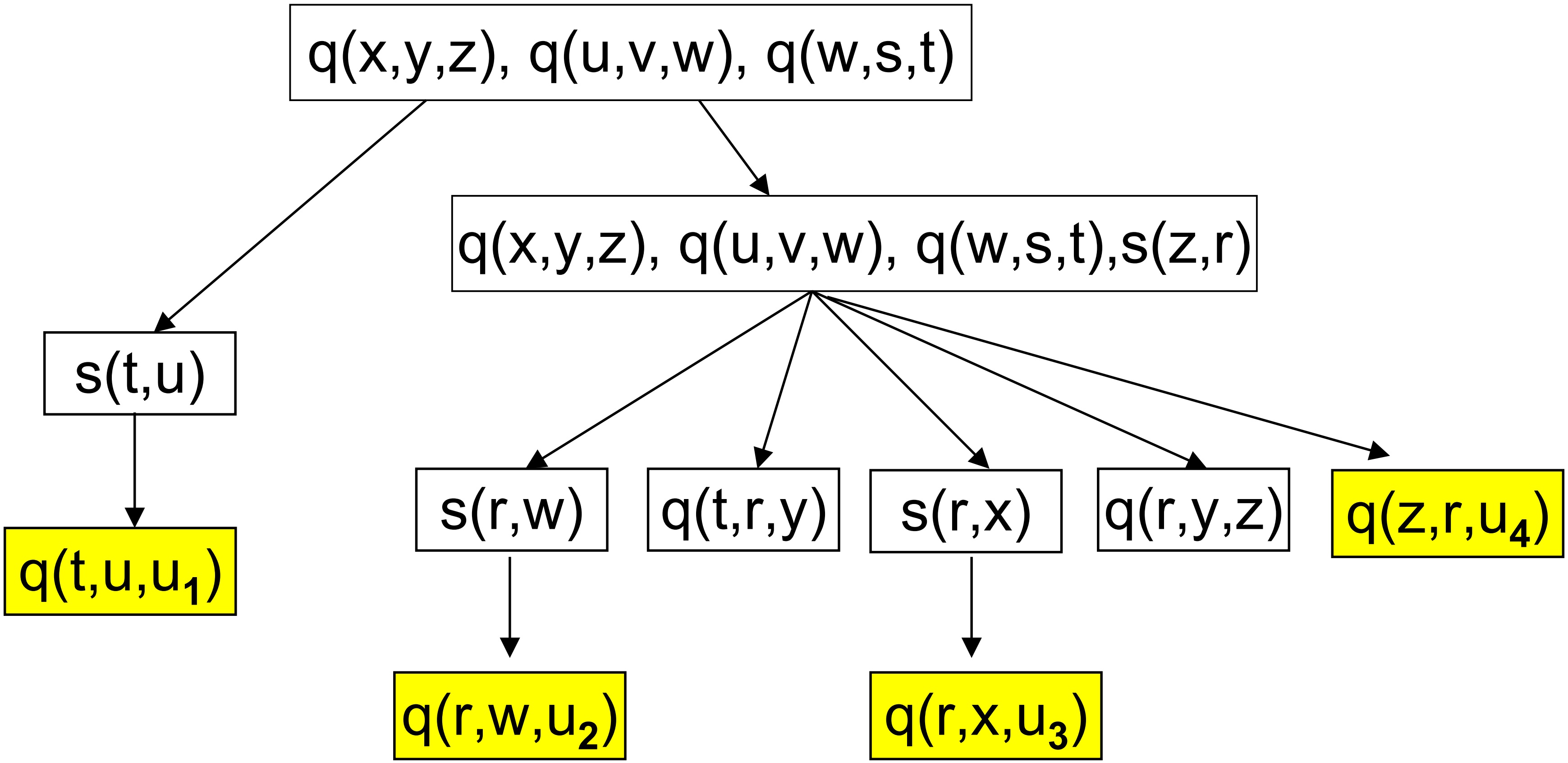

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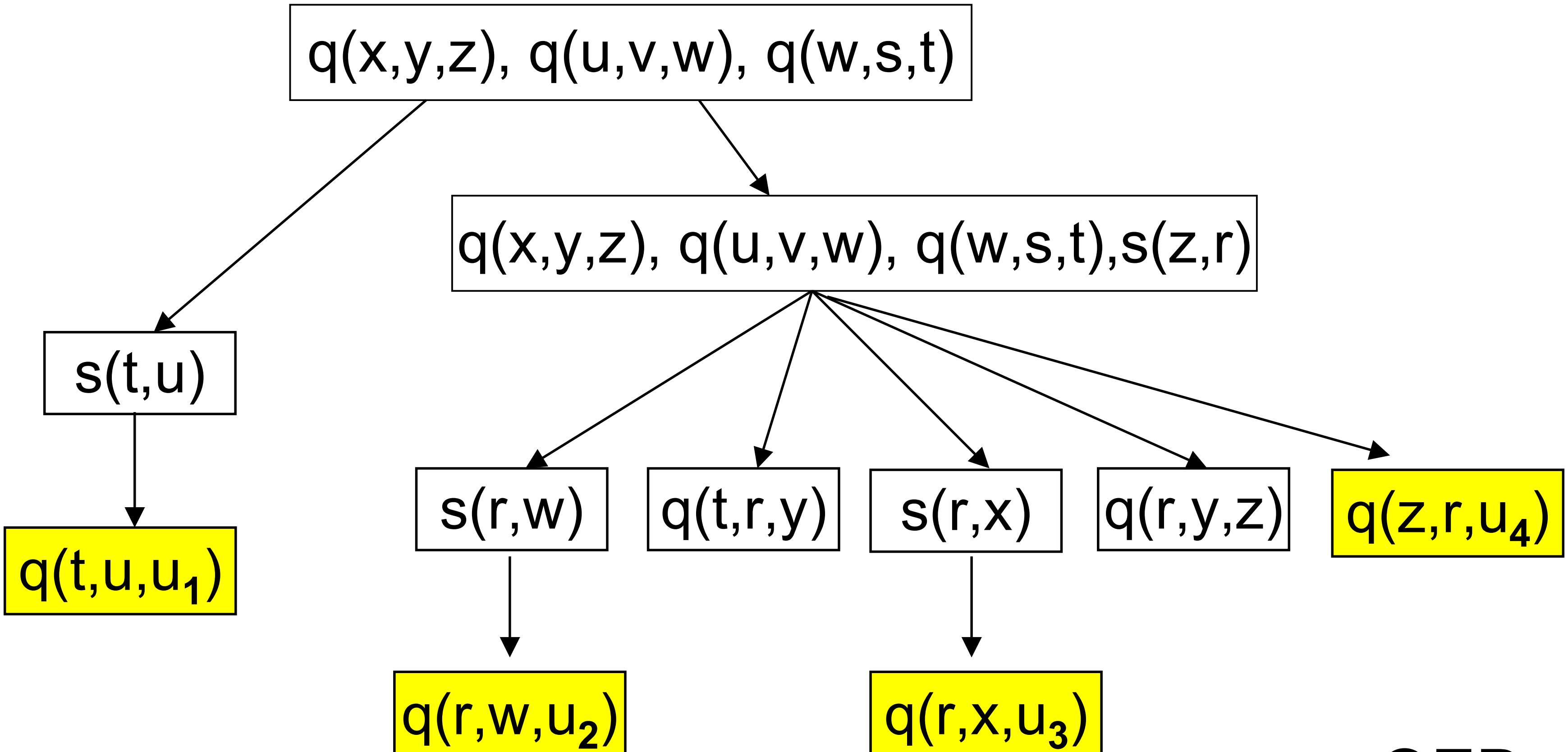
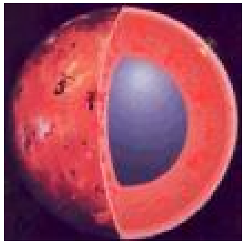
Variables from different blocks are never mixed!



Blocksize can increase, but hw remains bounded !

QED

Corollary: Core computation in data exchange is tractable if Σ_t consists of FDs and acyclic inclusion dependencies.



Tractability Result 2

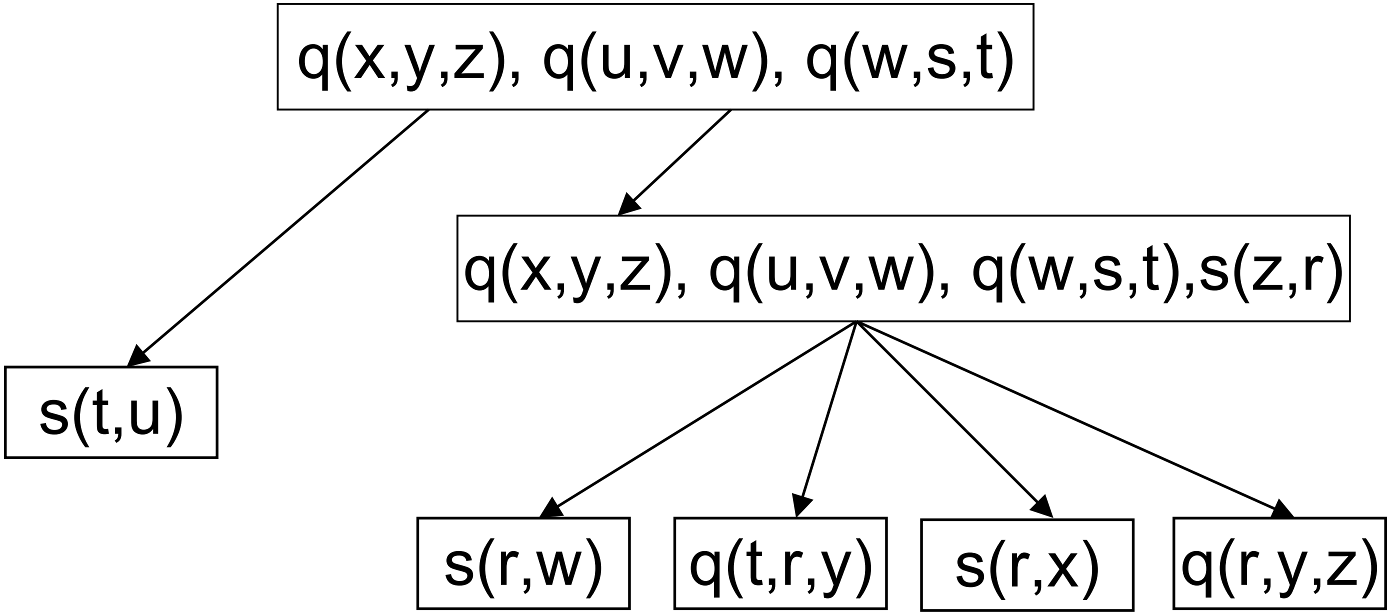
THEOREM: Data exchange with full TGDs + EGDs is tractable.

Difficulties

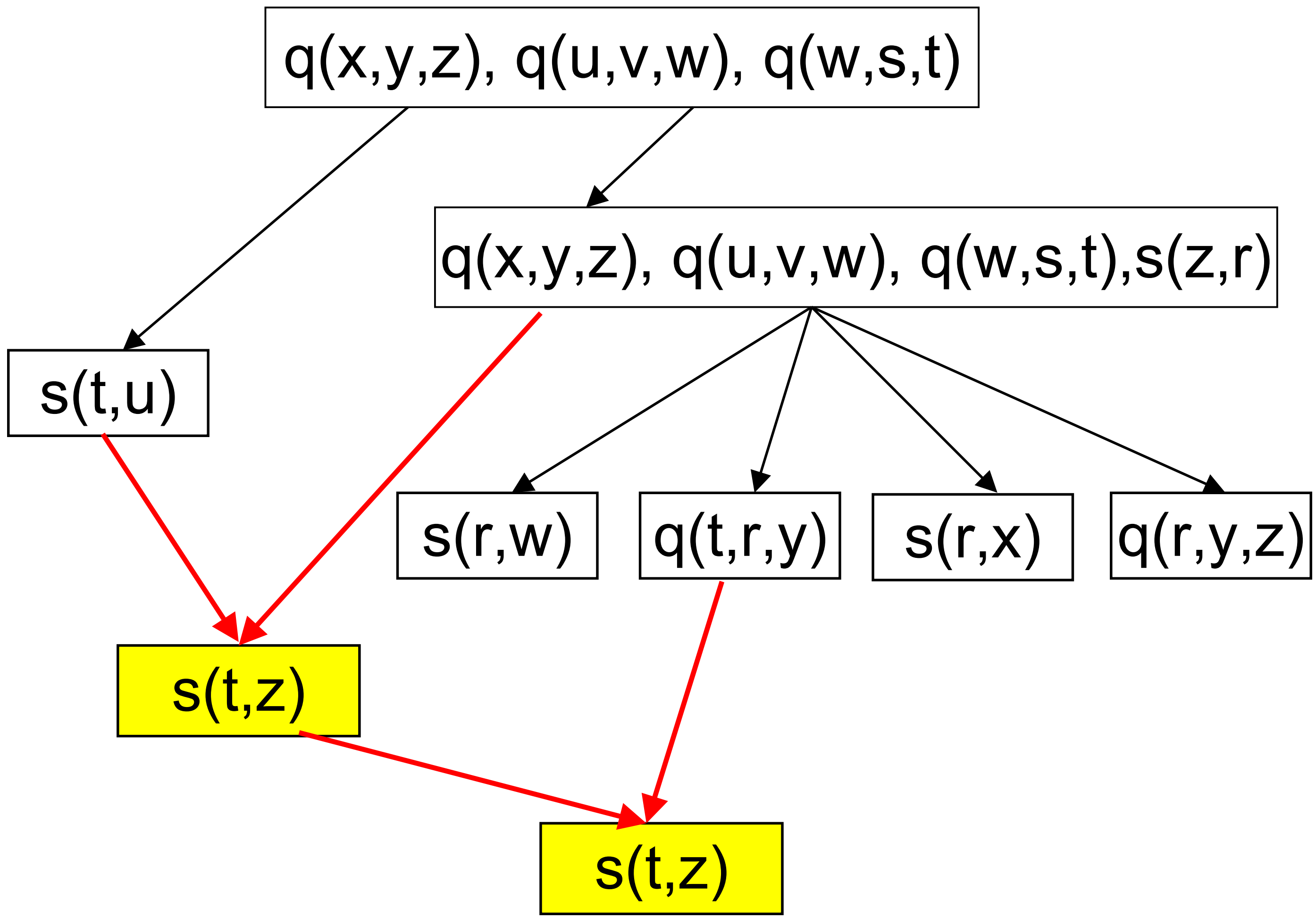
Blocks are merged when applying TGDs, thus no tree-structure possible !!!

➔Unbounded hypertee width.

Full TGD: $s(x_1,x_2)$ & $q(x_3,x_4,x_5) \rightarrow s(x_1,x_5)$



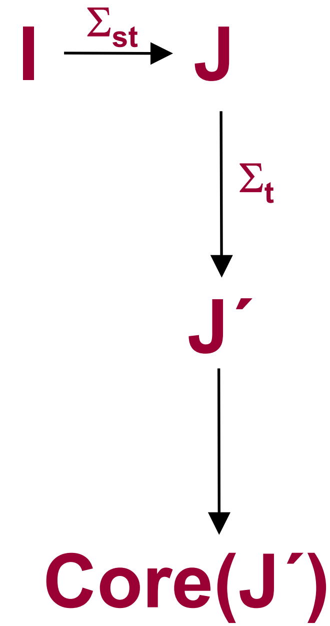
Full TGD: $s(x_1,x_2)$ & $q(x_3,x_4,x_5) \rightarrow s(x_1,x_5)$



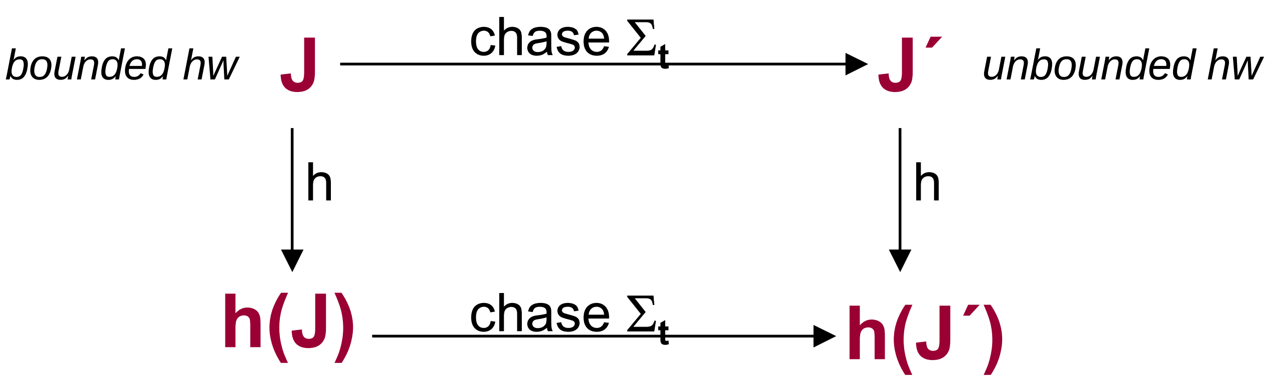
Solution:

Find useful homomorphism h: J→J'

such that h is a useful endomorphism J'→J'

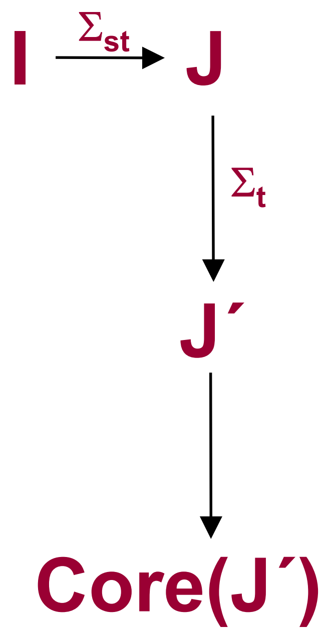


Possible, if the following diagram commutes:

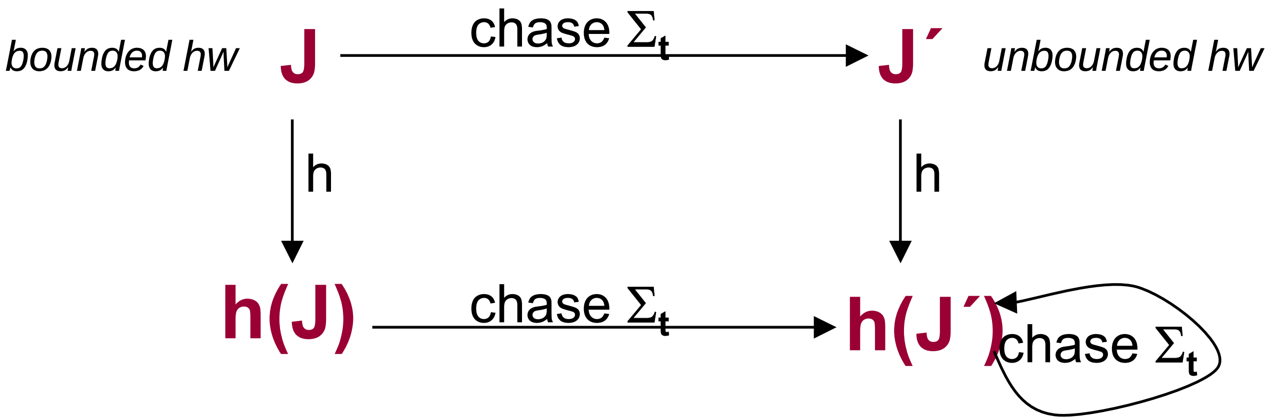


Solution:

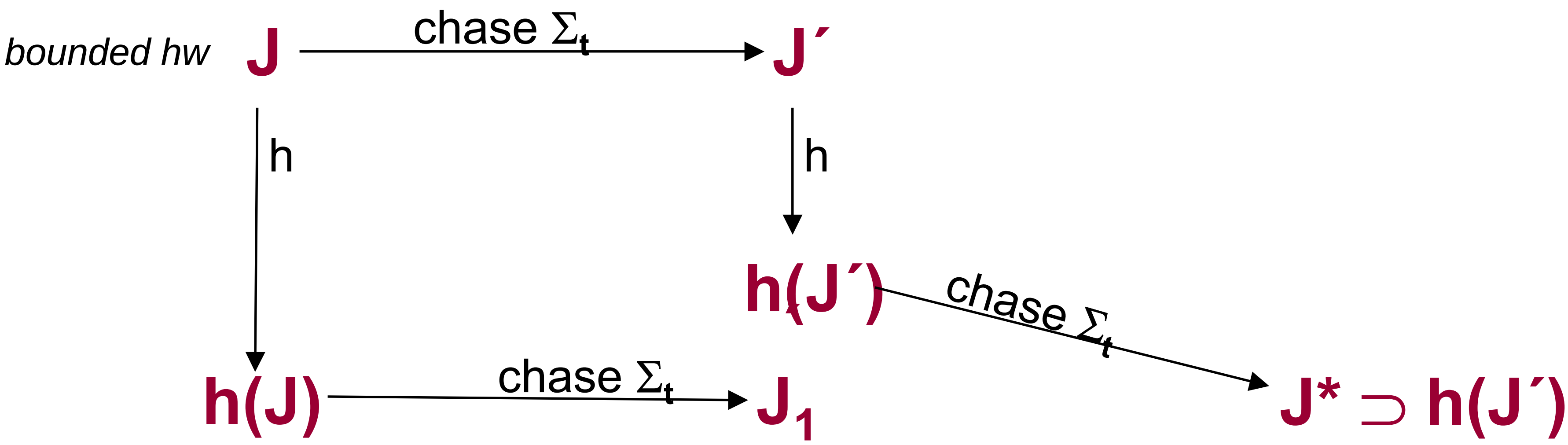
Find useful homomorphism h: J→J'
such that h is a useful endomorphism J'→J'



Possible, if the following diagram commutes:



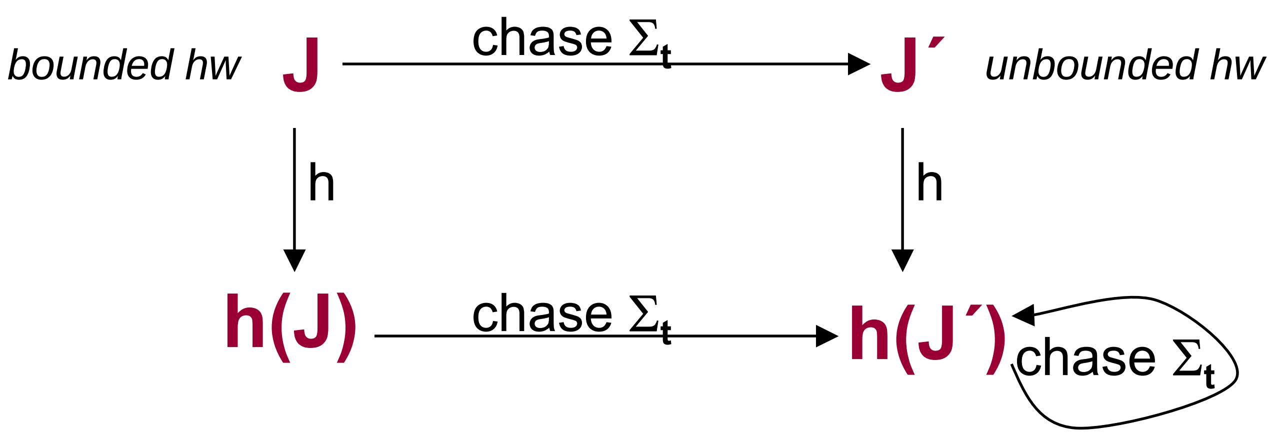
Unfortunately, in general, the diagram does not commute:



Definition: h idempotent if $h^2=h$, i.e., if $\forall x$: $h(h(x))=h(x)$

Lemma: If h is a useful endomorphism J→J', then h can be transformed in polynomial time into useful idempotent endomorphism J→J'.

Lemma: The diagram commutes whenever $h: J \rightarrow J'$ is an idempotent endomorphism.



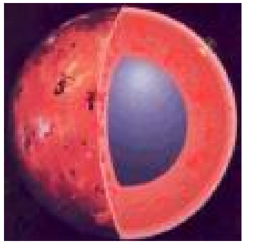
This allows us to design an algorithm that successively refines h via “local changes” as long as possible.
At the end, core(J) is computed.

To deal properly with EGDs, we simulate EGDs by full TGDs:

Assume the schema {p(.,.,.) s(.,.)}

p(x,y,z) → x=y

is simulated by the TGDs



p(x,y,z) & s(x,y) → s(x,x)
p(x,y,z) & s(x,y) → s(y,y)
p(x,y,z) & p(x,u,y) → p(x,u,x)
p(x,y,z) & p(x,u,y) → p(y,u,y)
etc. …..

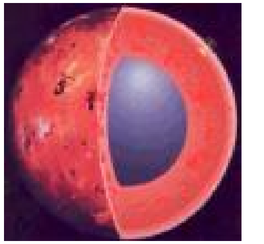
We can show that after computing the core these TGDs have the same effect as the EGD.

Limits of Tractability

Theorem: Data exchange with full TGDs and a Null predicate is NP-complete.

P(X,a) & q(X,Y) & Null(X) → r(X,Y)

Conclusions



- New general algorithm for computing cores based on hypertree decompositions

- A relevant class of target dependencies, for which data exchange is tractable:

  EGDs + weakly acyclic simple TGDs
  (encompassing FDs + w.a. INDs)

- A more involved tractable class: EGDs + full TGDs

Open: EGDs + weakly acyclic TGDs.