

5. Basics of Description Logics

Exercise 5.1 Translate the following DL expressions and axioms into first-order logic:

1. $\text{Father} \sqcap \forall \text{child}. (\text{Doctor} \sqcup \exists \text{managedBy}^-. (\text{Company} \sqcap \neg \text{MoreThan3Employees}))$
2. $\text{Person} \sqcap \forall \text{child}. \text{HappyPerson} \sqsubseteq \exists \text{child}. \forall \text{child}. \text{HappyPerson}$
3. $\text{Person} \sqcap \exists \text{child}. \text{HappyPerson} \sqsubseteq \text{Happy} \sqcap (\text{Father} \sqcup \text{Mother})$

Exercise 5.2 Translate the following sentences and first-order logic formulas into DL syntax, if possible:

1. Only humans have children that are humans.
2. A node cannot have two distinct P -successors, such that one is a B and the second one is not a B .
3. $\forall x_1, x_2, y_1, y_2. P(x_1, y_1) \wedge P(x_1, y_2) \wedge P(x_2, y_2) \rightarrow x_1 = x_2 \vee y_1 = y_2$
4. $\forall x, y, z. P(x, y) \wedge P(y, z) \wedge P(z, x) \rightarrow A(x)$
5. $\forall x, y, z. P(x, y) \wedge Q(y, z) \rightarrow R(x, z)$
6. $\neg(\forall x. A(x) \rightarrow B(x)) \vee (\forall x. A(x) \rightarrow C(x))$

Exercise 5.3 Compute the certain answers to the query q over the KB $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$.

1. $q(x) = B(x), \quad \mathcal{A} = \{A(a), B(b), C(c)\}, \quad \mathcal{T} = \{A \sqsubseteq B, C \sqsubseteq \exists R, \exists R^- \sqsubseteq B\}.$
2. $q() = \exists x. B(x), \quad \mathcal{A} = \{A(a)\},$
 - (a) $\mathcal{T} = \{A \sqsubseteq \exists R, \exists R^- \sqsubseteq B\}.$
 - (b) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \exists R^- \sqsubseteq B\}.$
 - (c) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcap (\exists S \sqcup \exists Q), \exists R^- \sqsubseteq B, \exists Q^- \sqsubseteq B\}.$
 - (d) $\mathcal{T} = \{A \sqsubseteq \exists R \sqcup \exists S, \exists R^- \sqsubseteq B, \exists S^- \sqsubseteq \exists R \sqcup \exists Q, \exists Q^- \sqsubseteq \exists R\}.$
3. $q(x) = \exists y. R(x, y), \quad \mathcal{A} = \{A(a), R(b, c)\}, \quad \mathcal{T} \text{ as in Item 2.}$
4. $q(x) = \exists y. R(x, y), \quad \mathcal{A} = \{A(a), R(a, c)\}, \quad \mathcal{T} \text{ as in Item 2.}$